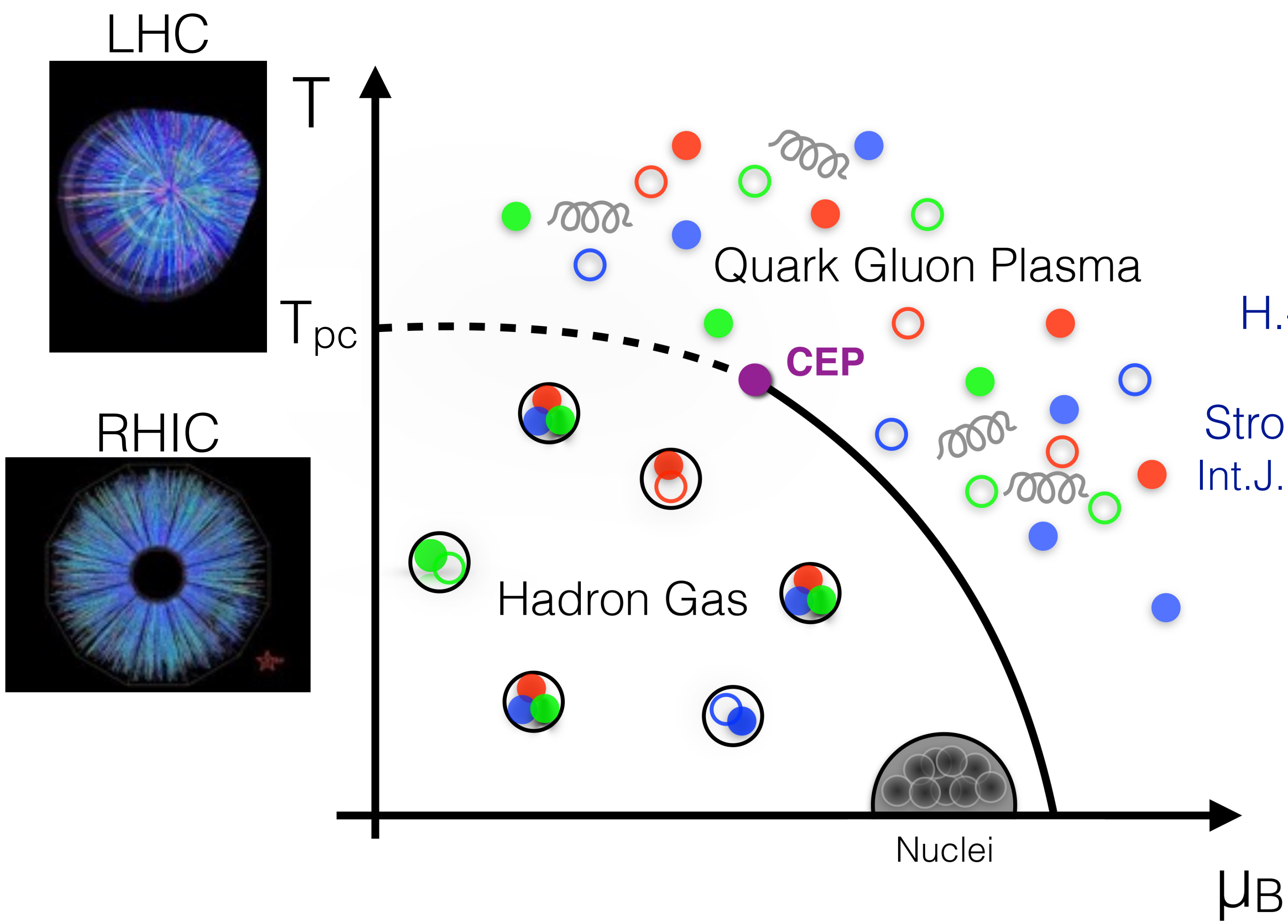
QCD phase diagram from LQCD

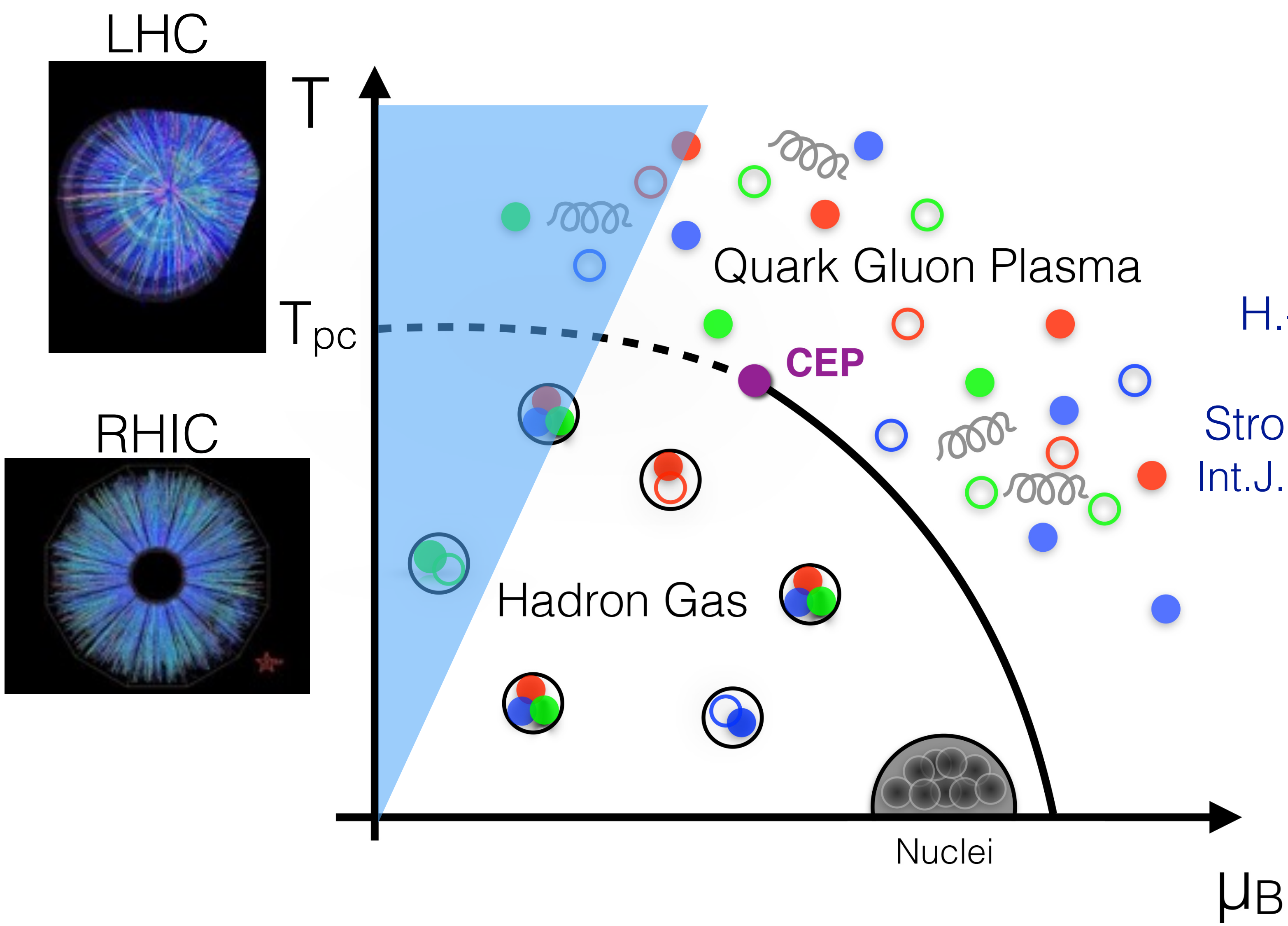


H.-T. Ding, F. Karsch, S. Mukherjee
"Thermodynamics of
Strong-Interaction Matter from LQCD",
Int.J.Mod.Phys. E24 (2015) no.10, 1530007

Heng-Tong Ding(丁亨通) from CCNU

Jan. 5, 2017@Peking University

QCD phase diagram from LQCD



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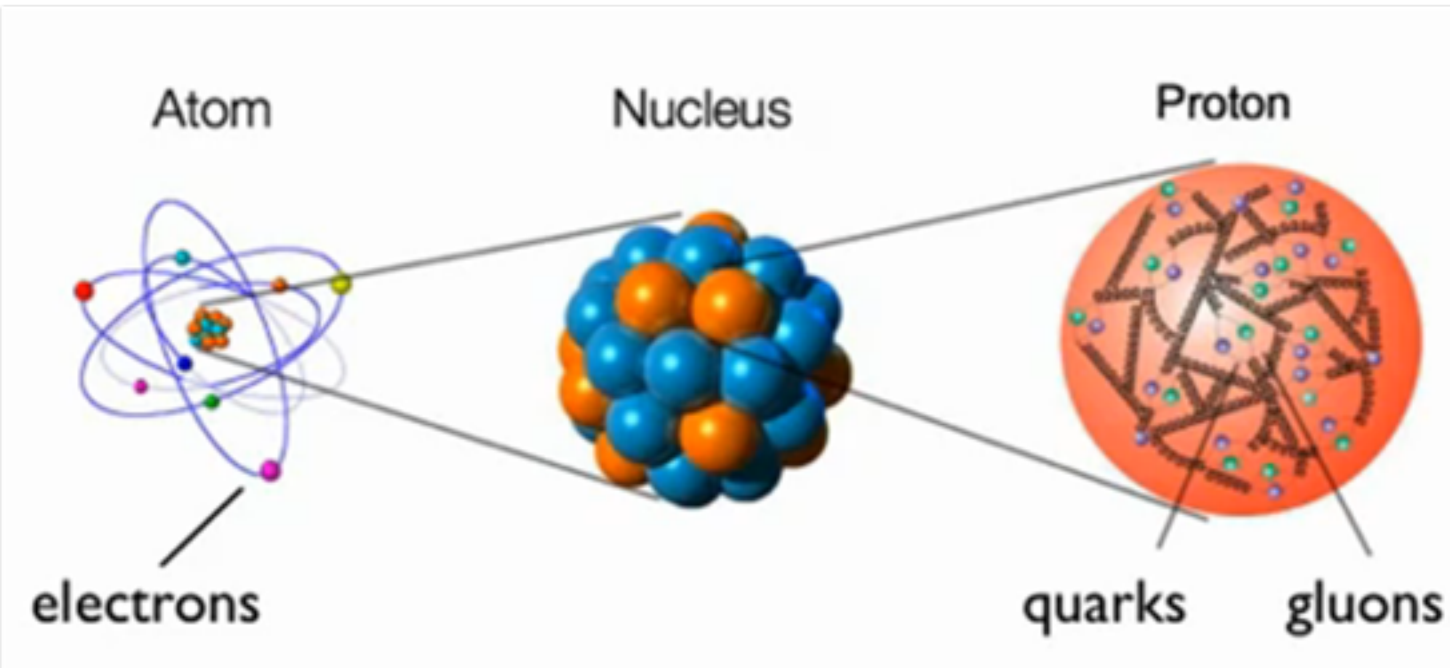
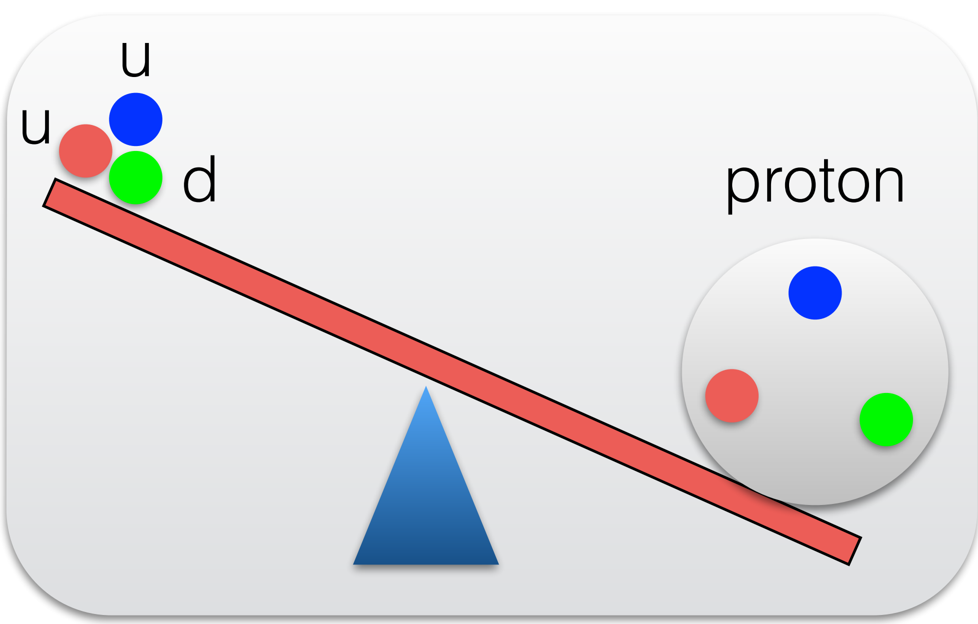
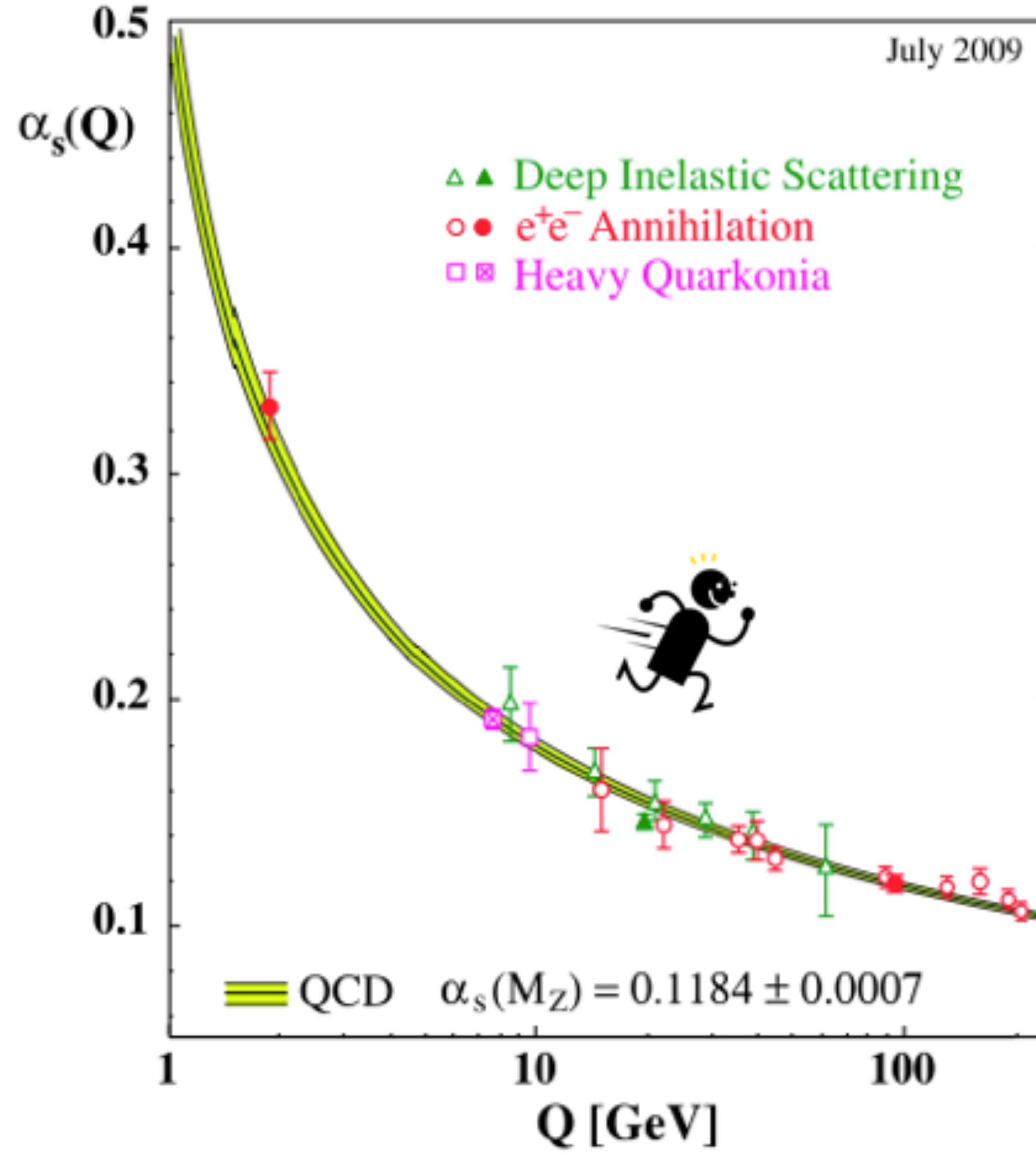
Quantum ChromoDynamics





Running coupling



Confinement

non-perturbative

Asymptotical free

perturbative

Symmetries of QCD in the vacuum

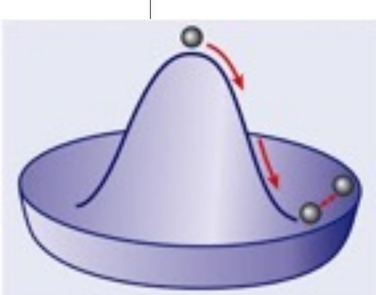
$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\Big[i\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) - m_q\Big]q$$

Classical QCD symmetry (m_q=0)

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

Quantum QCD vacuum (m_q=0)

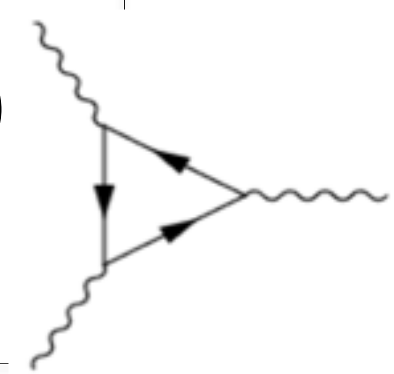
Chiral condensate:
spontaneous mass generation



$$\langle \bar{q}_R q_L \rangle \neq 0$$

Axial anomaly:
quantum violation of $U(1)_A$

$$\partial_{\mu} j_5^{\mu} = \frac{g^2 N_f}{16\pi^2} tr(\tilde{F}_{\mu\nu}F^{\mu\nu})$$

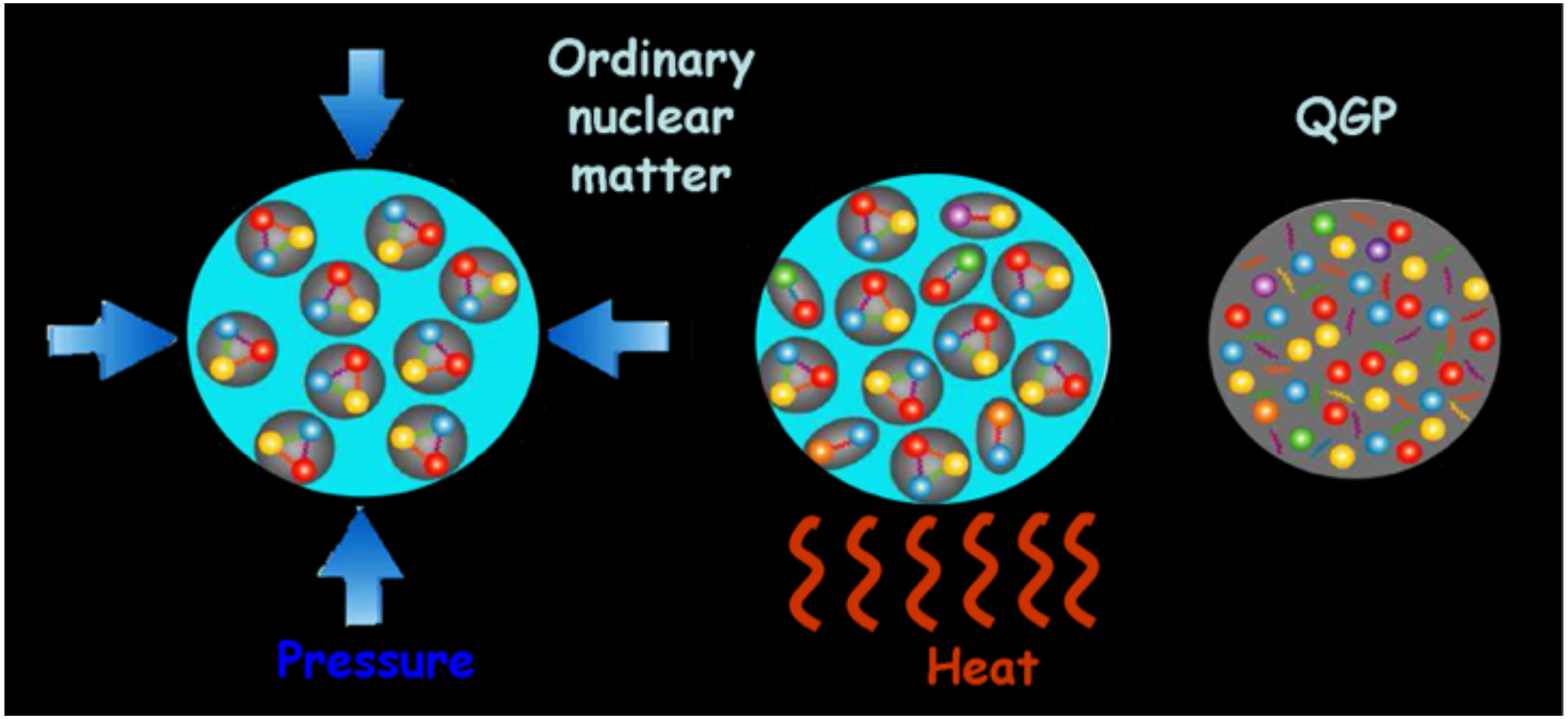


$$SU(N_f)_V \times U(1)_V$$

Symmetry restoration in extreme conditions?

"The whole is more than sum of its parts."
Aristotle, Metaphysica 10f-1045a

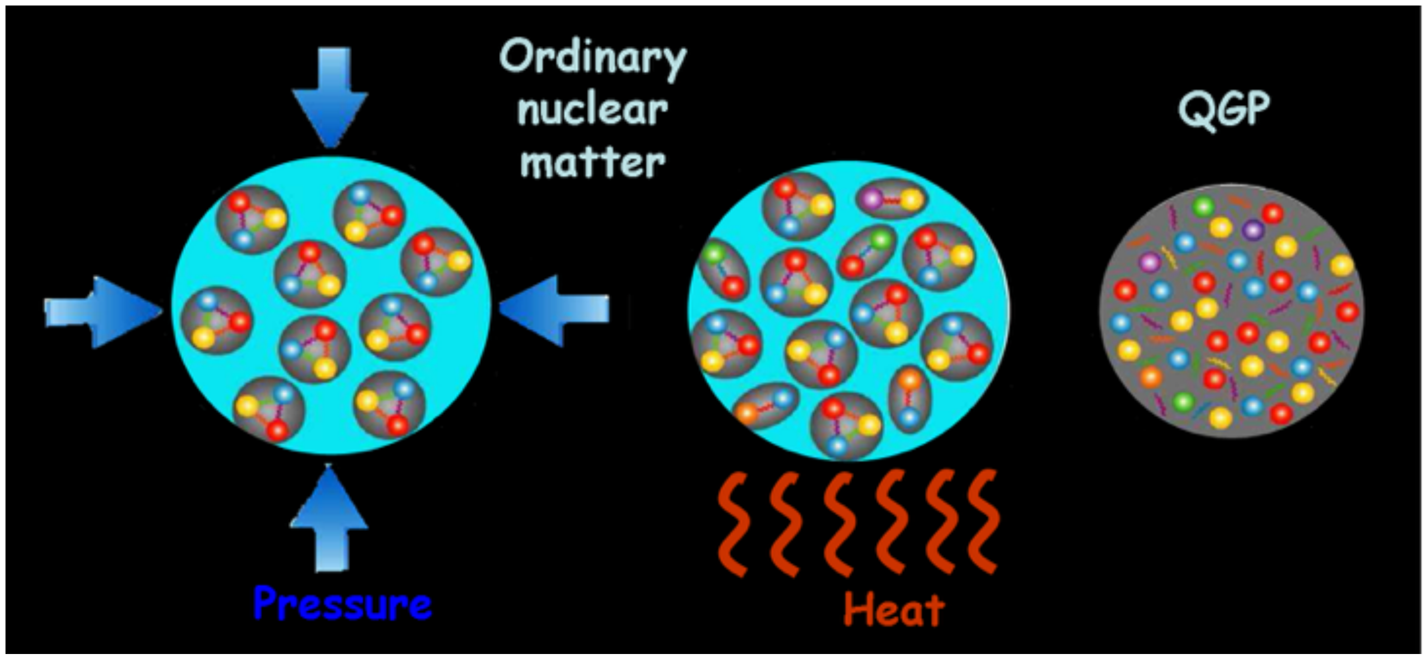
$\mu_B \gg \Lambda_{QCD}$ or $T \gg \Lambda_{QCD}$

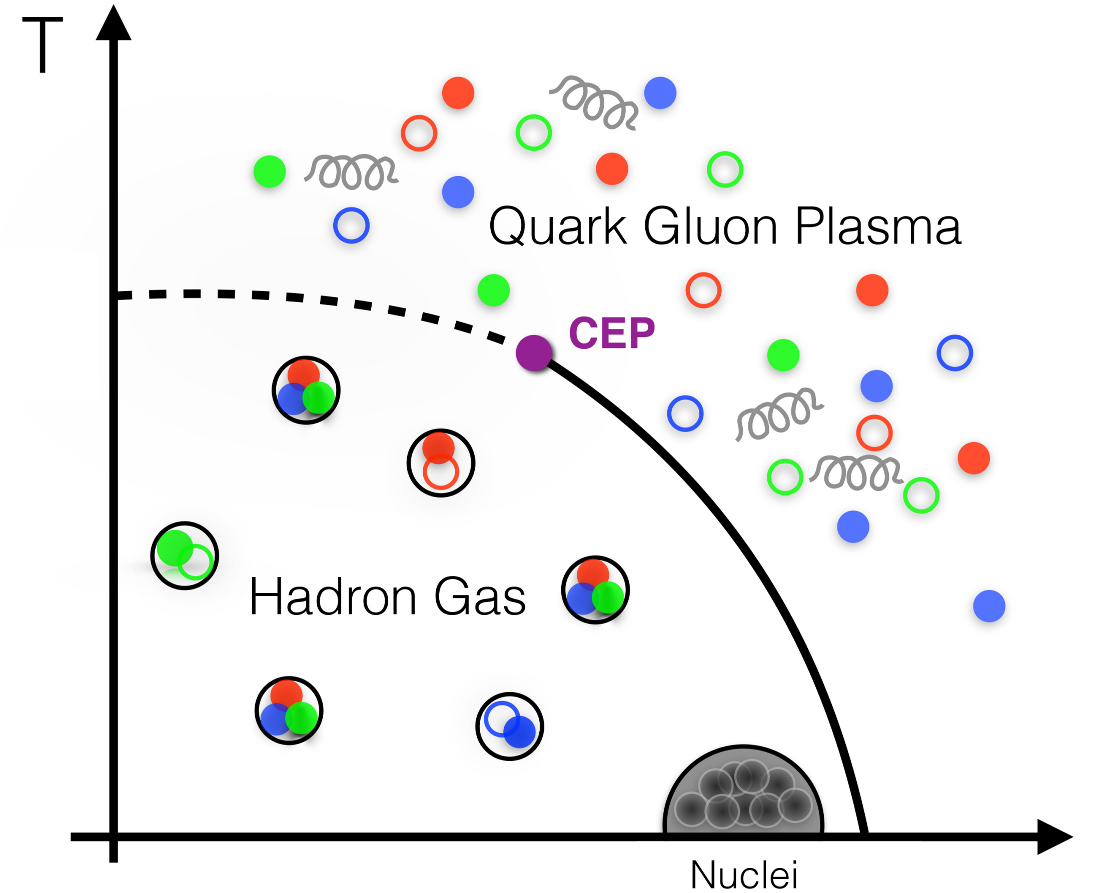


Quarks & Gluons get liberated from nucleons

From hadronic phase to
A new state of matter: Quark Gluon Plasma (QGP)

QCD phase transitions





HTD, F. Karsch, S. Mukherjee, arXiv:1504.0527

What are the orders of QCD phase transitions?

What are the T_c , critical temperatures of these transitions?

What will be the observable phenomena associated with the transitions?

Ginzburg-Landau-Wilson approach

Partition function: $$Z = \int [\mathrm{d}\sigma] \exp\Big(- \int \mathrm{dx}\, \mathcal{L}_{eff}\, (\sigma(\mathrm{x}); K) \Big)$$

Landau function: $$\mathcal{L}_{eff} = \frac{1}{2}(\nabla\sigma)^2 + \sum_n a_n(K)\sigma^n$$

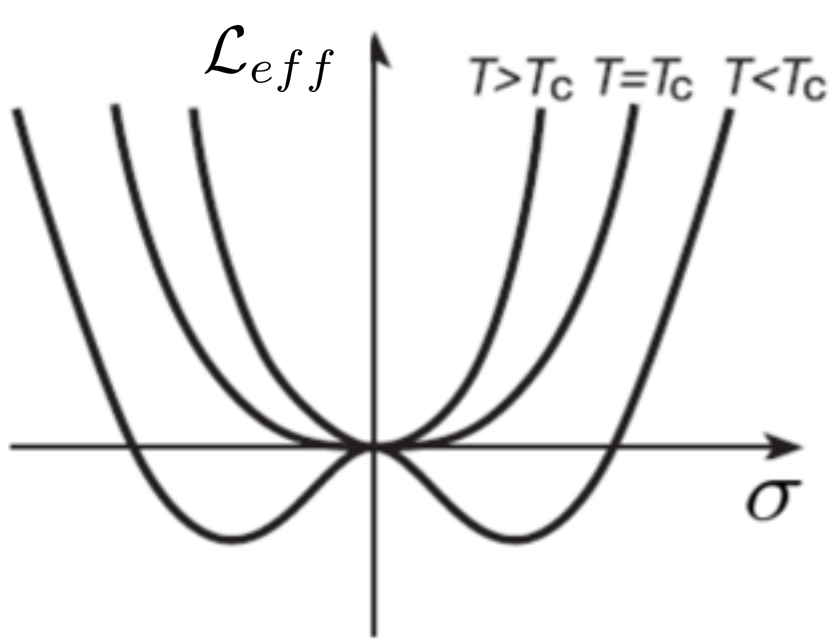
Same symmetry with the underlying theory

σ(x): order parameter field; K={m,μ}: external parameters

2nd order phase transition

Z(2) Ising model, Nf=2 QCD

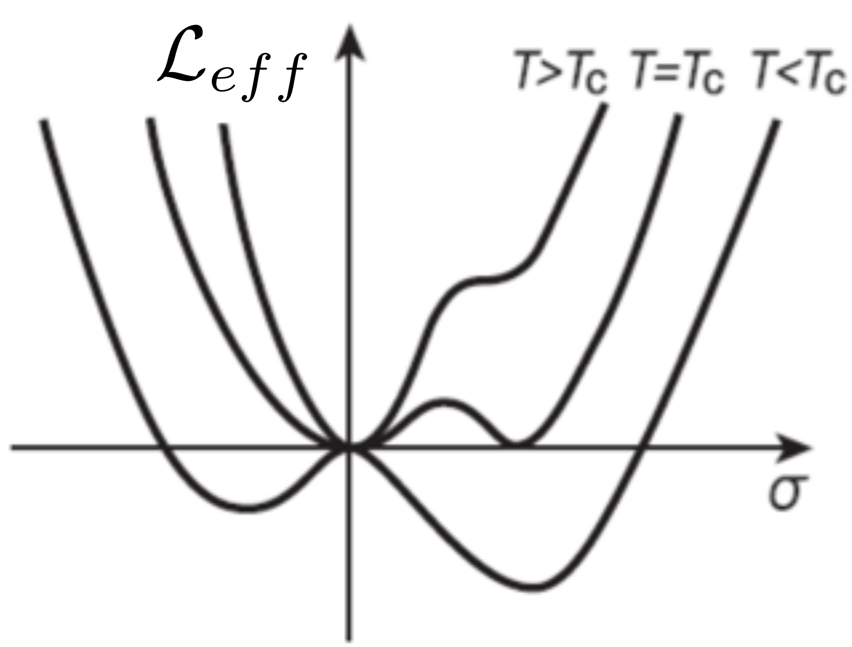
$$\mathcal{L}_{eff} = \frac{1}{2}a\sigma^2 + \frac{1}{4}b\sigma^4$$



1st order phase transition

Z(3) Potts model, Nf=3 QCD

$$\mathcal{L}_{eff} = \frac{1}{2}a\sigma^2 - \frac{1}{3}c\sigma^3 + \frac{1}{4}b\sigma^4$$



Ginzburg-Landau-Wilson approach

Partition function: $Z = \int [\mathrm{d}\sigma] \exp\left(- \int \mathrm{dx}\, \mathcal{L}_{eff}\, (\sigma(\mathrm{x}); K) \right)$

Landau function: $\mathcal{L}_{eff} = \frac{1}{2} (\nabla \sigma)^2 + \sum_n a_n(K) \sigma^n$ Same symmetry with the underlying theory

σ(x): order parameter field; K={m,μ}: external parameters

2nd order phase transition

order parameter M:
continuous in T

fluctuations of M:

$$\chi(T) = \frac{T}{V} (\langle M^2 \rangle - \langle M \rangle^2)$$

$$\chi(T_c) \sim V^{(2-\eta)/3}$$

1st order phase transition

M:
discontinuous in T

fluctuations of M:

$$\chi(T_c) \sim V$$

Landau functional of QCD

Pisarski & Wilczek (84)

Symmetry: $SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$

Chiral field: $\Phi_{ij} \sim \frac{1}{2}\bar{q}^j(1-\gamma_5)q^i = \bar{q}^j_R q^i_L$

Chiral transformation: $\Phi \to e^{-2i\alpha_A} V_L \, \Phi \, V_R^\dagger$

$$\mathcal{L}_{eff} = \frac{1}{2}\,\mathrm{tr}\,\partial\Phi^\dagger\partial\Phi + \frac{a}{2}\,\mathrm{tr}\,\Phi^\dagger\Phi + \frac{b_1}{4!}\left(\mathrm{tr}\,\Phi^\dagger\Phi\right)^2 + \frac{b_2}{4!}\,\mathrm{tr}\,\left(\Phi^\dagger\Phi\right)^2$$
→ $SU(N_f)_L \times SU(N_f)_R \times U(1)_A$

$$- \frac{c}{2}\left(\det\Phi + \det\Phi^\dagger\right)$$
→ $SU(N_f)_L \times SU(N_f)_R$

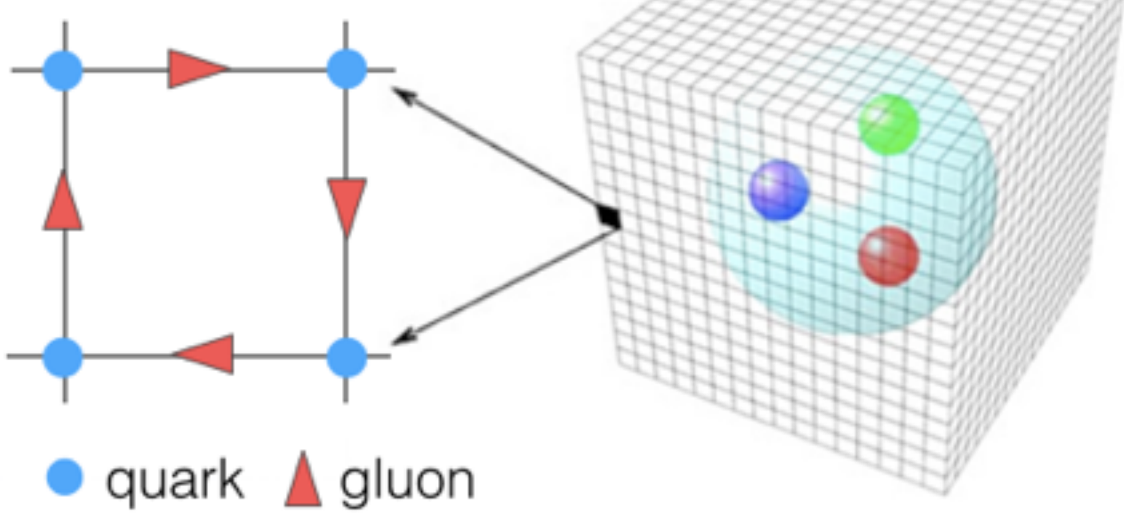
$$- \frac{d}{2}\mathrm{tr}\,h\left(\Phi + \Phi^\dagger\right).$$
→ Quark mass term

Results on phase transitions should be eventually checked by Lattice QCD

Lattice QCD

QCD Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q}\left[i\gamma^{\mu}(\partial_{\mu} - igA_{\mu}) - m_q\right]q$$



Discretization in Euclidean space

quarks: lattice sites
gluons: lattice links

Supercomputing the QCD matter:

structural equivalence between statistical mechanics & QFT on the lattice

$$< \mathcal{O} >= \frac{1}{Z}\int \mathcal{D}\mathcal{U}\ \mathcal{D}\psi\ \mathcal{D}\bar{\psi}\ \mathcal{O}\ e^{-S_{lat}}$$

$$S_{lat} = S_g + S_f$$

$$Z = \int \mathcal{D}\mathcal{U}\ \mathcal{D}\psi\ \mathcal{D}\bar{\psi}\ e^{-S_{lat}} = \int \mathcal{D}\mathcal{U}\ e^{-S_g}\ \det M_f$$

$N_c \otimes N_f \otimes N_{spin} \otimes N_d \otimes N_\sigma \otimes N_T^3 \gtrsim 10^6$

chiral condensate: $\langle \bar{q}q \rangle = \frac{\partial \ln Z}{\partial m_q} = \frac{n_f}{4}\langle \mathrm{Tr} M^{-1} \rangle$

重大科学计算装置

天津超算中心天河一号
GPU

广州超算中心天河二号
Xeon Phi Coprocessors

无锡神威太湖之光
申威-64-CPU

德国Jülich超算科研中心
IBM Bluegene Q

日本理化研究所
京超级计算机
CPU

美国橡树岭国家实验室
Titan超级计算机GPU

NSC3 Nuclear Science Computing Center at CCNU

华中师范大学
核科学计算中心

- **N**: Nuclear
- **S**: Science
- **C³**: Color 3 -> QCD

计算决定未来！

“道生一，一生二，二生三，三生万物” —- 《道德经》老子 600 BC

High Performance, Low Power Consumption
~100% utilized

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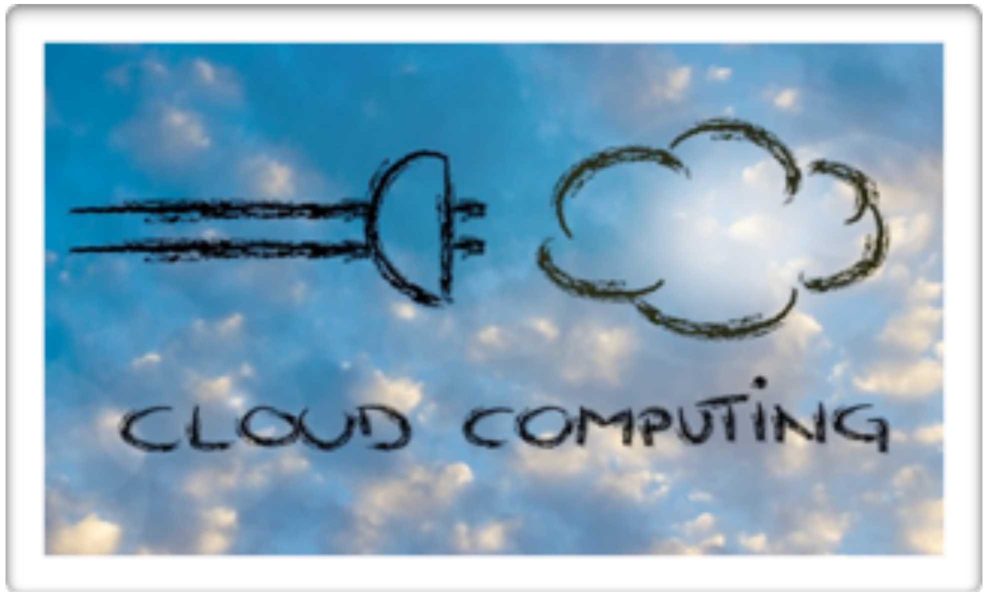
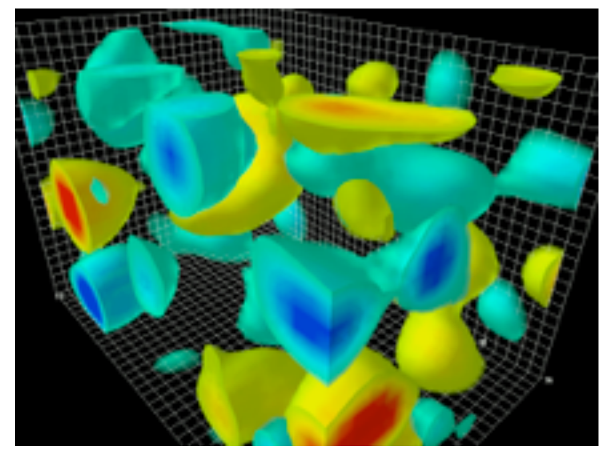


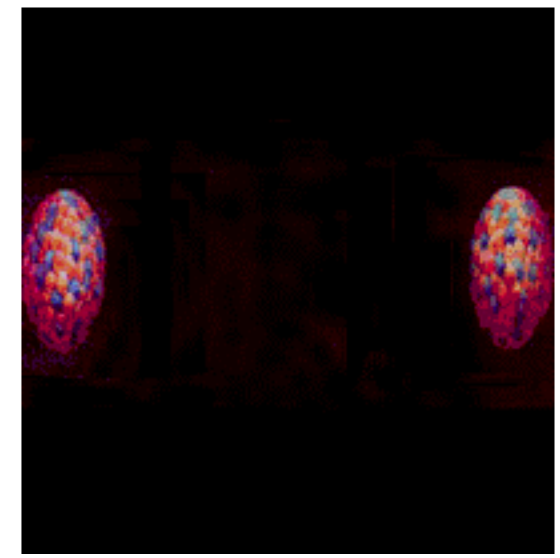
Nuclear Science Computing Center at CCNU

华中师范大学 核科学计算中心

First installation (2017):
Peak performance 1 PFlops/s, 400 TB storage

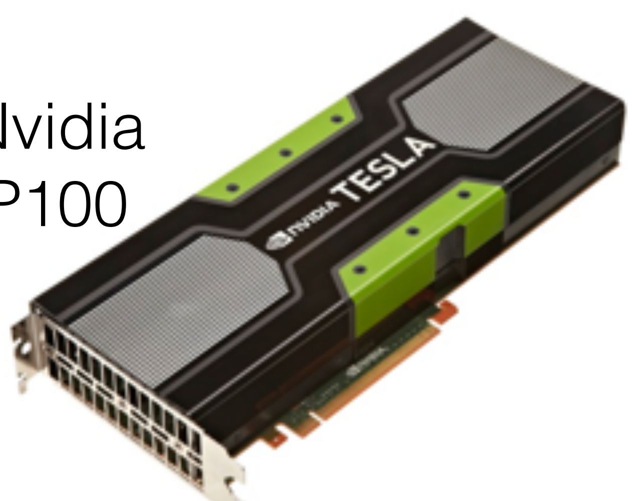
- Lattice QCD
- Heavy Ion Experiment
- Phenomenological calculations
- ……





High Performance, Low Power Consumption

Nvidia P100



Intel KNL

~1/2 中科院'元'超级计算中心@2015的理论峰值：2.3 PFlops/s(处理器：混搭)

~ 1/10 Italian Cineca Supercomputer 'MARCONI'@2016

NuclearScience
Computing CenteratCCNU

华中师范大学
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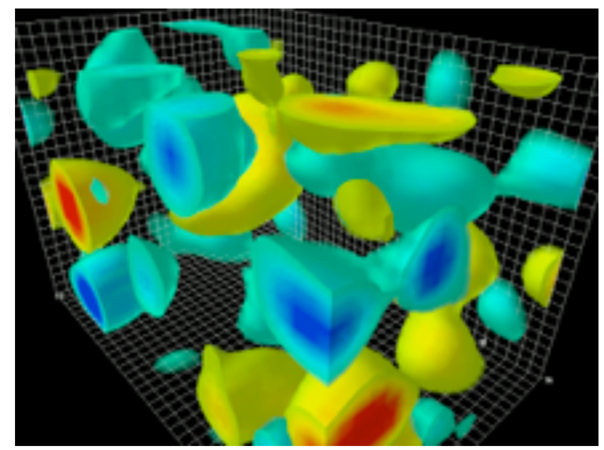
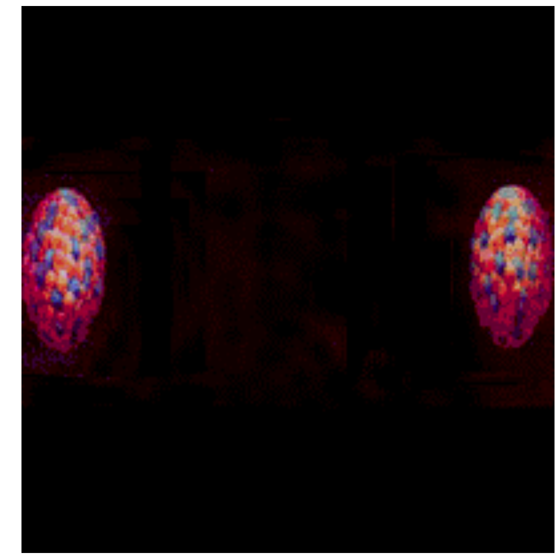
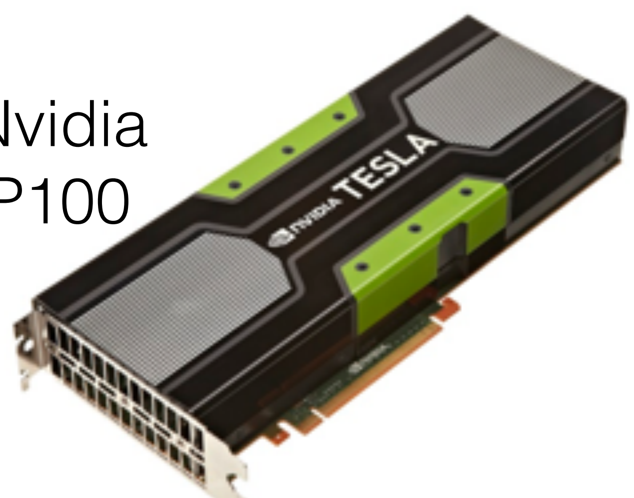




High Performance, Low Power Consumption

Nvidia
P100



Intel
KNL

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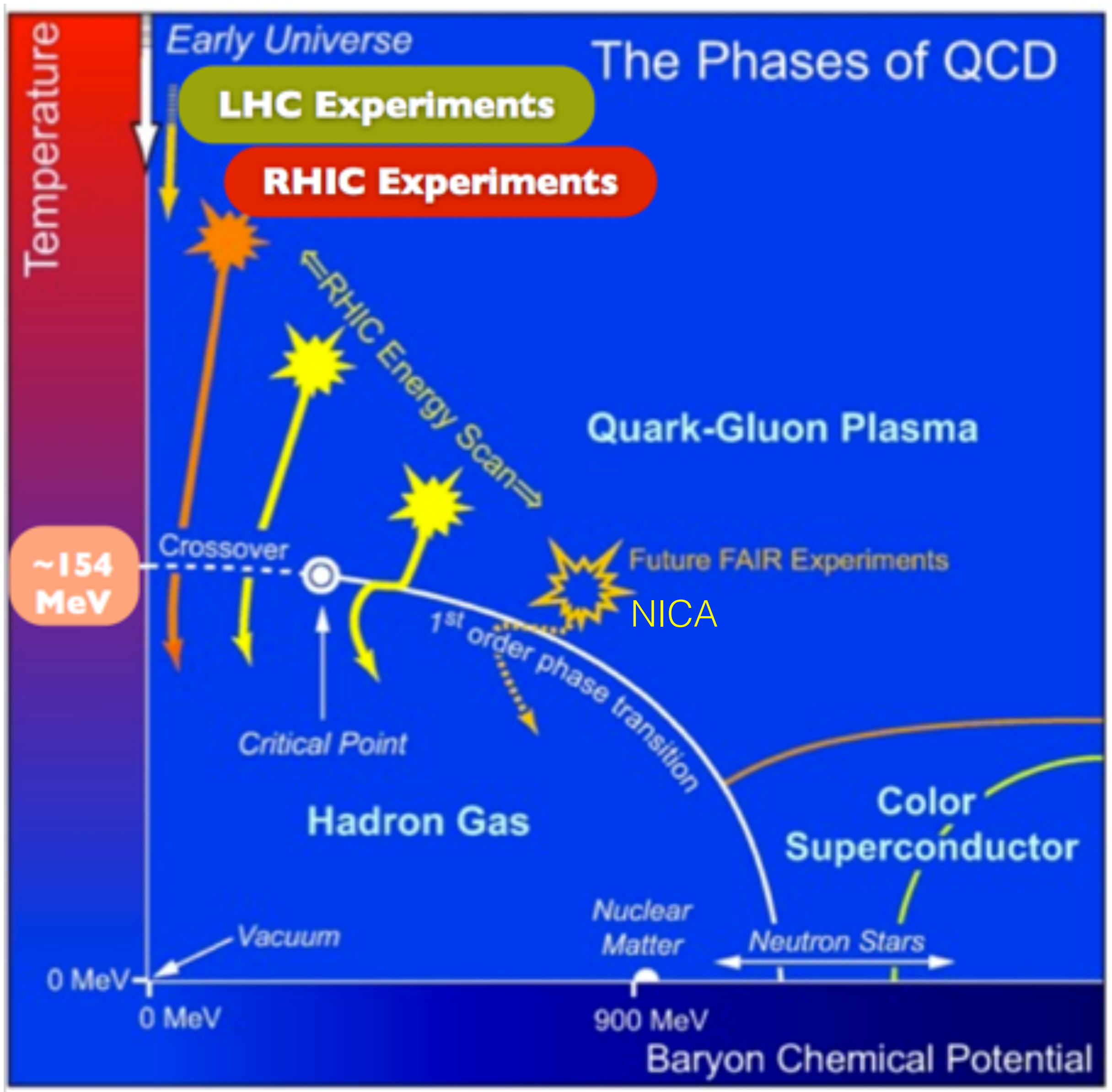
~ 1/10 Italian Cineca Supercomputer 'MARCONI'@2016

QCD Phase Diagram

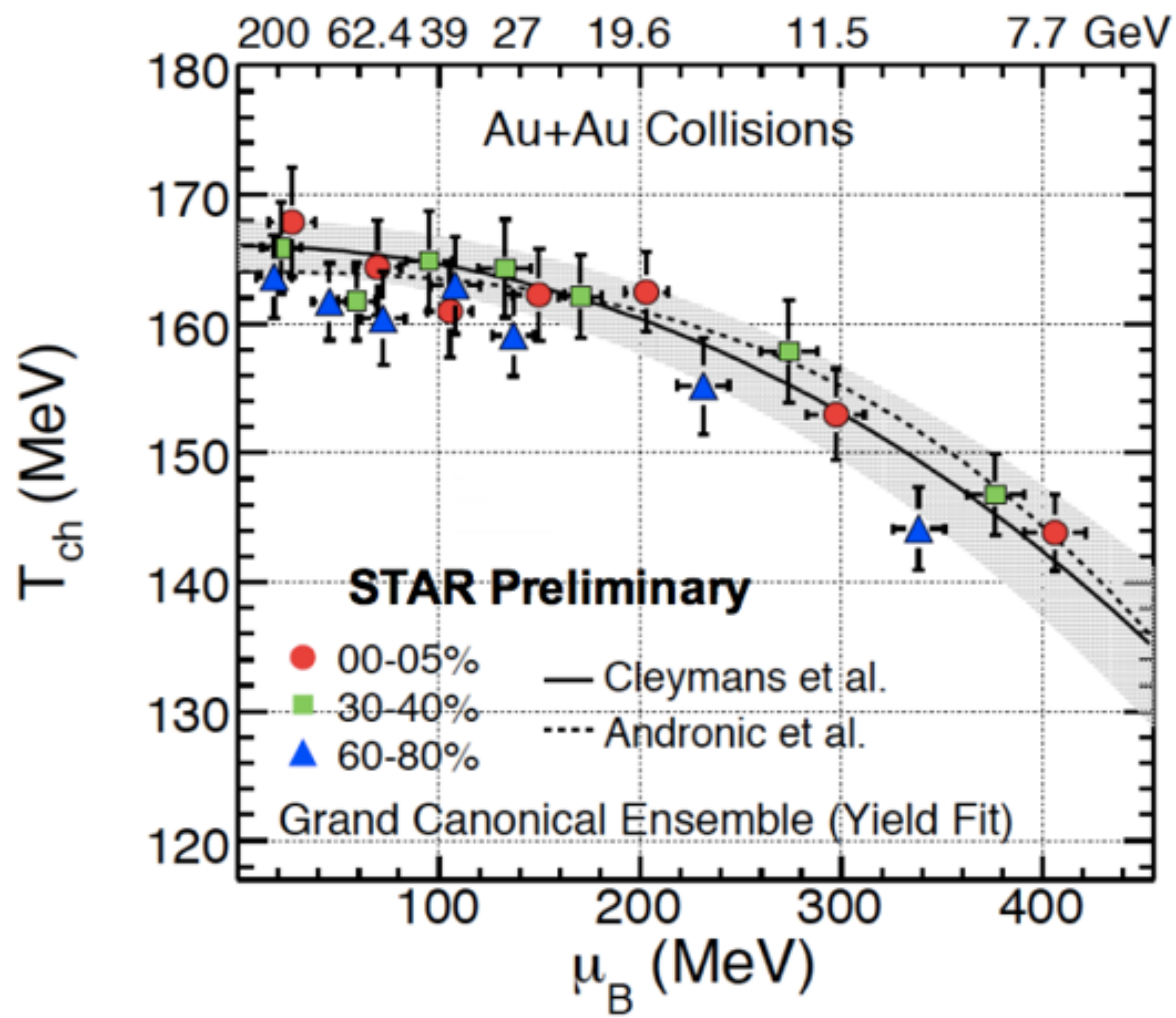


- The QCD phase structure is under extensively investigation by Heavy Ion Collision (HIC) Experiment

- Hadronic fluctuations and abundance are measured at freezeout

Beam Energy Scan at RHIC

$\sqrt{s_{NN}}$ (GeV)	Events (10^6)	Year	*μ_B (MeV)	*T_{CH} (MeV)
200	350	2010	25	166
62.4	67	2010	73	165
39	39	2010	112	164
27	70	2011	156	162
19.6	36	2011	206	160
14.5	20	2014	264	156
11.5	12	2010	316	152
7.7	4	2010	422	140



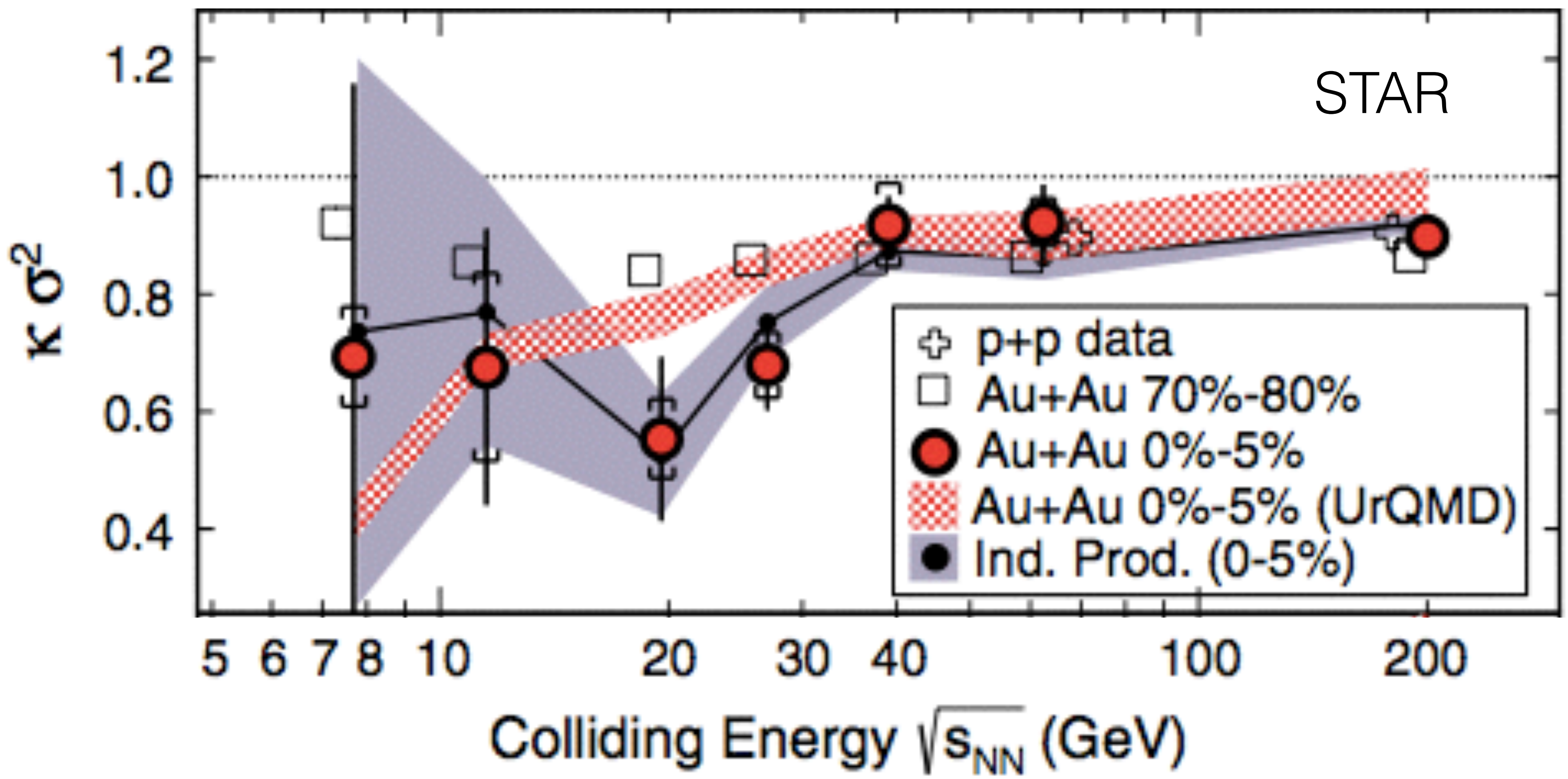
Hydrodynamics & Thermodynamics in

$$0 \lesssim \mu_B/T \lesssim 3$$

(T_{ch},μ_B): J. Cleymans et al., PRC 73 (2006)034906

Beam Energy Scan at RHIC

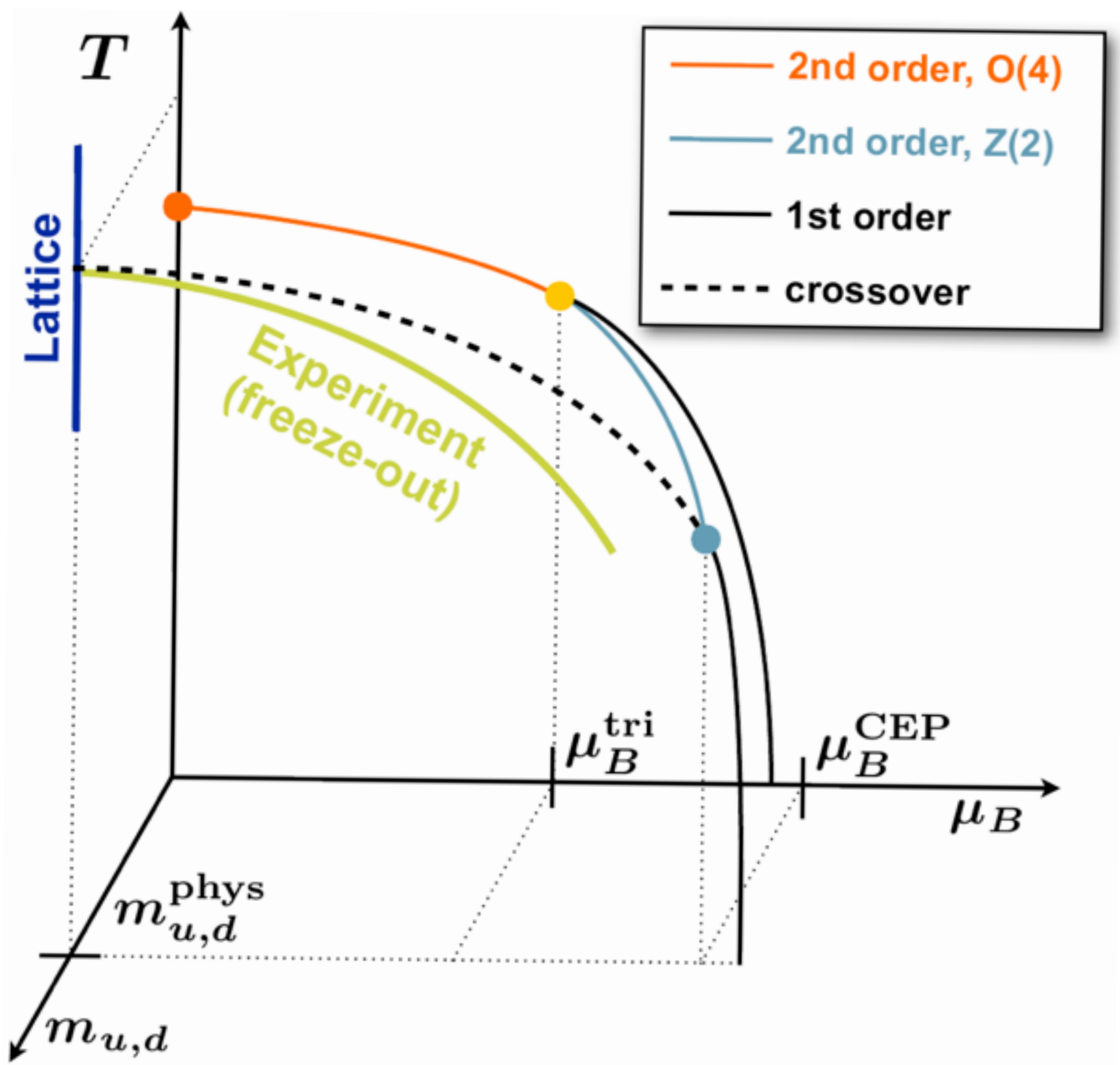
Ratio of the 4th to 2nd order proton number fluctuations



Can this non-monotonic behavior be understood in terms of the QCD thermodynamics in equilibrium?

What is the relation of this intriguing phenomenon to the critical behavior of QCD phase transition?

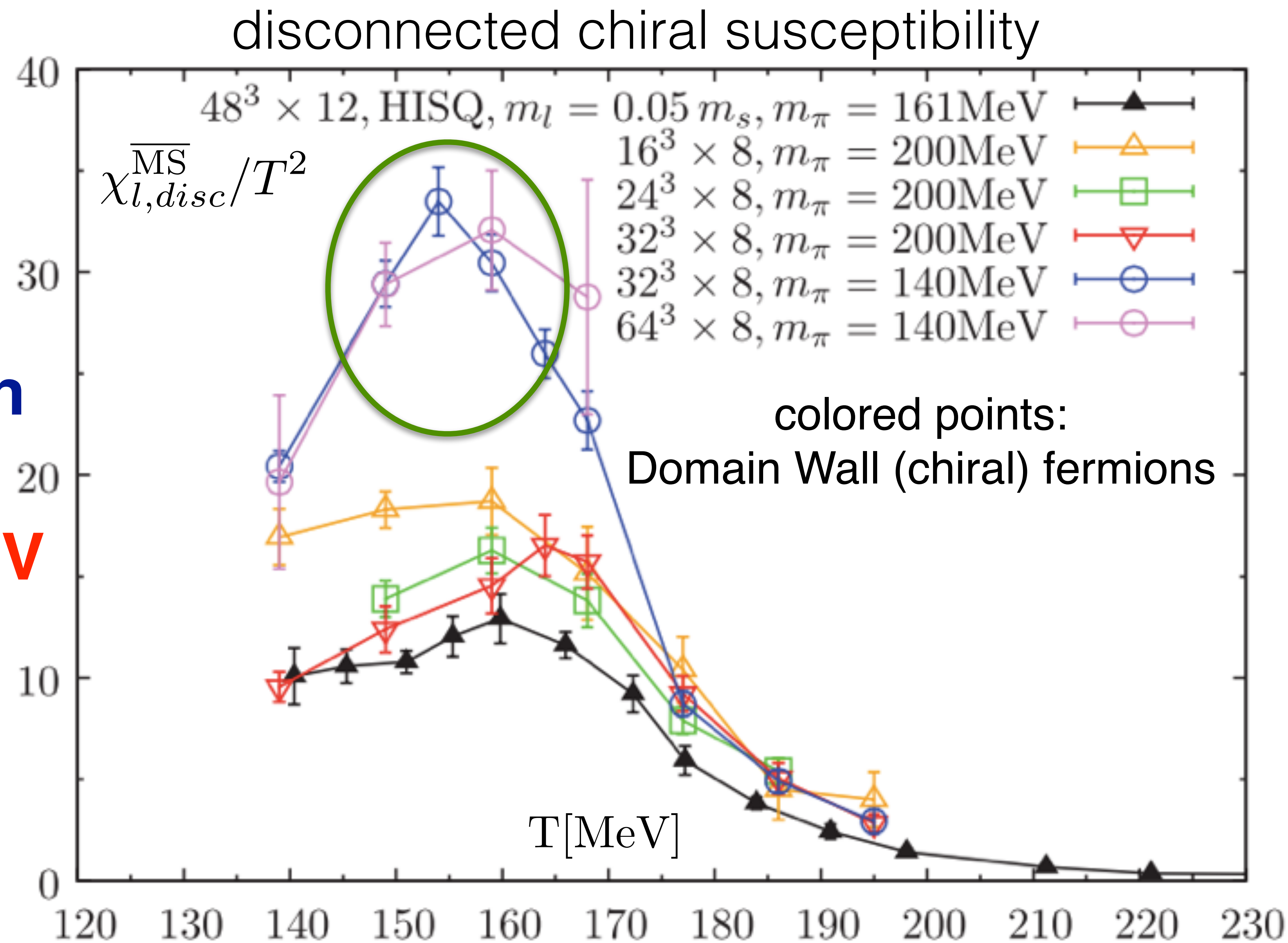
QCD phase diagram



QCD transition with m_π =140 MeV at μ_B=0

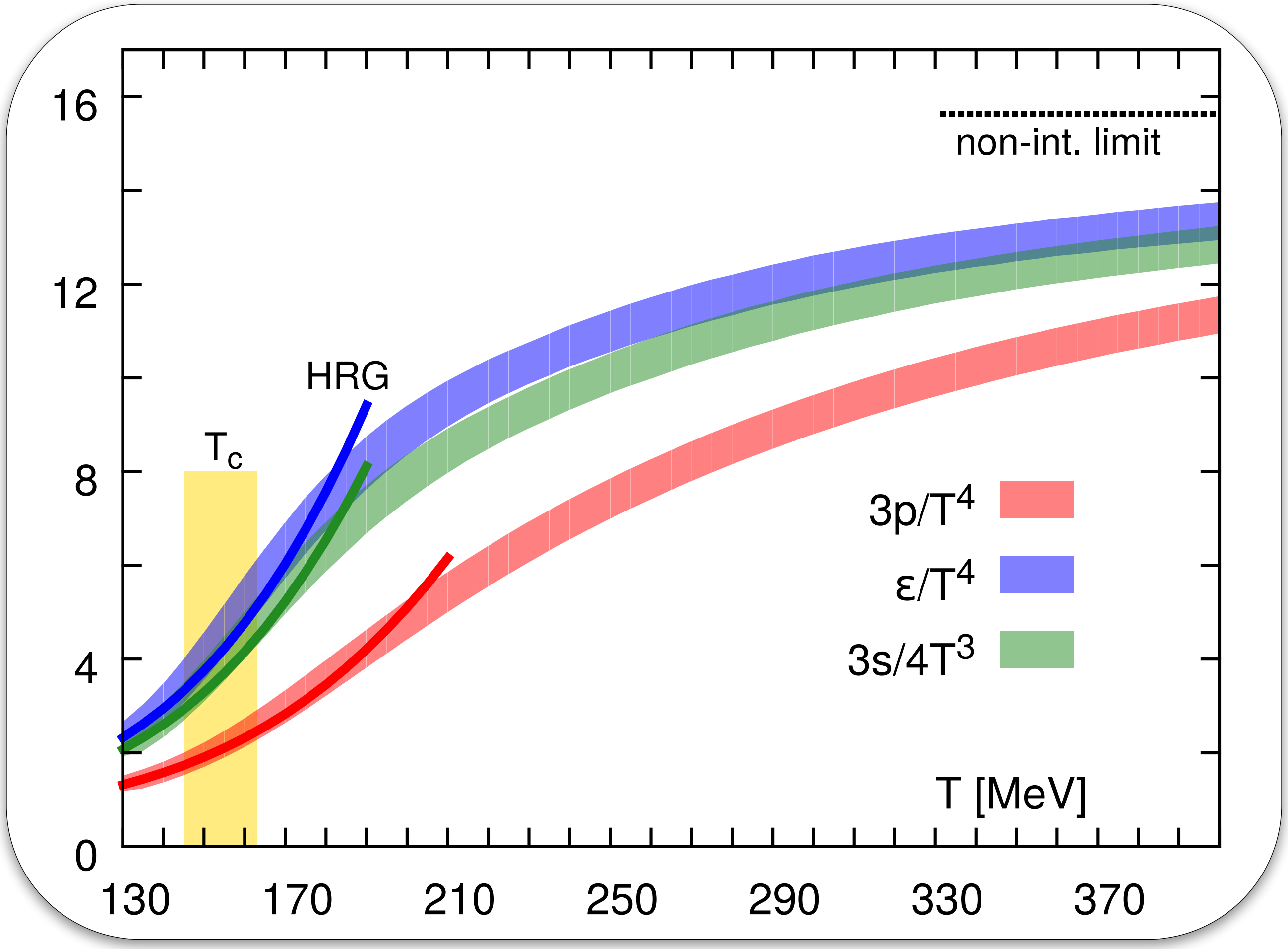
"cross over" type of transition

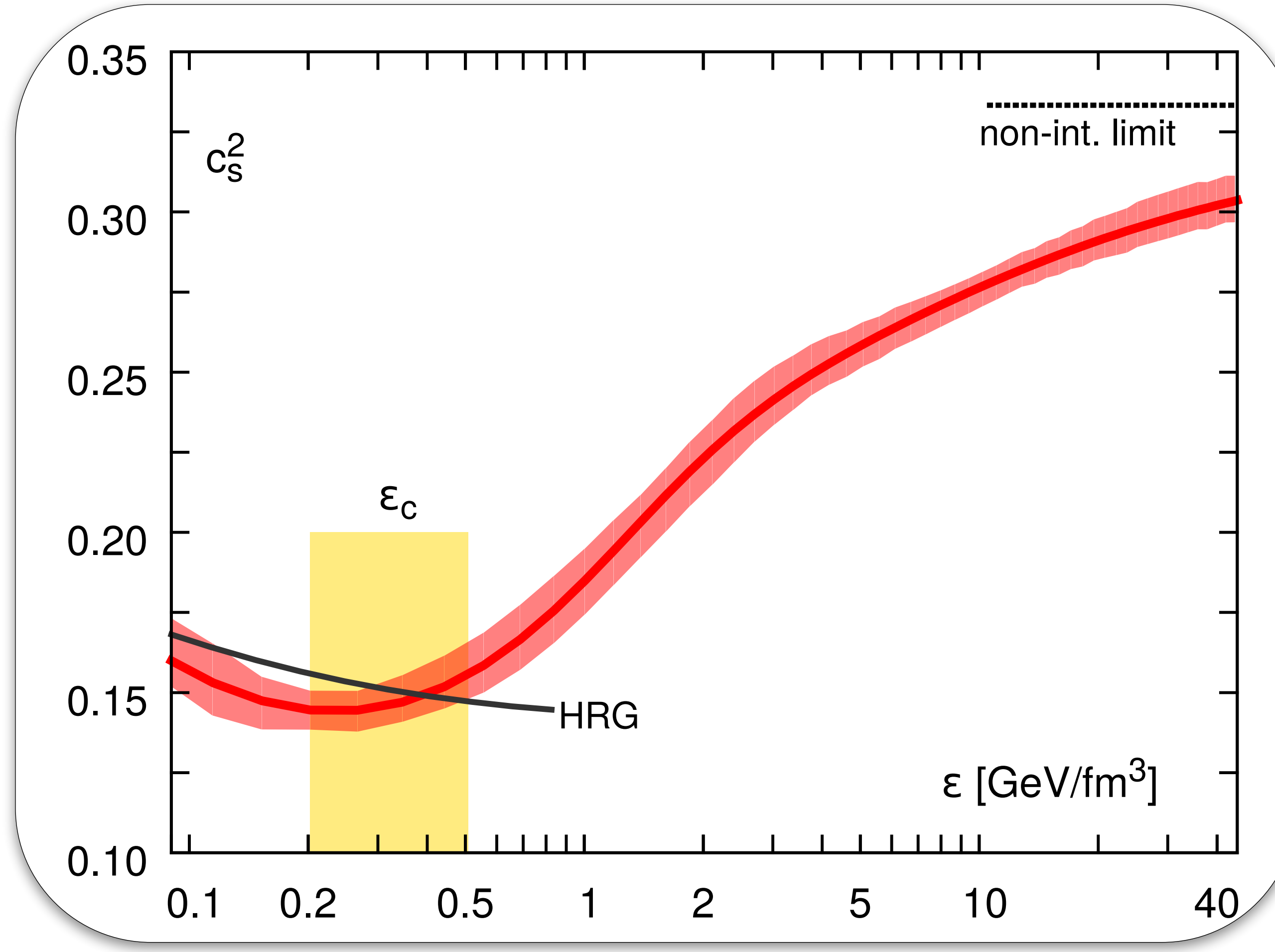
T_{pc} =155(1)(8) MeV



T. Bhattacharya,…丁亨通,…et al.[HotQCD collaboration],
Phys. Rev. Lett.,113(2014)082001 (编辑推荐阅读Editor's suggestion)

QCD Thermodynamics at $\mu_B=0$





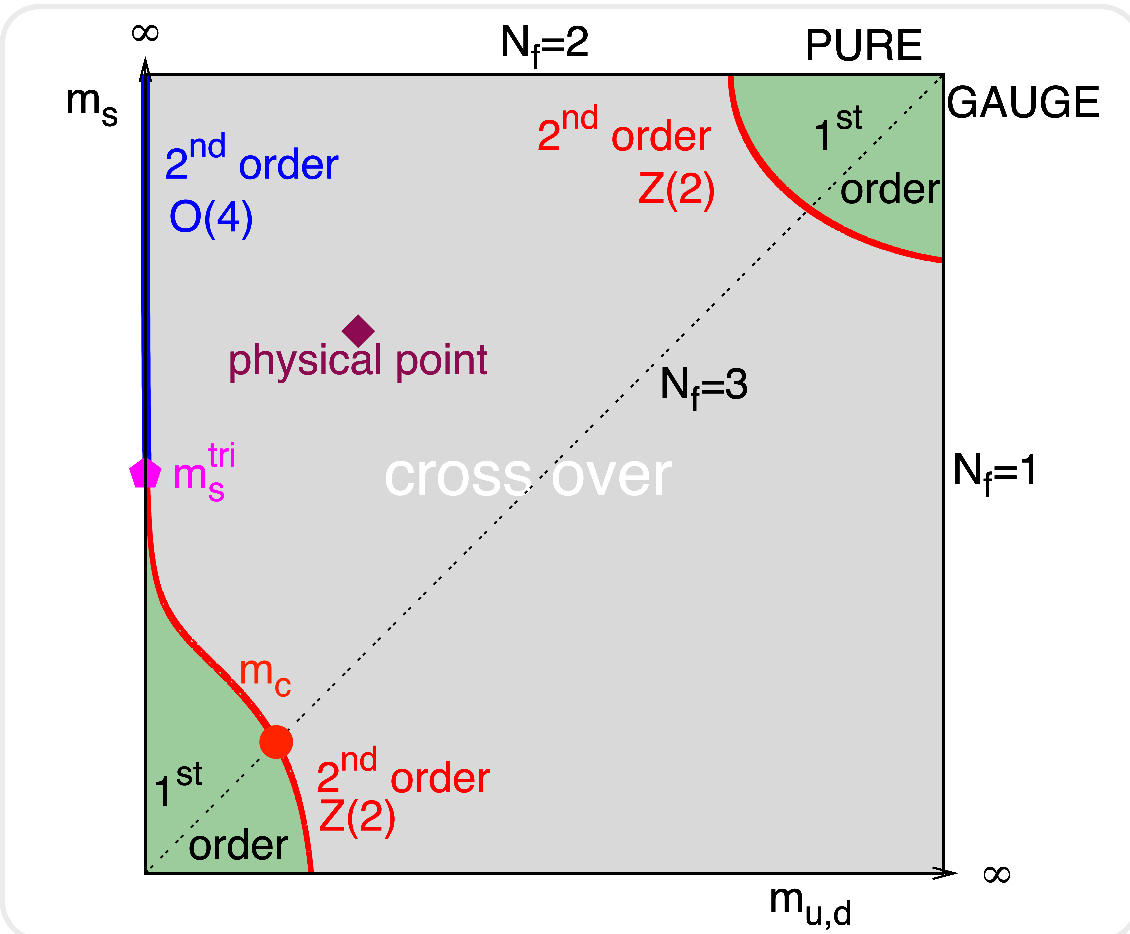
HotQCD: PRD 90(2014)094503

- pressure, energy and entropy densities rise rapidly in the cross over temperature region approaching to the non-interaction limit

- In the critical T region, $\boldsymbol{\varepsilon_c \in (180,500)}$ **MeV/fm³**, note that $\varepsilon_{nuclear}=150$ MeV/fm³ and $\varepsilon_{proton}=450$ MeV/fm³

QCD phase structure in the quark mass plane

columbia plot, PRL 65(1990)2491



HTD, F. Karsch, S. Mukherjee, arXiv:1504.0527

RG arguments:

- $m_q=0$ or ∞ with $N_f=3$: a first order phase transition Pisarski & Wilczek, PRD29 (1984) 338
- Critical lines of second order transition
  - $N_f=2$: O(4) universality class
  - $N_f=3$: Z(2) universality class

K. Rajagopal & F. Wilczek, NPB 399 (1993) 395

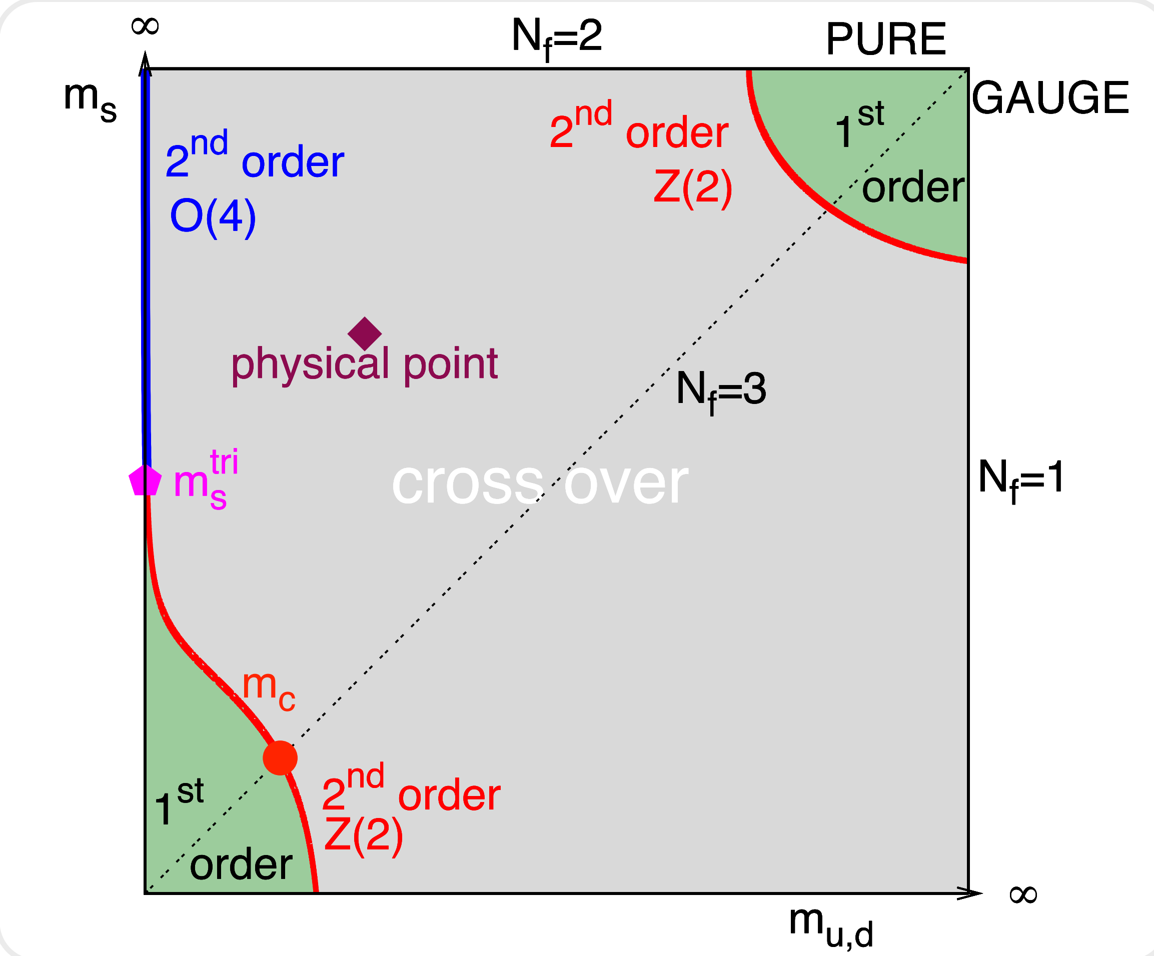
F. Wilczek, Int. J. Mod. Phys. A 7(1992) 3911,6951

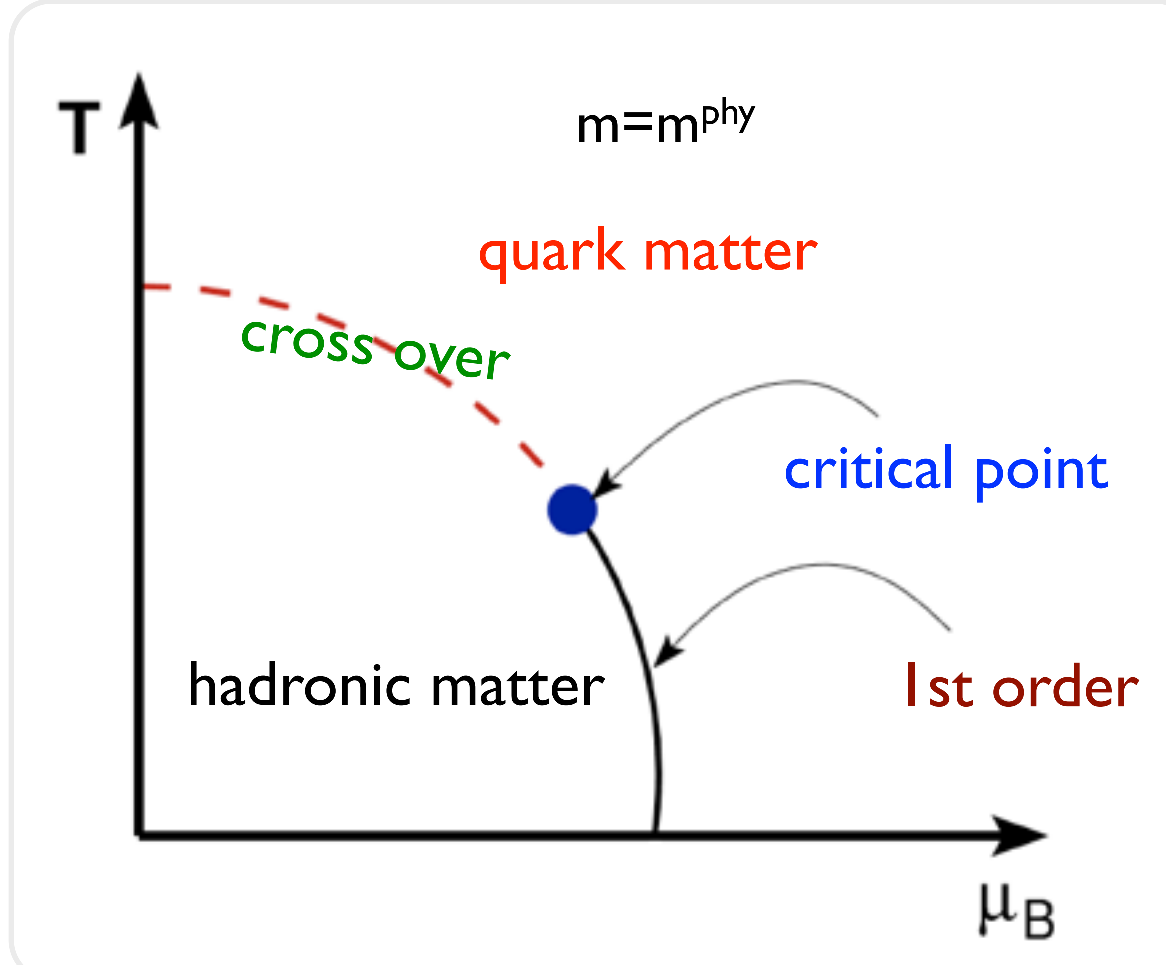
Gavin, Gocksch & Pisarski, PRD 49 (1994) 3079

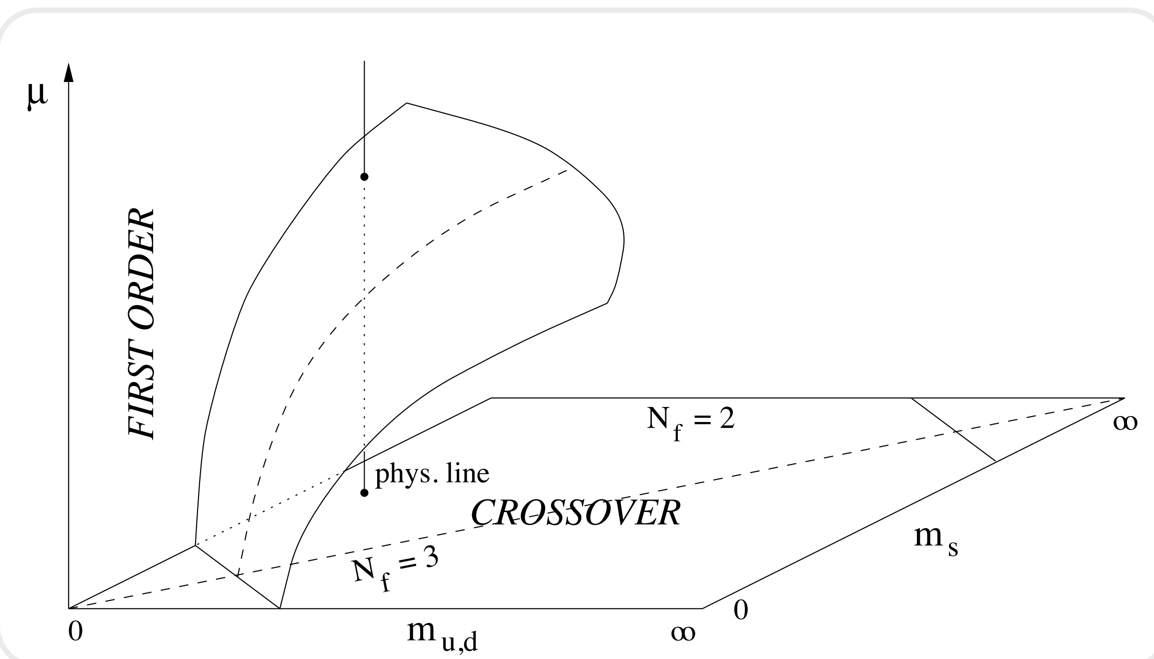
Lattice QCD calculations:

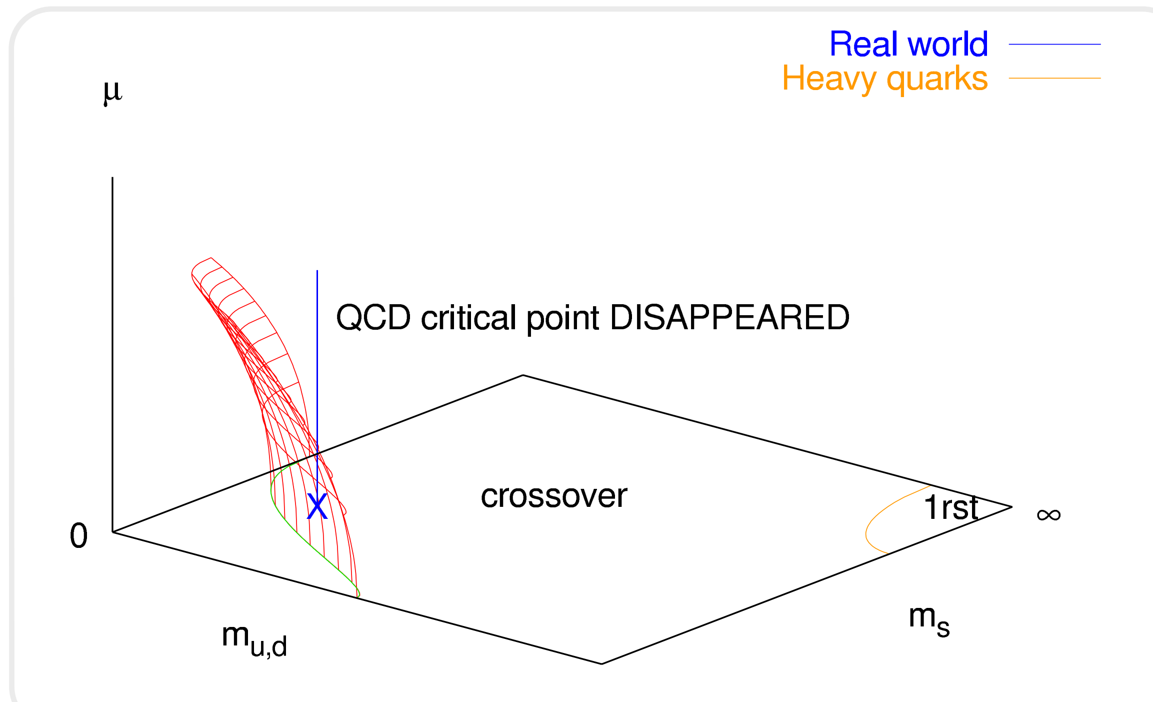
- The value of tri-critical point (m_s^{tri}) ?
- The location of 2^{nd} order Z(2) lines ?
- The influence of criticalities to the physical point ?

QCD transitions at the physical point





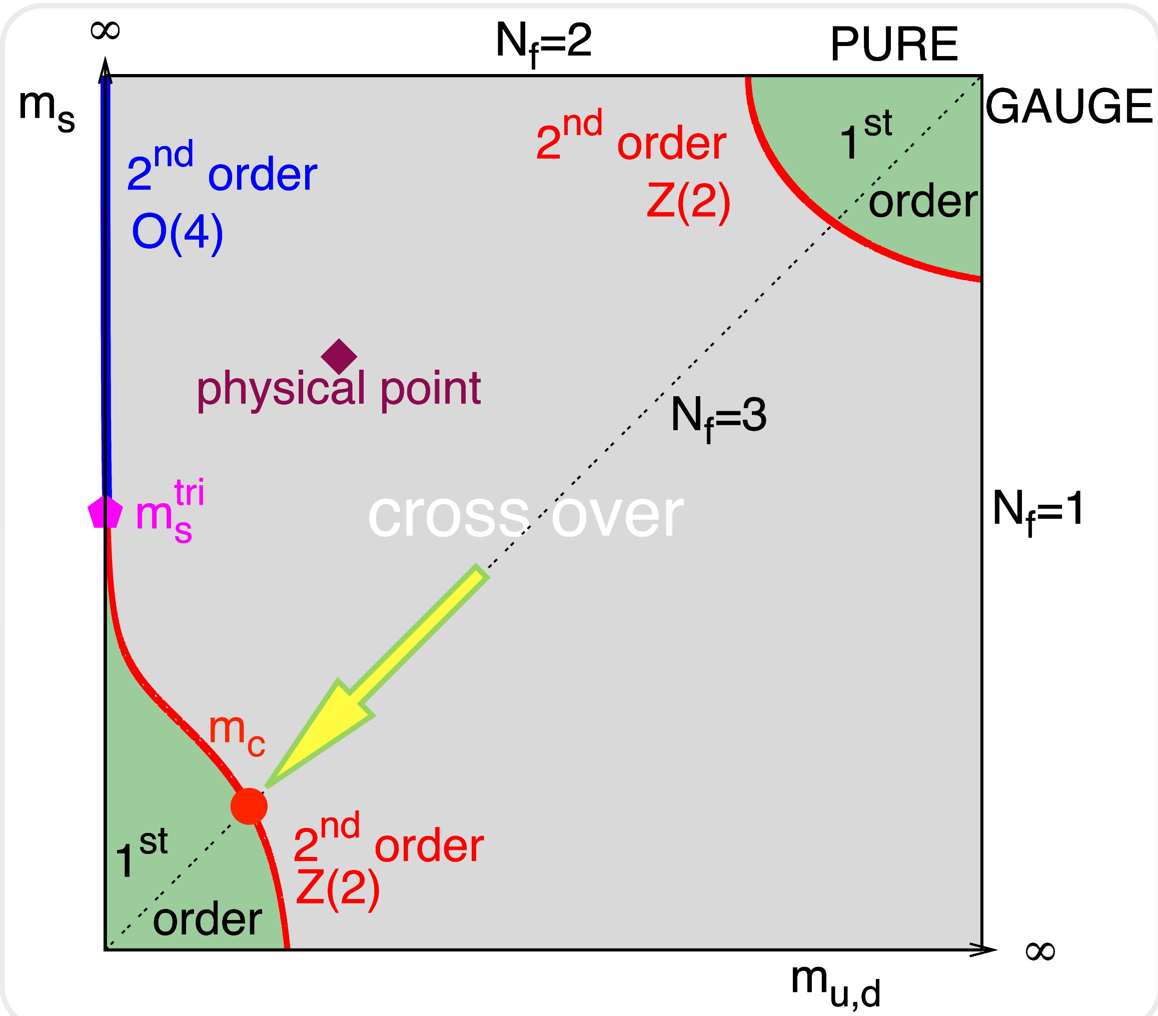


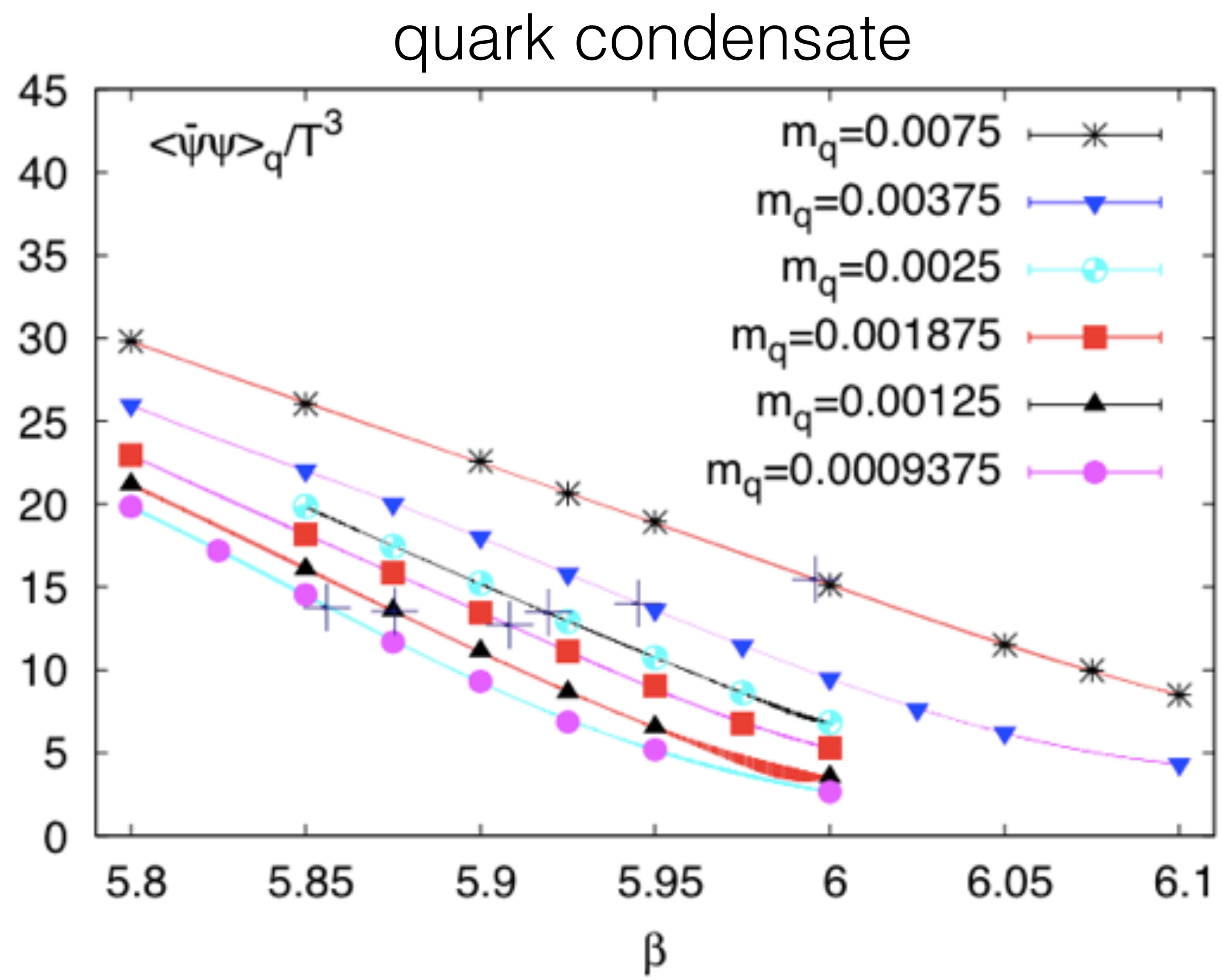


Karsch et al., '03, X.-Y. Jin et al., '15

de Forcrand & Philipsen, '07

chiral phase transition in Nf=3 QCD at μ_B=0



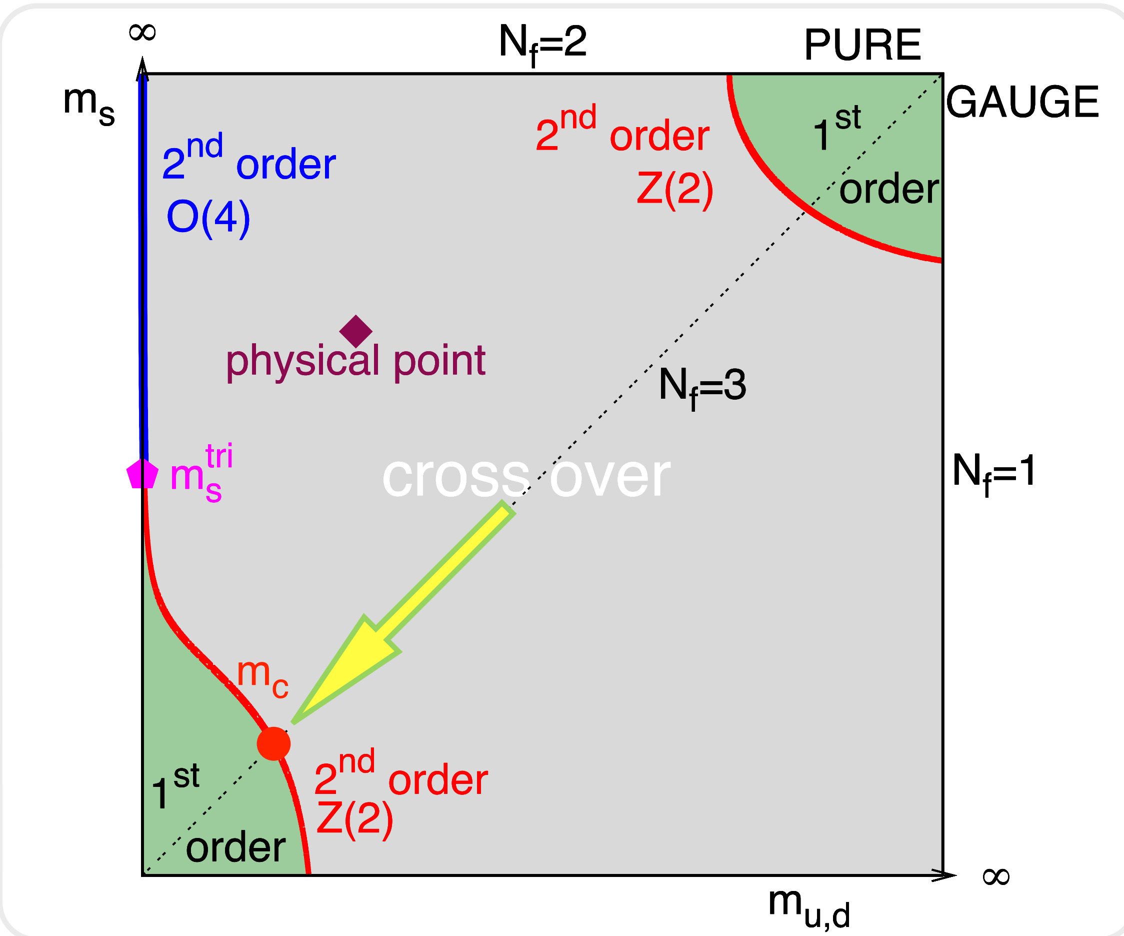


mass region: $200\text{MeV} \lesssim m_\pi \lesssim 80\text{MeV}$

No evidence of a first order phase transition

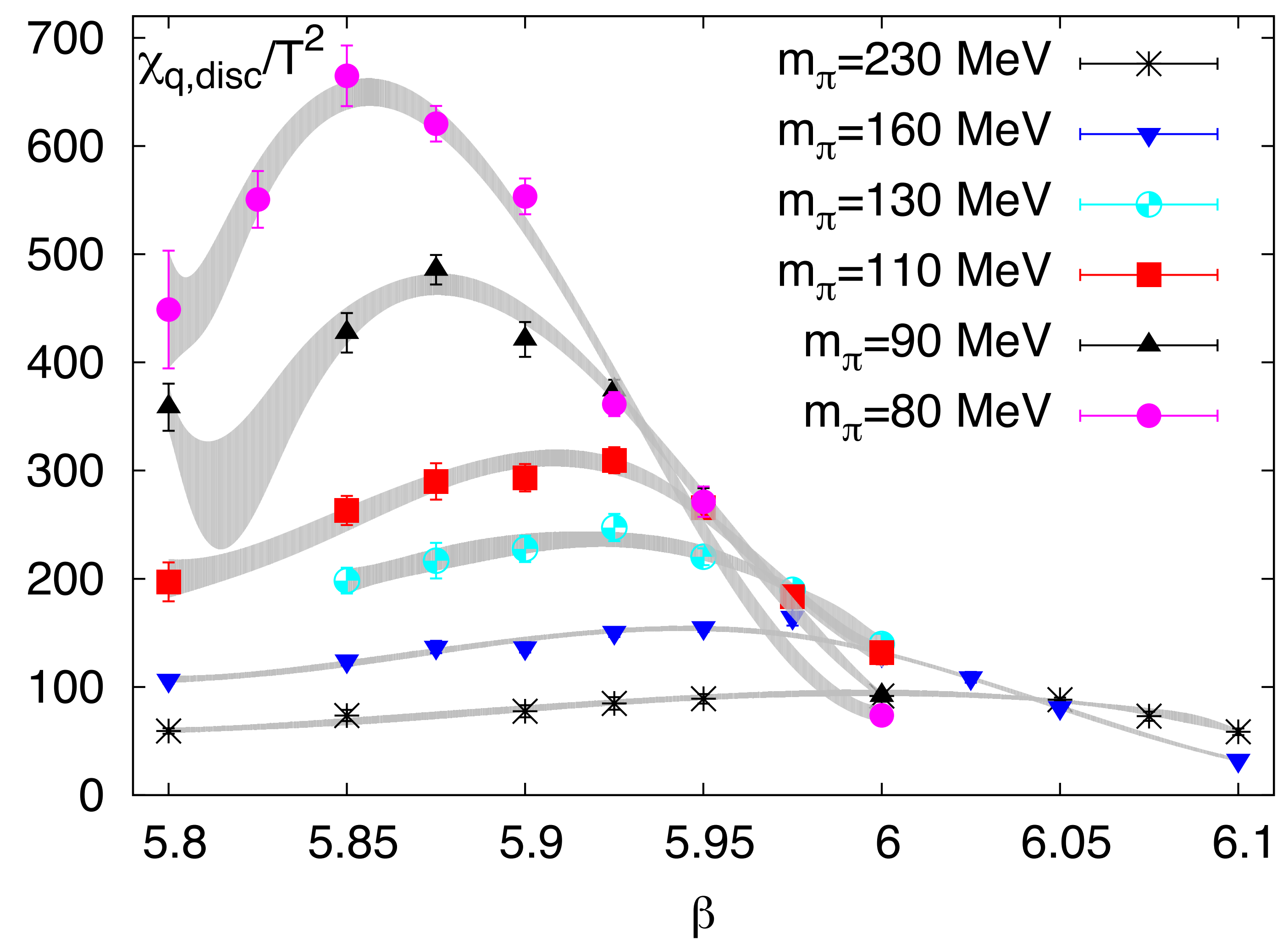
HTD, lattice 2015, Bielefeld-BNL-CCNU, to appear soon

Chiral phase transition in Nf=3 QCD at μ_B=0



physical point:
(ml, ms):(0.00375,0.10125)

disconnected chiral susceptibility

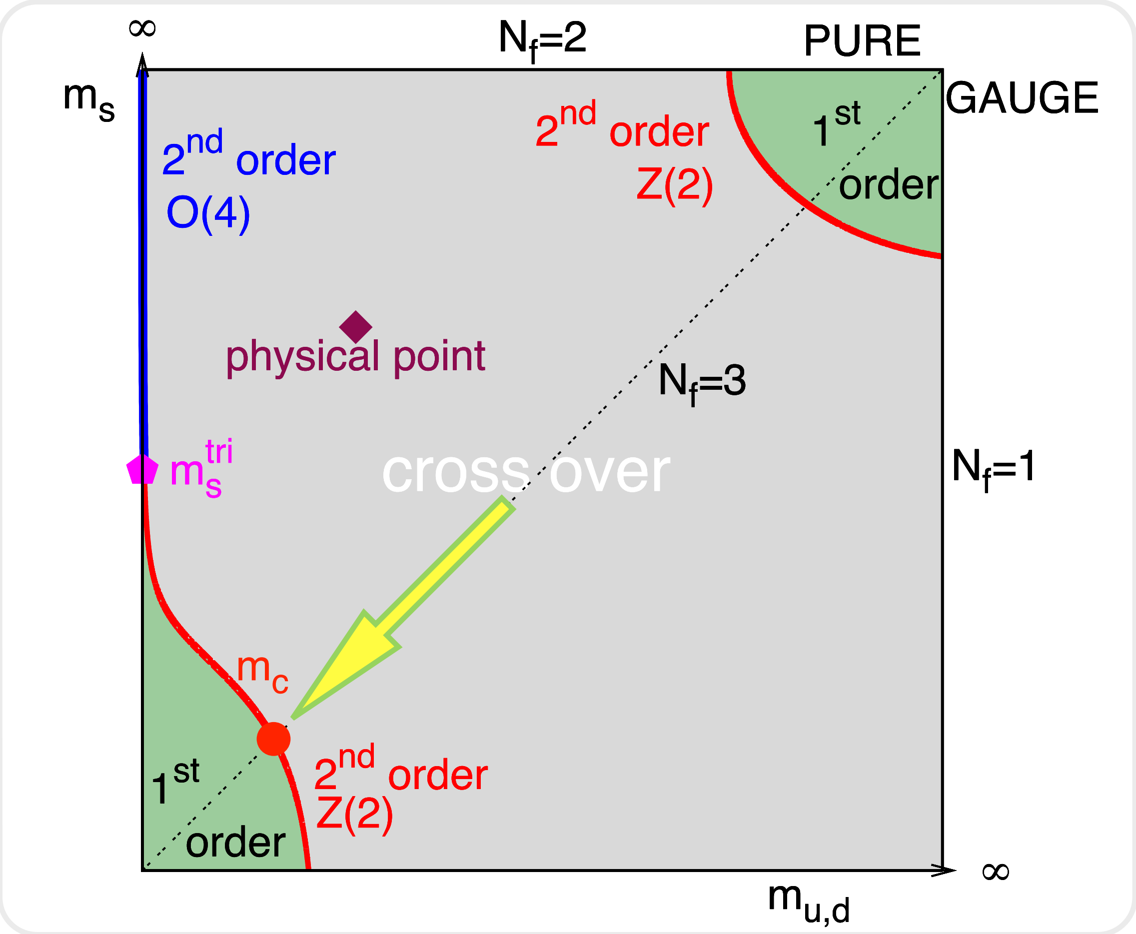


HTD, Lattice 2015, Bielefeld-BNL-CCNU, to appear soon

Close to Z(2) phase transition line: $\chi_{q,disc}^{max} \sim (m - m_c)^{1/\delta - 1}$

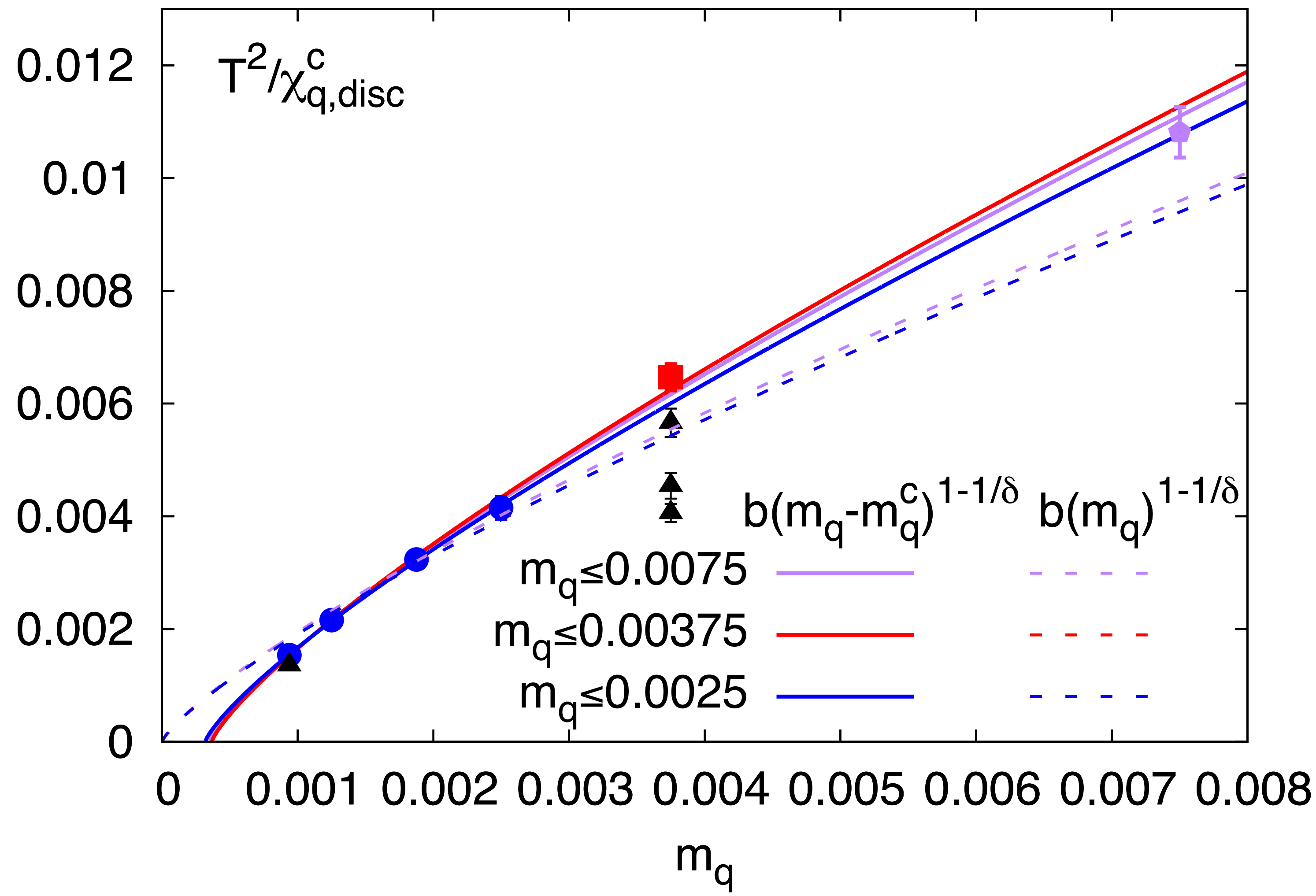
critical quark mass m_c ~ 0.0004 $\Rightarrow$ $m_\pi^c \lesssim$ 50MeV

Chiral phase transition in Nf=3 QCD at μ_B=0



physical point:
(ml, ms):(0.00375,0.10125)

inverse peak height of
disconnected chiral susceptibility

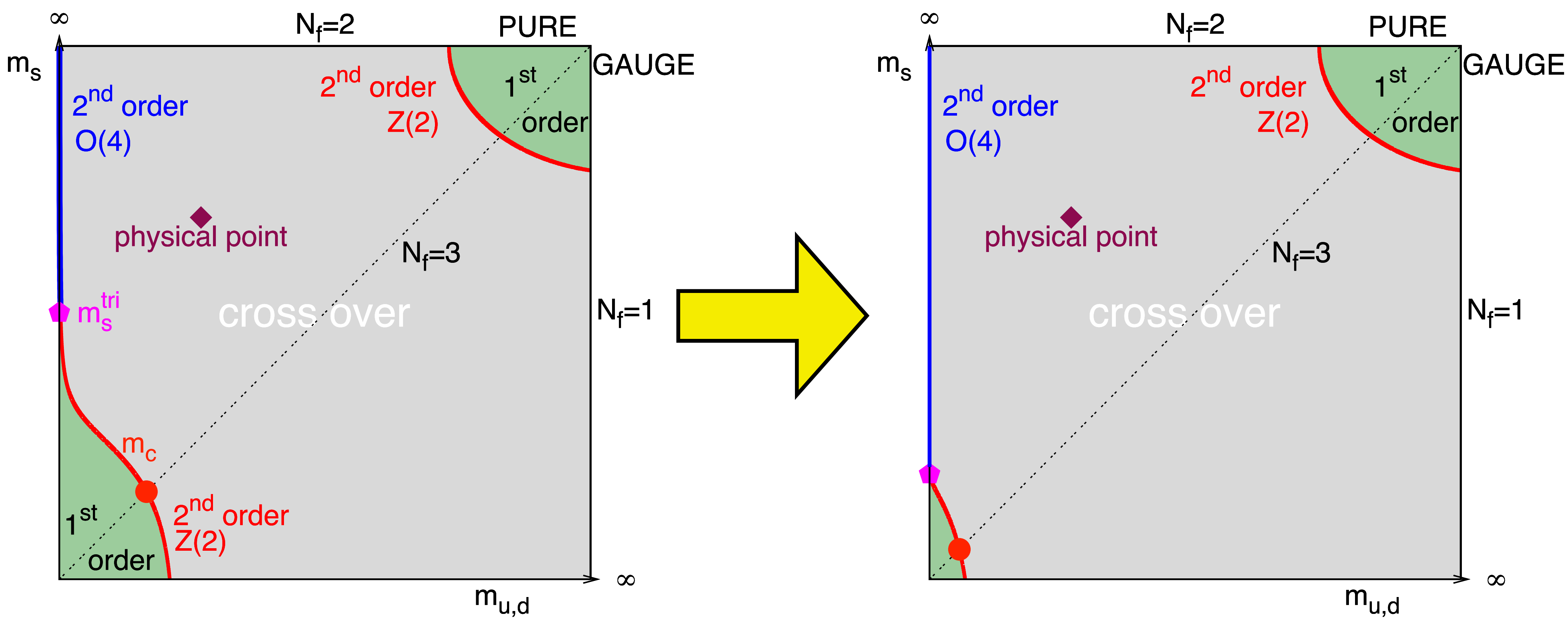


HTD, Lattice 2015, Bielefeld-BNL-CCNU, to appear soon

Close to Z(2) phase transition line: $$\chi^{max}_{q,disc} \sim (m - m_c)^{1/\delta - 1}$$

critical quark mass $m_c \sim 0.0004$ $\Rightarrow$ $m^c_\pi \lesssim$ 50MeV

Chiral phase transition region in Nf=3 QCD at μ_B=0



Whether the 1st order chiral phase transition is relevant for the physical point at all?

Universal behavior of chiral phase transition in Nf=2+1 QCD at μ_B=0

Behavior of the free energy close to critical lines

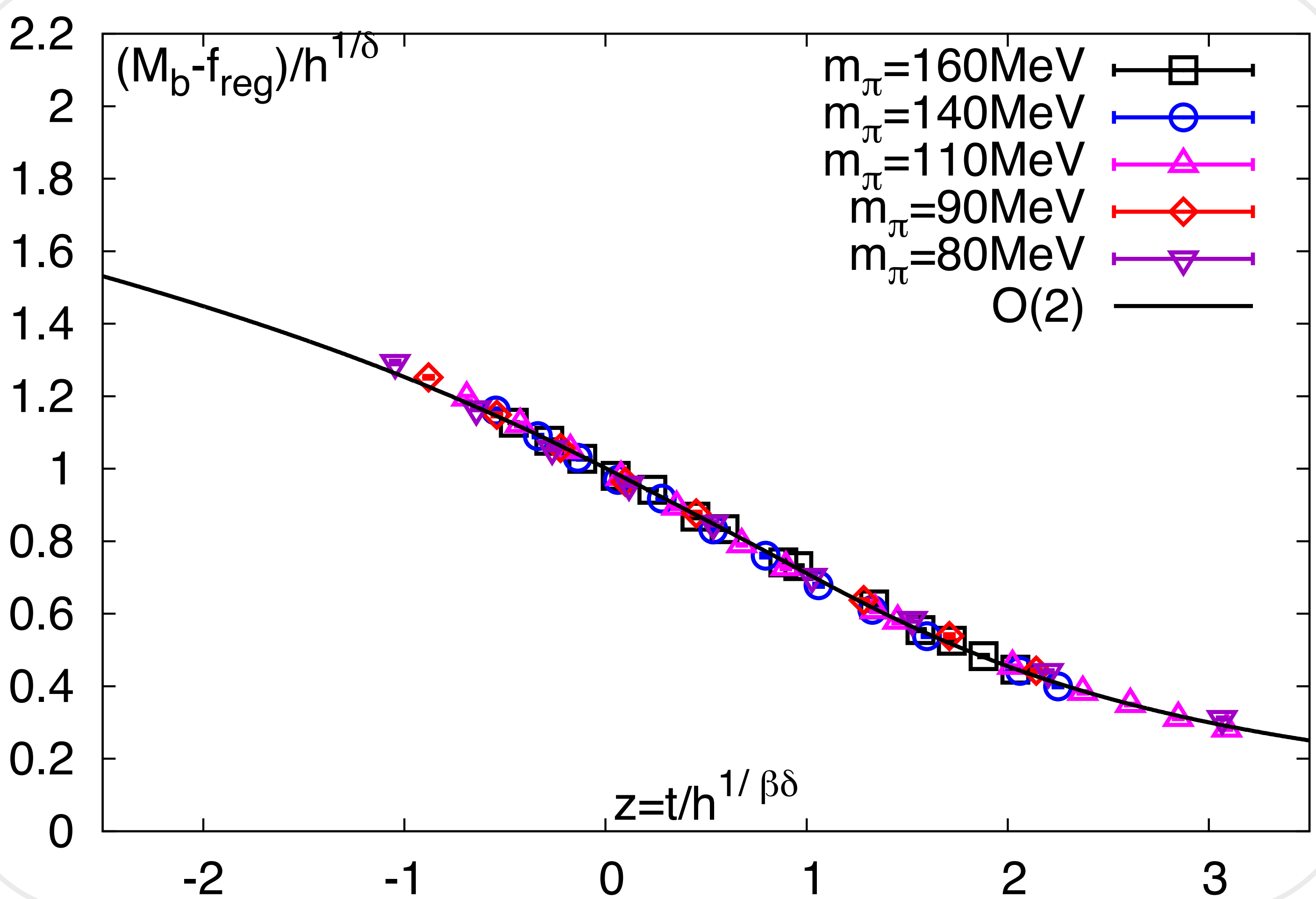
$$f(m,T) = h^{1+1/\delta} f_s(z) + f_{reg}(m,T), \qquad z = t/h^{1/\beta\delta}$$

h: external field, t: reduced temperature, β,δ: universal critical exponents

$$M = -\partial f(t,h)/\partial h = h^{1/\delta} f_G(z) + f_{reg}(t,h)$$

$h \sim m$; $t \sim T - T_c$

$f_G(z)$: (O(2))scaling functions



Good evidence of O(N) scaling for chiral phase transition

Sheng-Tai Li, Lattice 2016,
Bielefeld-BNL-CCNU, in preparation

role of $U_A(1)$ symmetry in Nf=2 QCD

$U_A(1)$ symmetry on the lattice:

- always broken in the Wilson/ Staggered discretization scheme

$U_A(1)$ symmetry:

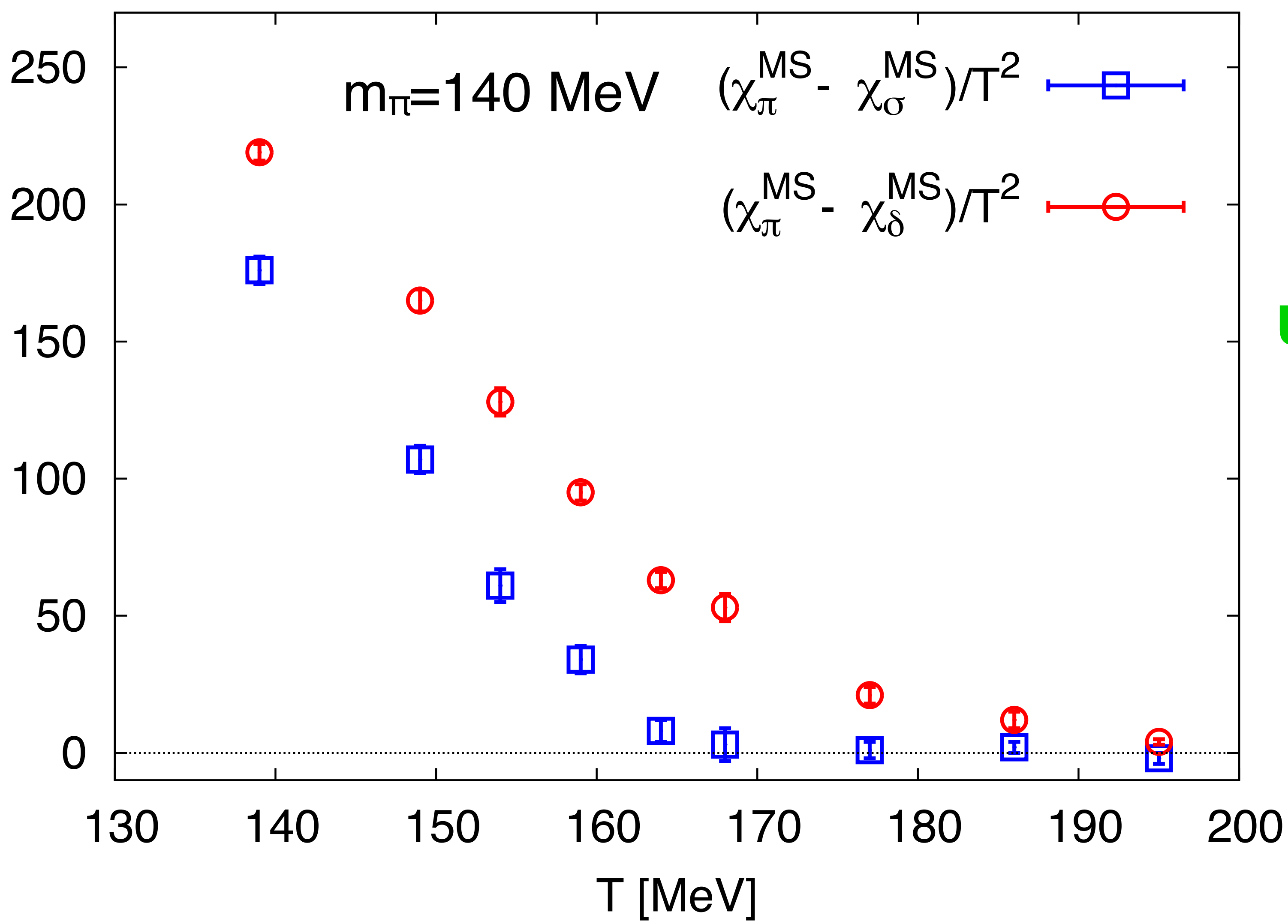
- restored, 1st or 2nd order ($U(2)_L \otimes U(2)_R / U(2)_V$)
- broken, 2nd order (O(4)) phase transition

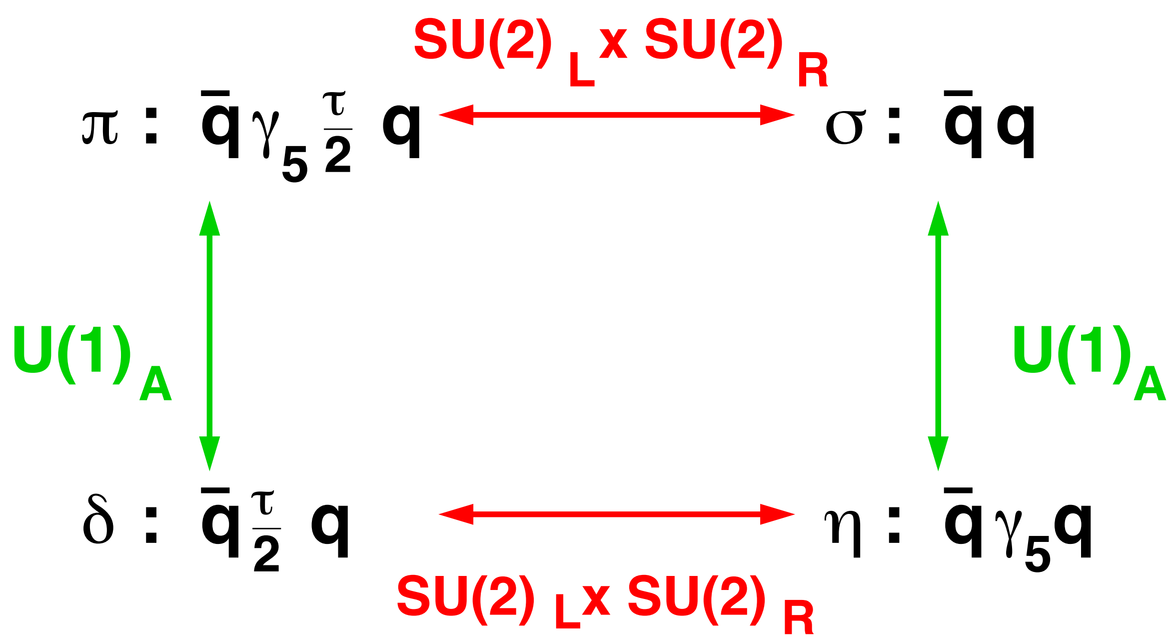
Pisarski and Wilczek, PRD 29(1984)338

Butti, Pelissetto and Vicar, JHEP 08 (2003)029

Fate of chiral symmetries at T=/=0: Nf=2+1 QCD

Domain Wall fermions, 32^3x8, Ls=24,16

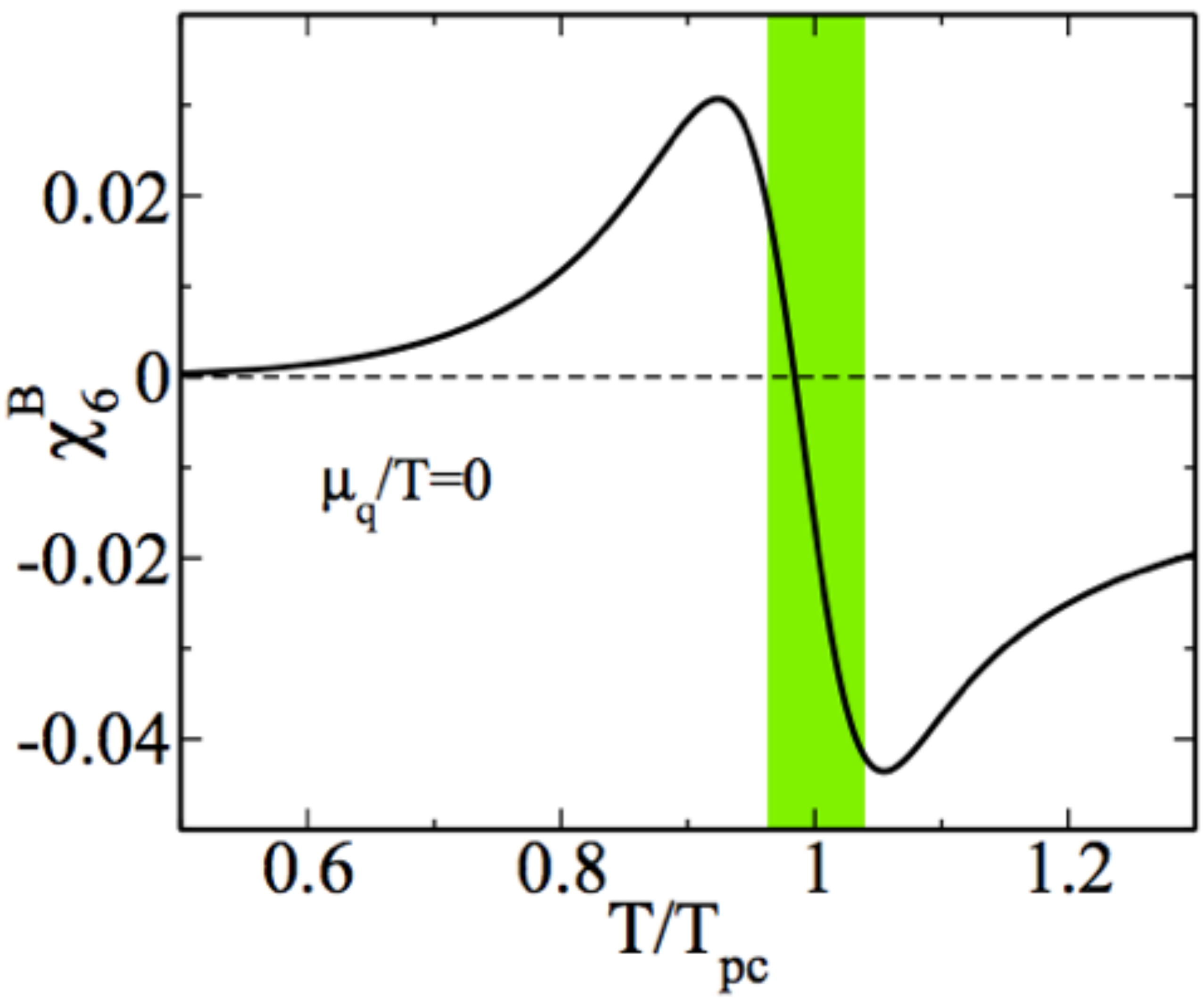


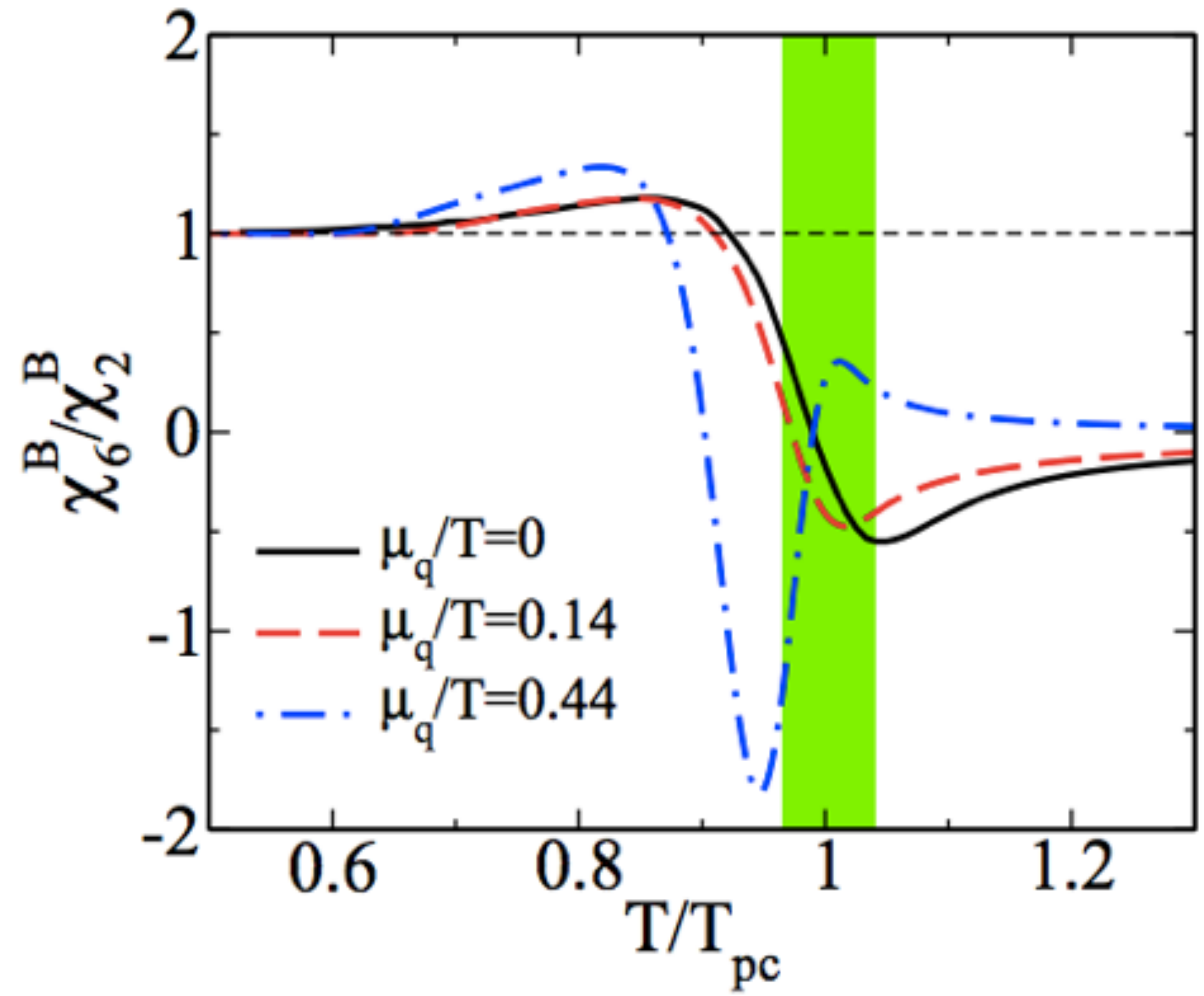


At the physical point, $U(1)_A$ does not restore at $T_{\chi SB}$~170 MeV, remains broken up to 195 MeV ~ $1.16T_{\chi SB}$

HotQCD, PRL 113 (2014) 082001, PRD 89 (2014) 054514

Baryon number fluctuations according to 3-d O(4) universality class





$$\chi_n^B \sim \begin{cases} -(2\kappa_q)^{n/2}|t|^{2-\alpha-n/2}f_{\pm}^{(n/2)} & , \text{ for } \mu_q/T = 0, \text{ and } n \text{ even} \\ -(2\kappa_q)^n \left(\frac{\mu_q}{T}\right)^n |t|^{2-\alpha-n} f_{\pm}^{(n)} & , \text{ for } \mu_q/T > 0\,, \end{cases}$$

3-d O(4) : $\boldsymbol{\alpha}$=-0.21

B. Friman et al., Eur.Phys.J. C71 (2011) 1694

Lattice QCD simulation at μ_B =/=0

fermion sign problem

QCD:

$$Z = \mathrm{Tr}\left[e^{-(H-\mu N)/T}\right] = \int [\mathrm{d}A]\, \det[D + m_q + i\mu\gamma_4]\, e^{-S(A)}$$

complex

Toy model: Yagi, Hatsuda & Miake, "Quark-Gluon Plasma — From Big Bang to Little Bang"

$$Z = \sum_{\{\phi(x)=\pm 1\}} \mathrm{sign}(\phi)\, e^{-S(\phi)}\,; \qquad Z_0 = \sum_{\{\phi(x)=\pm 1\}} e^{-S(\phi)}$$

$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O}(\phi)\, \mathrm{sign}(\phi) \rangle_0}{\langle \mathrm{sign}(\phi) \rangle_0}\,, \quad \langle \mathrm{sign}(\phi) \rangle_0 = \frac{Z}{Z_0} = e^{-(f-f_0)V/T} \ll 1$$

$f(f_0)$: free energy density corresponding to $Z(Z_0)$

$$\frac{\Delta \mathrm{sign}(\phi)}{\langle \mathrm{sign}(\phi) \rangle_0} = \frac{\sqrt{\langle \mathrm{sign}^2 \rangle_0 - \langle \mathrm{sign} \rangle_0^2}}{\sqrt{N} \langle \mathrm{sign} \rangle_0} \simeq \frac{e^{(f-f_0)V/T}}{\sqrt{N}} \ll 1$$

$\Rightarrow$

$$N \gg e^{2(f-f_0)V/T}$$

EoS at nonzero μB from LQCD

Taylor expansion of the QCD pressure:

Allton et al., Phys.Rev. D66 (2002) 074507
Gavai & Gupta et al., Phys.Rev. D68 (2003) 034506

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

- Taylor expansion coefficients at μ=0 are computable in LQCD

fluctuations of conserved charges:

$$\chi_{ijk}^{BQS} \equiv \chi_{ijk}^{BQS}(T) = \frac{1}{VT^3} \frac{\partial P(T, \hat{\mu})/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \Big|_{\hat{\mu}=0}$$

- Other quantities can be obtained using relations, e.g.

$$\frac{\epsilon - 3p}{T^4} = T \frac{\partial P/T^4}{\partial T} = \sum_{i,j,k=0}^{\infty} \frac{T\, \mathrm{d}\chi_{ijk}^{BQS}/\mathrm{d}T}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

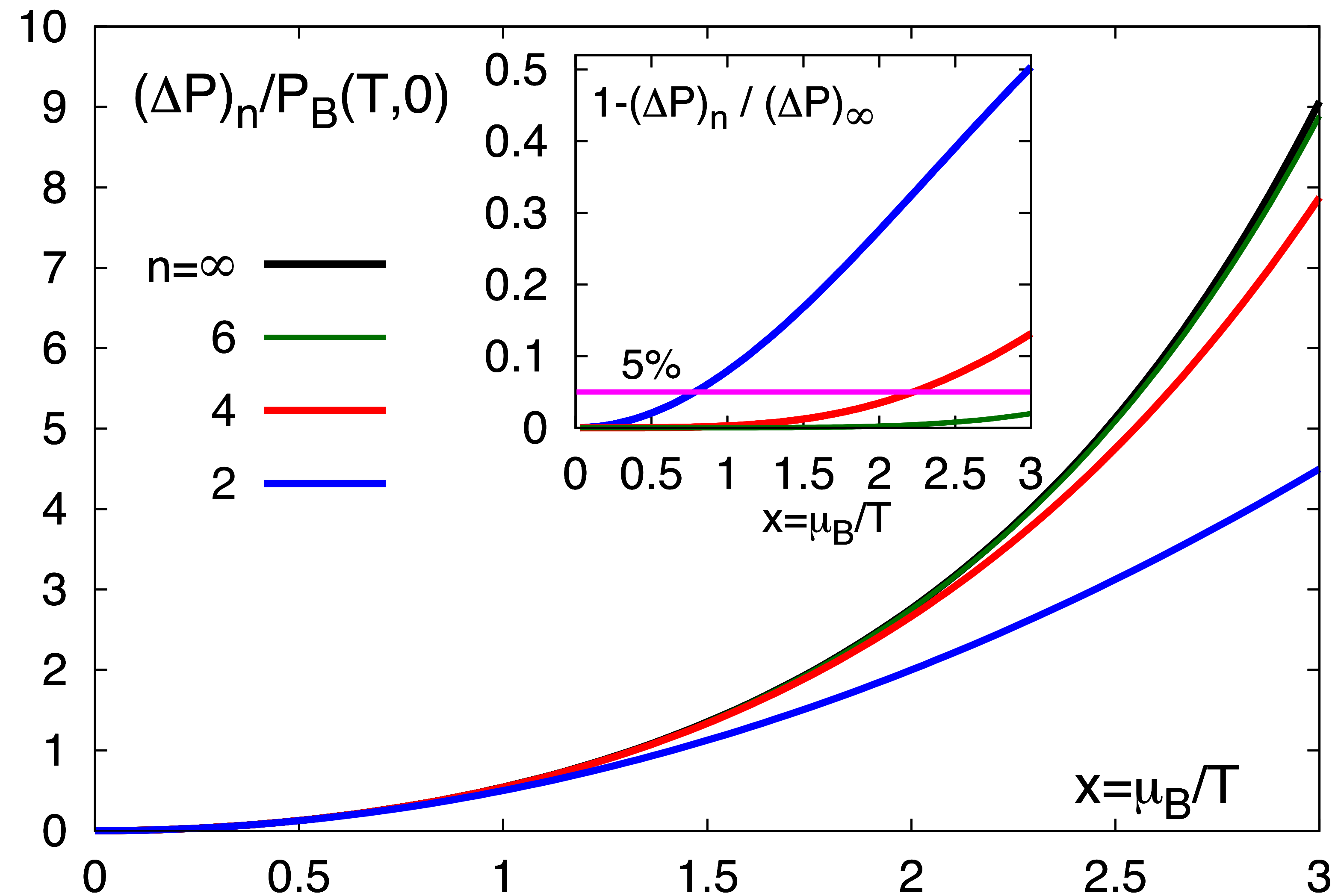
Truncation effects of pressure in HRG

Pressure of hadron resonance gas (HRG)

$$P(T,\mu_B) = P_M(T) + P_B(T,\hat{\mu}_B)$$
$$= P_M(T) + P_B(T,0) + P_B(T,0)(\cosh(\hat{\mu}_B) - 1)$$

Truncate the Taylor expansion at (2n)-th order:

$$(\Delta P)_n = \Big(P_B(T,\mu_B) - P_B(T,0)\Big)_n$$
$$= \sum_{k=1}^{n} \frac{\chi_{2k}^{B,HRG}(T)}{(2k)!}\hat{\mu}_B^{2k}$$
$$\simeq P_B(T,0)\sum_{i=1}^{n}\frac{1}{(2k)!}\hat{\mu}_B^{2k}$$



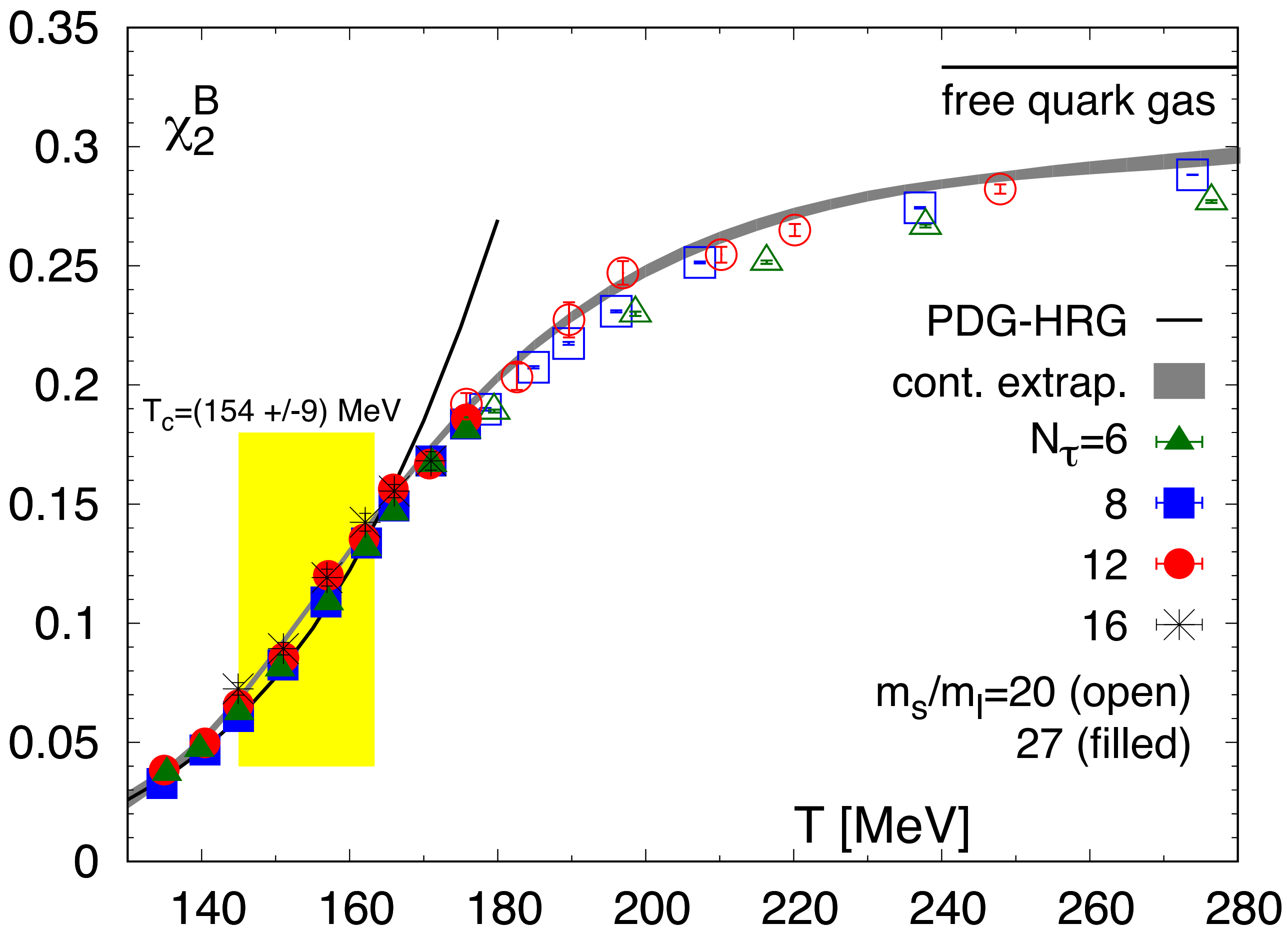
Radius of convergence from HRG is infinity

Pressure of QCD at μ_B =/=0

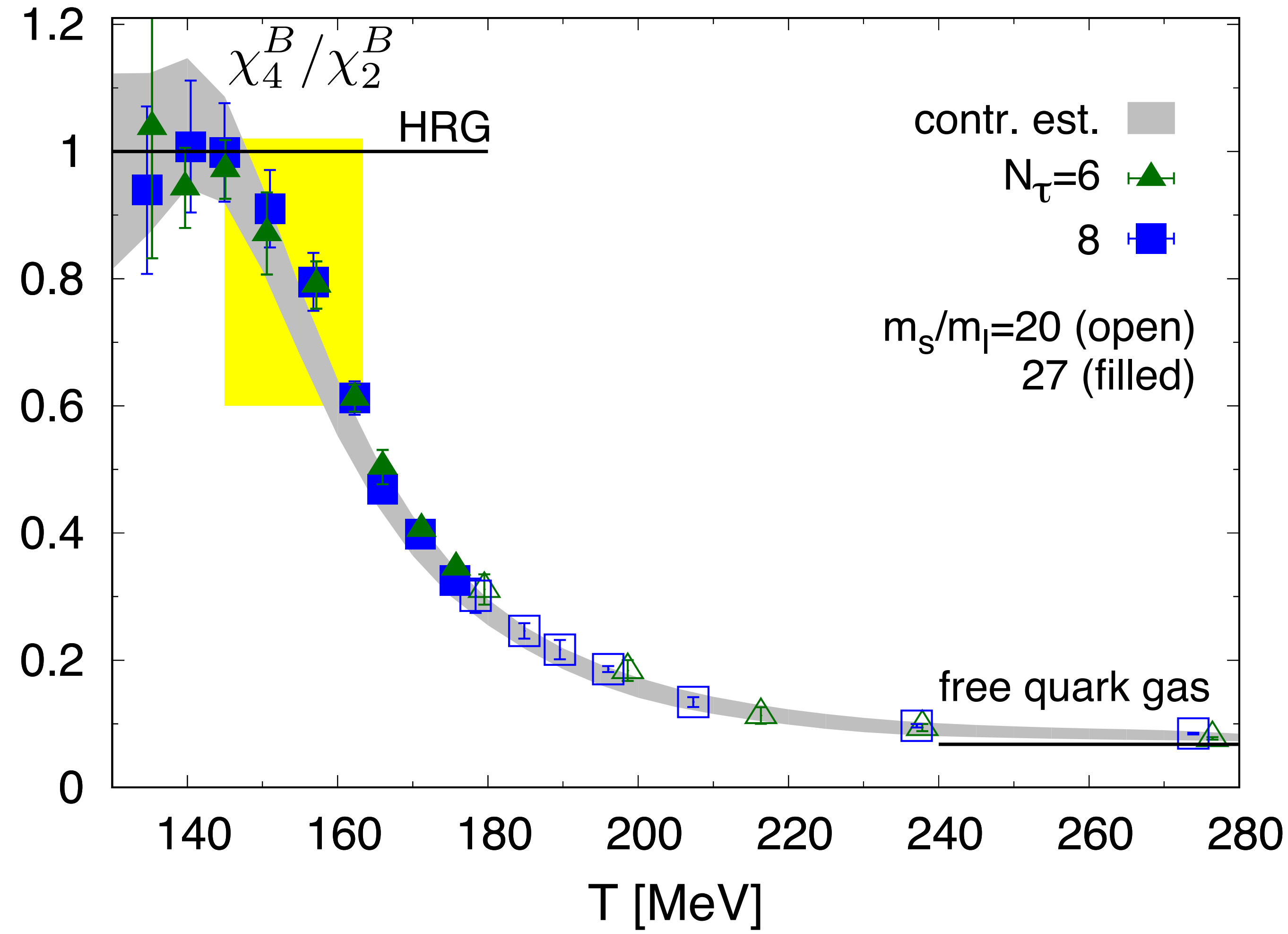
$\mu_Q=\mu_S=0$:

$$\Delta(P/T^4) = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n}$$
$$= \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2\left(1+\frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2+\frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4+\cdots\right)$$

LO expansion coefficient
variance of net-baryon number distri.



NLO expansion coefficient
kurtosis * variance



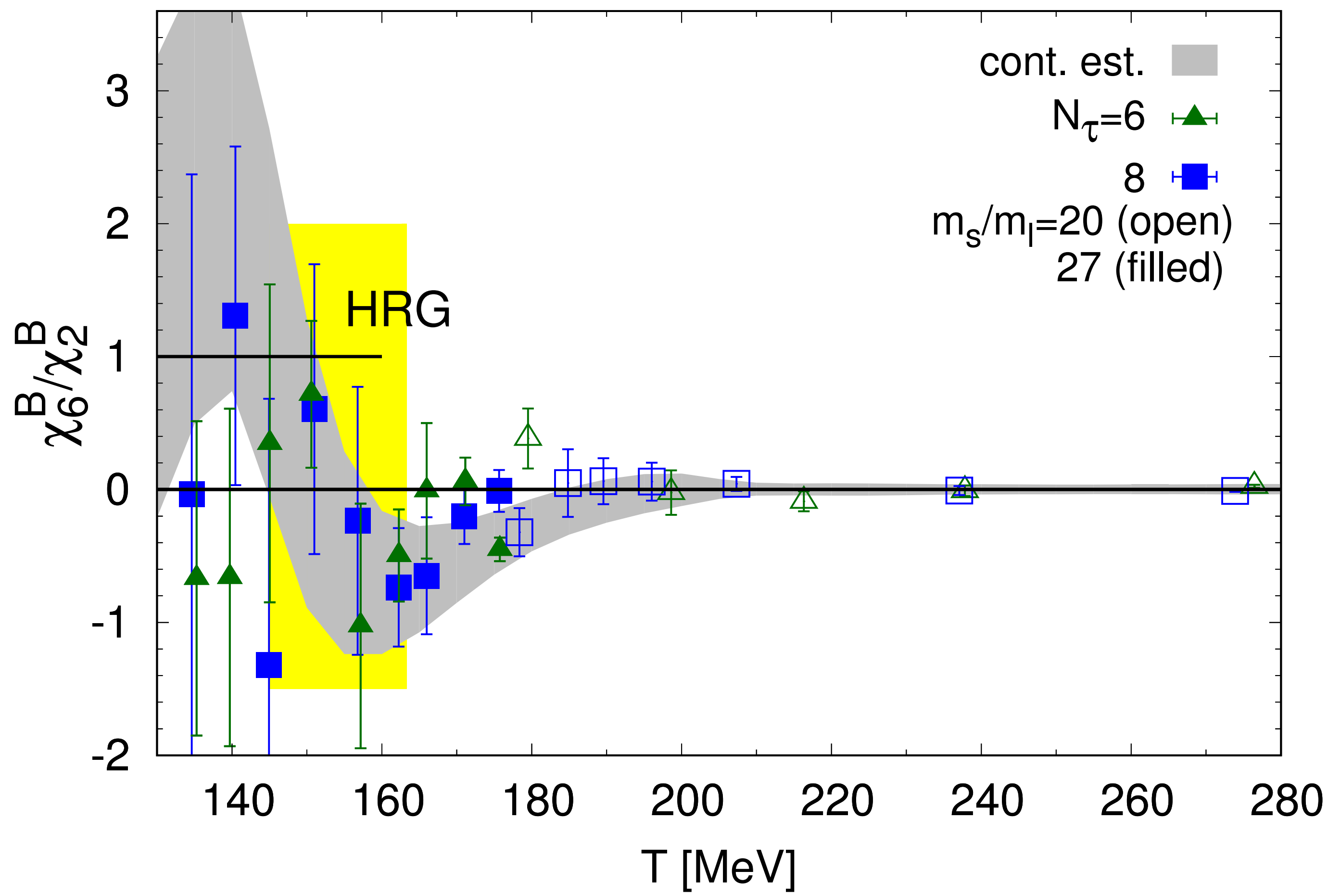
Bielefeld-BNL-CCNU, to appear soon

Pressure of QCD at $\mu_B =/=0$

$\mu_Q=\mu_S=0$:

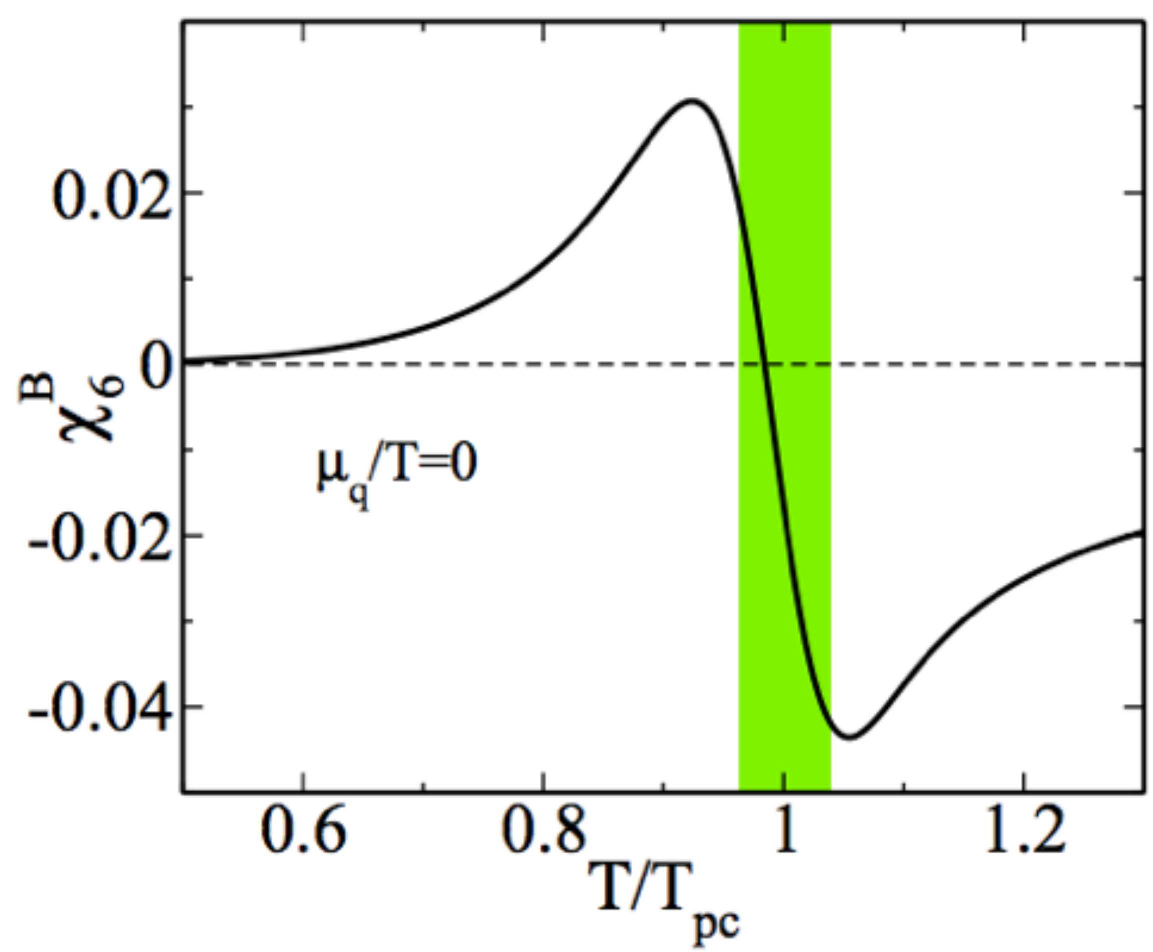
$$\Delta(P/T^4) = \frac{P(T,\mu_B)-P(T,0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n}$$
$$= \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2\left(1+\frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2+\frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4+\cdots\right)$$

NNLO expansion coefficient



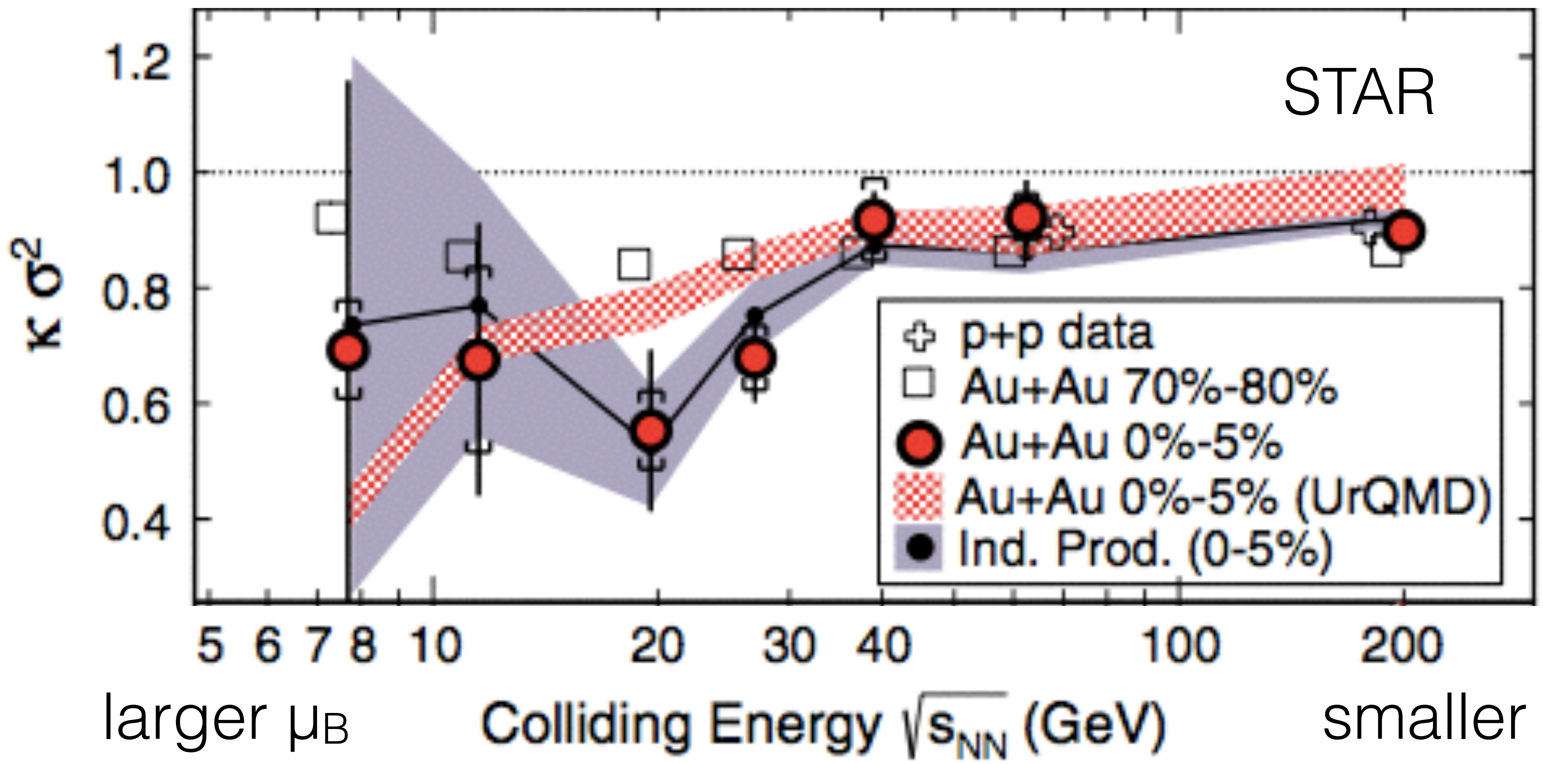
Bielefeld-BNL-CCNU, to appear soon

PQM with O(4) symmetry



B. Friman et al.,EPJC71 (2011) 1694

Cumulants of net proton number fluctuations



larger μ_B smaller μ_B

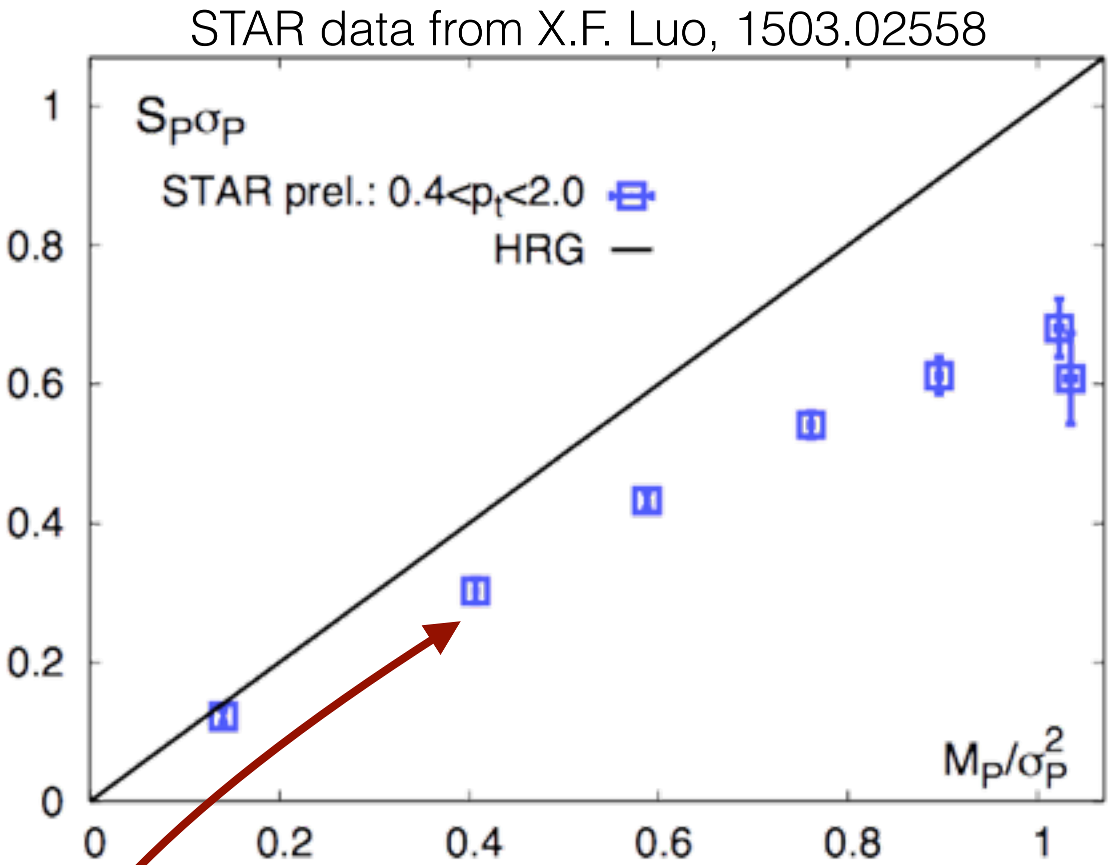
$$(\kappa\sigma^2)_B = \frac{\chi^B_{4,\mu}}{\chi^B_{2,\mu}} = \frac{\chi^B_4}{\chi^B_2}\left[1 + \left(\frac{\chi^B_6}{\chi^B_4} - \frac{\chi^B_4}{\chi^B_2}\right)\left(\frac{\mu_B}{T}\right)^2 + \cdots\right]$$

In HRG: $\chi^B_6/\chi^B_4 = \chi^B_4/\chi^B_2 = 1$

In the O(4) universality class and from LQCD data

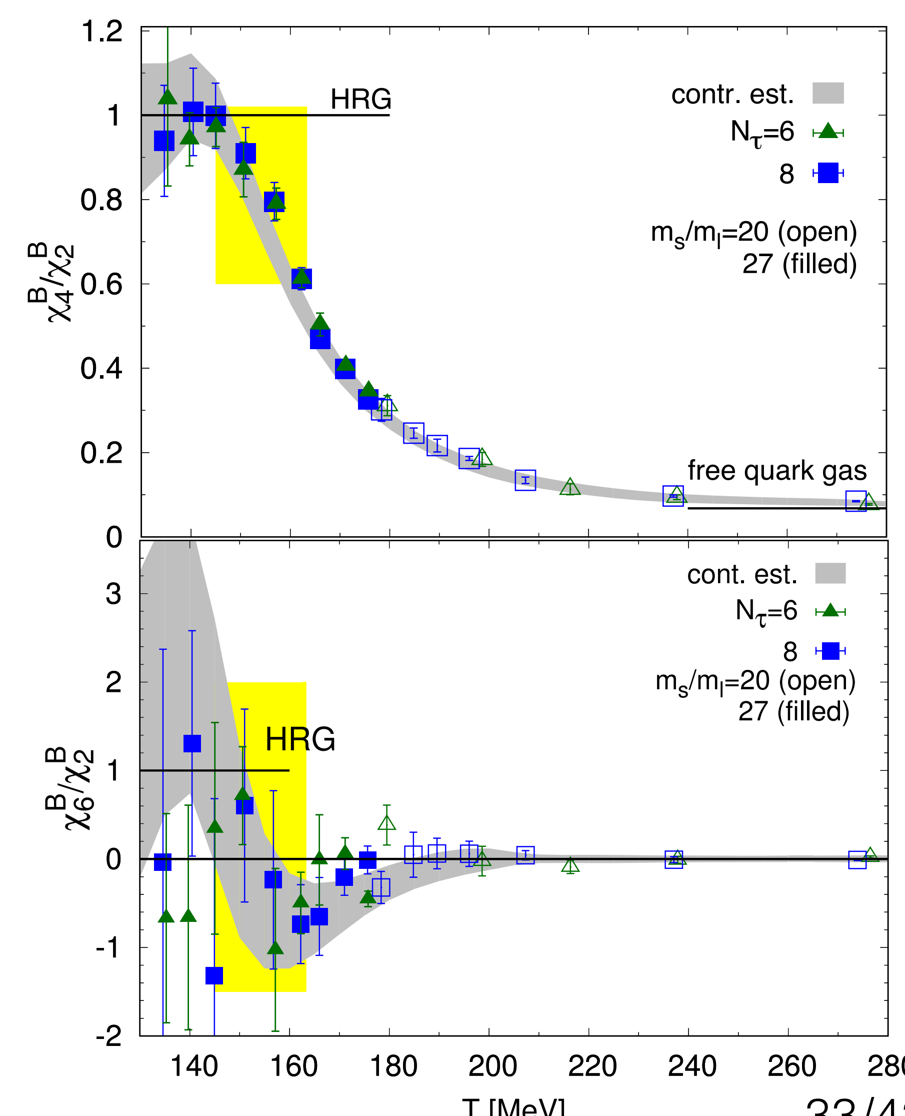
$$\chi^B_6 < 0\,, \qquad T \sim T_c$$

conserved charge fluctuations & freeze-out



$$S_B\sigma_B = \frac{\chi^B_{3,\mu}}{\chi^B_{2,\mu}} = \frac{\mu_B}{T}\frac{\chi^B_4}{\chi^B_2}\frac{1+\frac{1}{6}\frac{\chi^B_6}{\chi^B_4}(\frac{\mu_B}{T})^2+\cdots}{1+\frac{1}{2}\frac{\chi^B_4}{\chi^B_2}(\frac{\mu_B}{T})^2+\cdots}$$

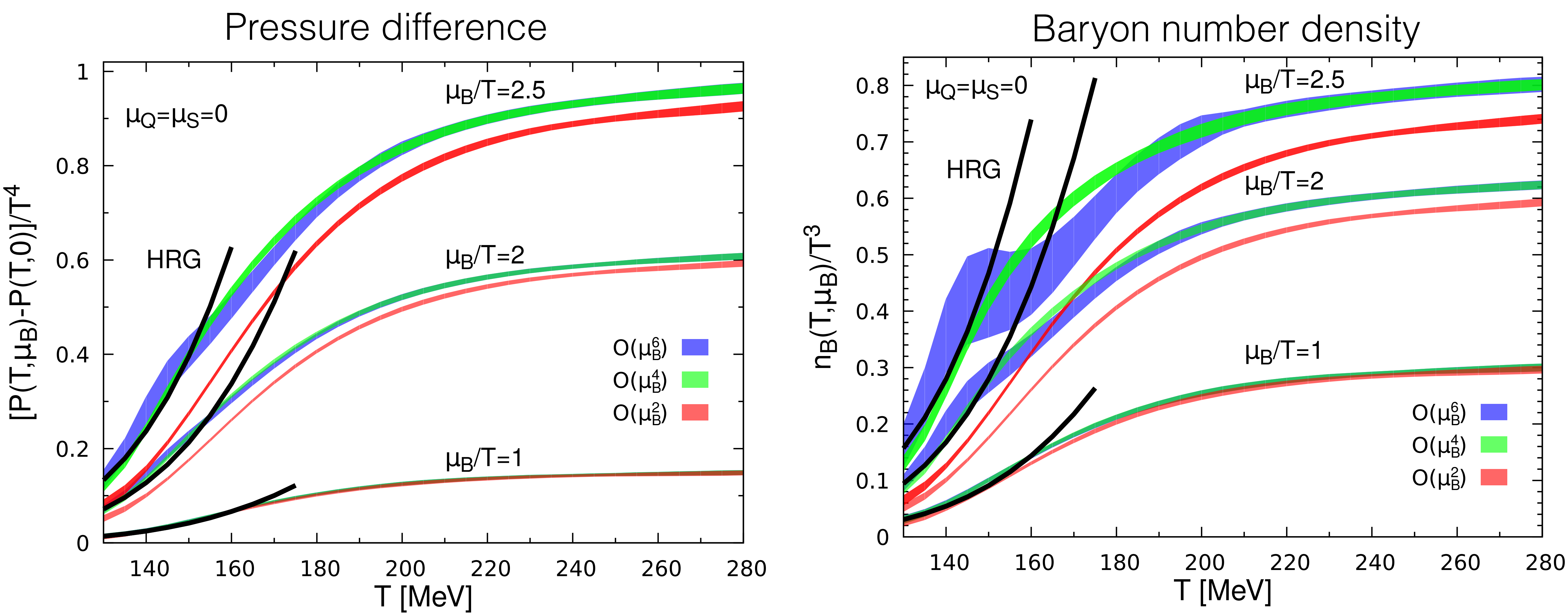
$$\frac{M_B}{\sigma^2_B} = \frac{\chi^B_{1,\mu}}{\chi^B_{2,\mu}} = \frac{\mu_B}{T}\frac{1+\frac{1}{6}\frac{\chi^B_4}{\chi^B_2}(\frac{\mu_B}{T})^2+\cdots}{1+\frac{1}{2}\frac{\chi^B_4}{\chi^B_2}(\frac{\mu_B}{T})^2+\cdots}$$



$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi^B_4}{\chi^B_2} + \mathcal{O}((\frac{\mu_B}{T})^3) = \frac{M_B}{\sigma^2_B}\frac{\chi^B_4}{\chi^B_2} + \mathcal{O}((\frac{\mu_B}{T})^3)$$

slope is smaller than 1 as $\frac{\chi^B_4}{\chi^B_2} < 1$

Pressure and baryon number density $\mu_Q=\mu_s=0$



Bielefeld-BNL-CCNU, to appear soon

$$\frac{P(T,\mu_B)-P(T,0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n)!}\left(\frac{\mu_B}{T}\right)^{2n} = \frac{1}{2}\chi_2^B(T)\hat{\mu}_B^2\left(1+\frac{1}{12}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2+\frac{1}{360}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4+\cdots\right)$$

$$\frac{n_B}{T^3} = \sum_{n=1}^{\infty} \frac{\chi_{2n}^B(T)}{(2n-1)!}\hat{\mu}_B^{2n-1} = \chi_2^B(T)\hat{\mu}_B\left(1+\frac{1}{6}\frac{\chi_4^B(T)}{\chi_2^B(T)}\hat{\mu}_B^2+\frac{1}{120}\frac{\chi_6^B(T)}{\chi_2^B(T)}\hat{\mu}_B^4+\ ...\right)$$

Conditions meet in heavy ion collisions

- Zero net strangeness $n_S=0$, and $n_Q/n_B=r=0.4$ as in PbPb collision systems

$$\frac{n_X}{T^3} = \frac{\partial P/T^4}{\partial \hat{\mu}_X} \quad , \text{X=B,Q,S}$$

,

$$n_S = n_S^{(1)}\mu_B + n_S^{(3)}\mu_B^3 + \cdots = 0\,, \quad n_Q = n_Q^{(1)}\mu_B + n_Q^{(3)}\mu_B^3 + \cdots$$

$$n_I = n_I^{(1)}\mu_B + n_I^{(3)}\mu_B^3 + \cdots = (\frac{1}{r} - 2)n_Q$$

E.g. 1st order coefficient in n_S: $$n_S^{(1)} = \chi_2^S \frac{\mu_S}{\mu_B} + \chi_{11}^{QS} \frac{\mu_Q}{\mu_B} + \chi_{11}^{BS}$$

μ_S, μ_Q and μ_B are correlated

Conditions meet in heavy ion collisions

Taylor expansion of the QCD pressure:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z}(T, V, \hat{\mu}_u, \hat{\mu}_d, \hat{\mu}_s) = \sum_{i,j,k=0}^{\infty} \frac{\chi_{ijk}^{BQS}}{i!j!k!} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$\mu_Q = \mu_S = 0$:

$$\frac{p}{T^4} = \sum_{n=0} \frac{\chi_{2n}^B}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

strangeness neutral case:

$$\frac{p}{T^4} = \sum_{n=0} \frac{\chi_{2n,\mathrm{SN}}^B}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

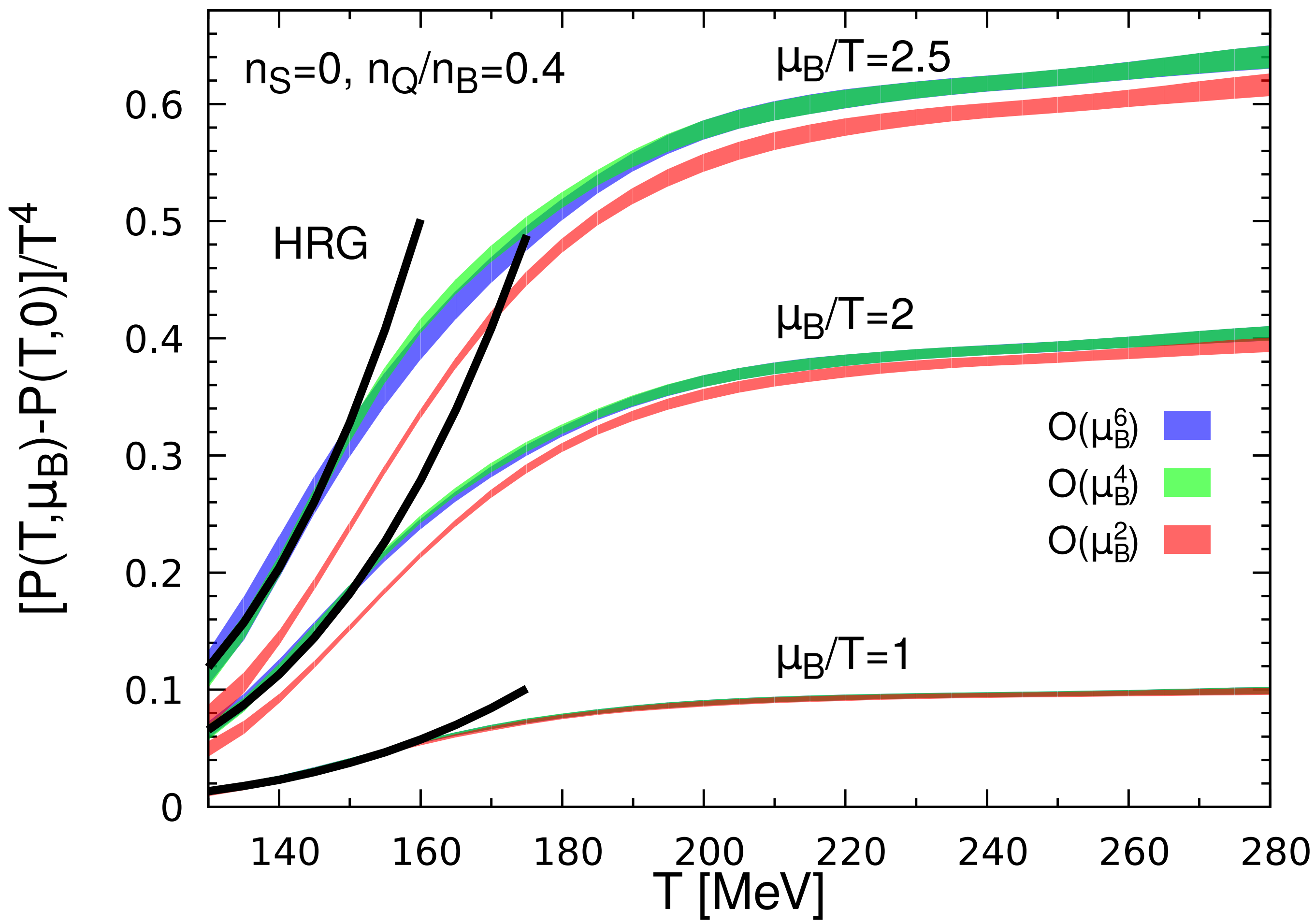
- Expand μ_Q and μ_S in terms of μ_B

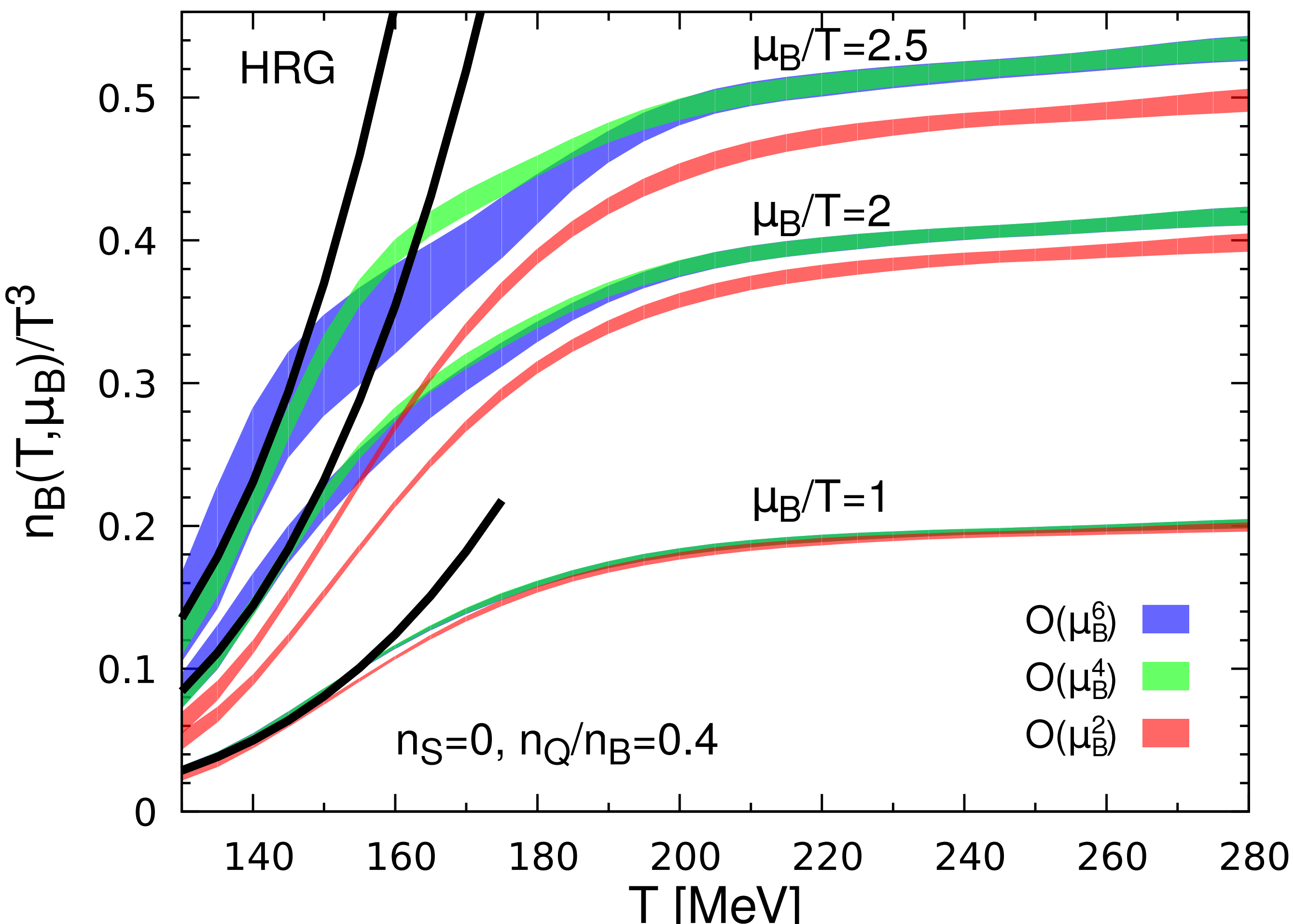
$$\frac{\mu_Q}{T} = q_1 \frac{\mu_B}{T} + q_3 \left(\frac{\mu_B}{T}\right)^3 + \cdots \; , \quad \frac{\mu_S}{T} = s_1 \frac{\mu_B}{T} + s_3 \left(\frac{\mu_B}{T}\right)^3 + \cdots$$

- With constrains from isospin symmetry etc., one can derive q_i and s_i order by order and then the pressure etc.

A. Bazavov, HTD et al., Phys. Rev. Lett. 109 (2012)192302

Pressure and baryon number density in the strangeness neutral case

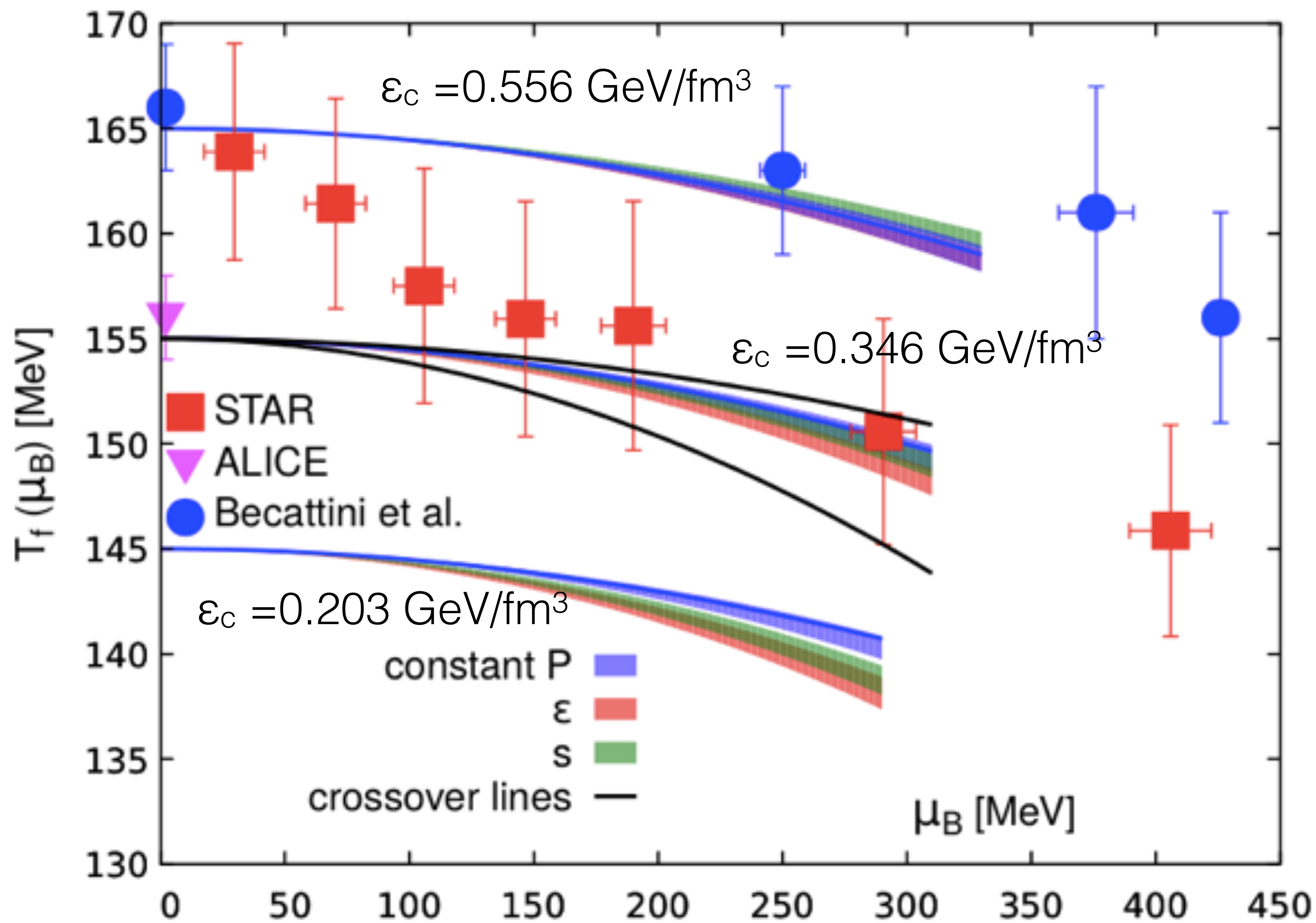




Bielefeld-BNL-CCNU, to appear soon

The EoS is well under control at $\mu_B/T \lesssim 2$ or $\sqrt{s_{NN}} \gtrsim 12$ GeV

Line of constant physics and freeze-out



Parameterization $T(\mu_B) = T(0)(1 - \kappa_2 \hat{\mu}_B^2 + \mathcal{O}(\hat{\mu}_B^4))$

curvature at constant pressure: $0.0064 \leq \kappa_2^P \leq 0.0101$

curvature at constant energy: $0.0087 \leq \kappa_2^\epsilon \leq 0.012$

curvature of the crossover line: $0.0074 \leq \kappa_2^s \leq 0.011$

Estimates of the radius of convergence

Taylor expansion of the pressure: $\frac{P}{T^4} = \sum_0^\infty \frac{1}{n!} \chi_n^B(T) \left(\frac{\mu_B}{T}\right)^n$

radius of convergence = $\lim_{n\to\infty} r_{2n}^{\chi,a} = \lim_{n\to\infty} r_{2n-2}^{\chi,b}$

$$r_{2n}^{\chi,a} = \left| \frac{2n(2n-1)\chi_{2n}^B}{\chi_{2n+2}^B} \right|^{1/2} , \qquad r_{2n-2}^{\chi,b} = \left| \frac{(2n)!\, \chi_2^B}{\chi_{2n}^B} \right|^{1/2n}$$

- The Radius of Convergence corresponds to a critical point only if all χ_n >0 for all n>n_0

This forces P/T^4 and $\chi_{n,\mu}^B$ grows monotonically with μ_B/T

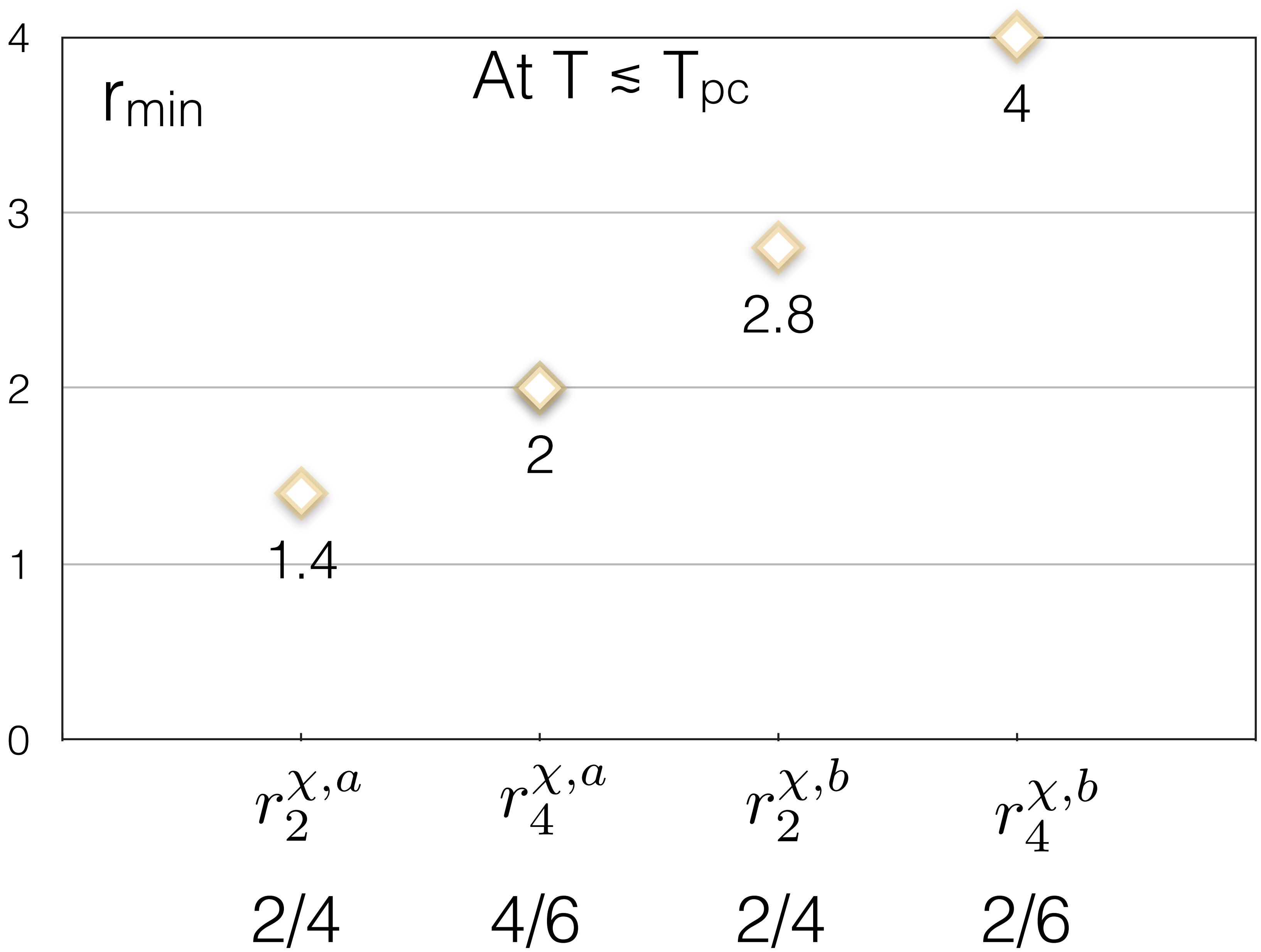
$$(\kappa\sigma^2)_B = \chi_{4,\mu}^B / \chi_{2,\mu}^B > 1$$

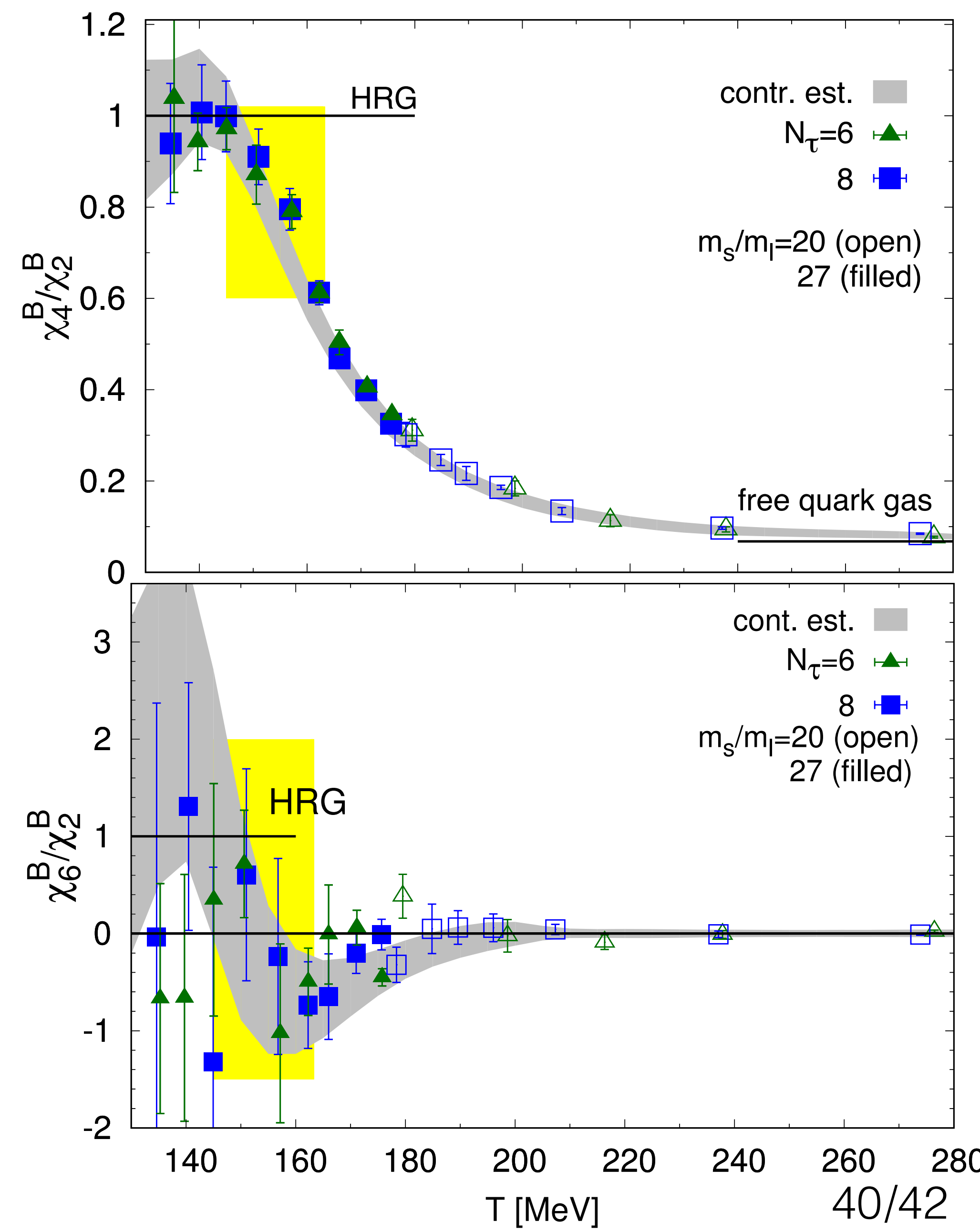
- Otherwise: 1) the ROC does not determine a critical point 2) Taylor expansion is not applicable near the critical point

Estimates of the radius of convergence

radius of convergence = $\lim_{n\to\infty} r_{2n}^{\chi,a} = \lim_{n\to\infty} r_{2n-2}^{\chi,b}$

$$r_{2n}^{\chi,a} = \left| \frac{2n(2n-1)\chi_{2n}^{B}}{\chi_{2n+2}^{B}} \right|^{1/2}, \quad r_{2n-2}^{\chi,b} = \left| \frac{(2n)!\,\chi_{2}^{B}}{\chi_{2n}^{B}} \right|^{1/2n}$$





Conclusions

- ☑ The 2nd O(4) chiral phase transition seems more relevant to the thermodynamics at the physical point at vanishing baryon density
- ☑ EoS from Taylor expansions of QCD partition functions are now reliable in the region $\mu_B/T \lesssim 2$ or $\sqrt{s_{NN}} \gtrsim 12$ GeV
- ☑ Properties of cumulants measured in BES-I for $\sqrt{s_{NN}} \gtrsim 20$ GeV clearly differs from HRG thermodynamics but are consistent to QCD thermodynamics close to the transition region
- ☑ A QCD critical point is strongly disfavored at $\mu_B/T \lesssim 2$

谢谢！

Thanks for your attention!