Short- and mid-distance physics (or quark-hadron duality) **from lattice QCD**

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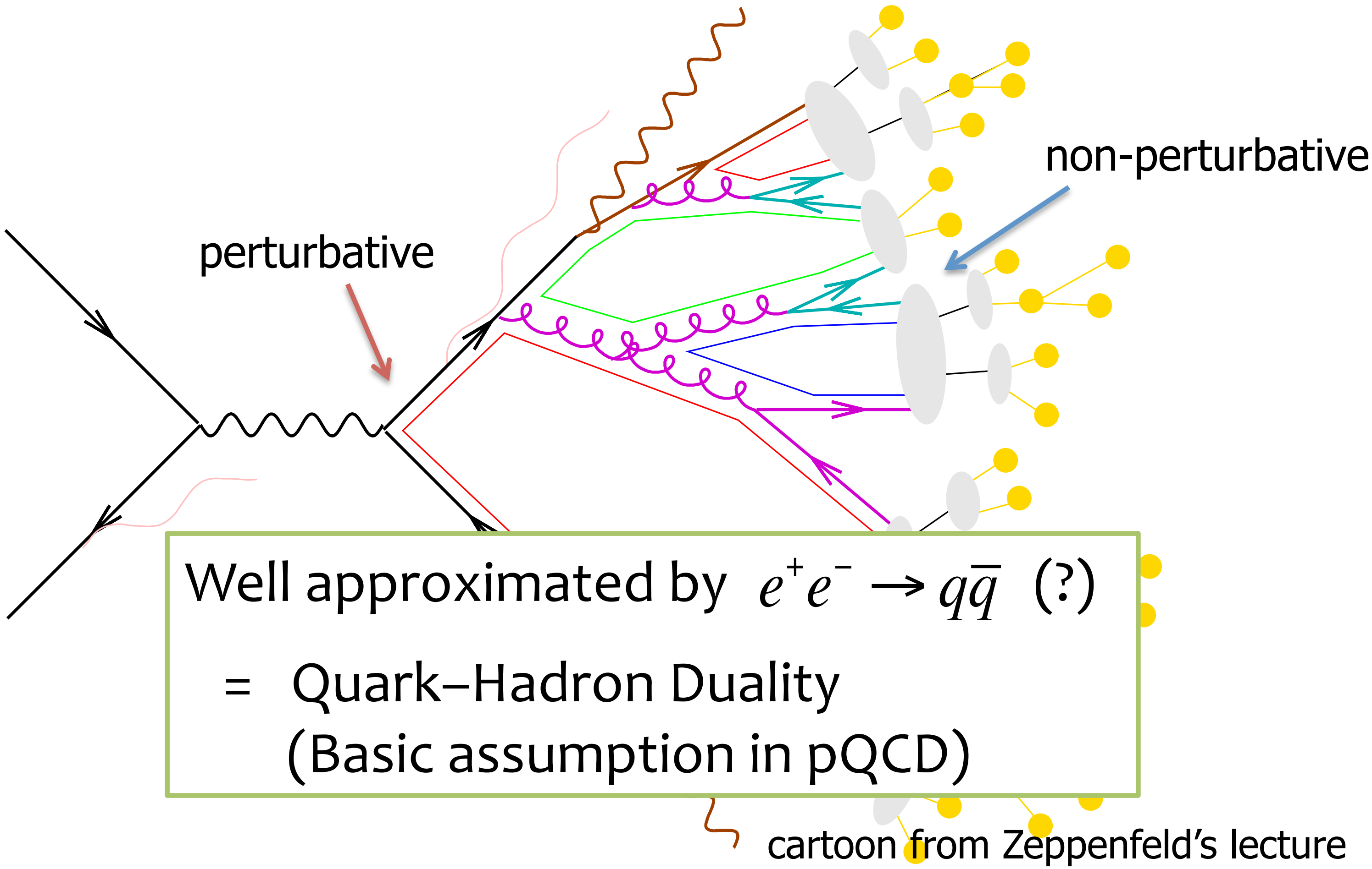
December 20, 2018

@ Peking University

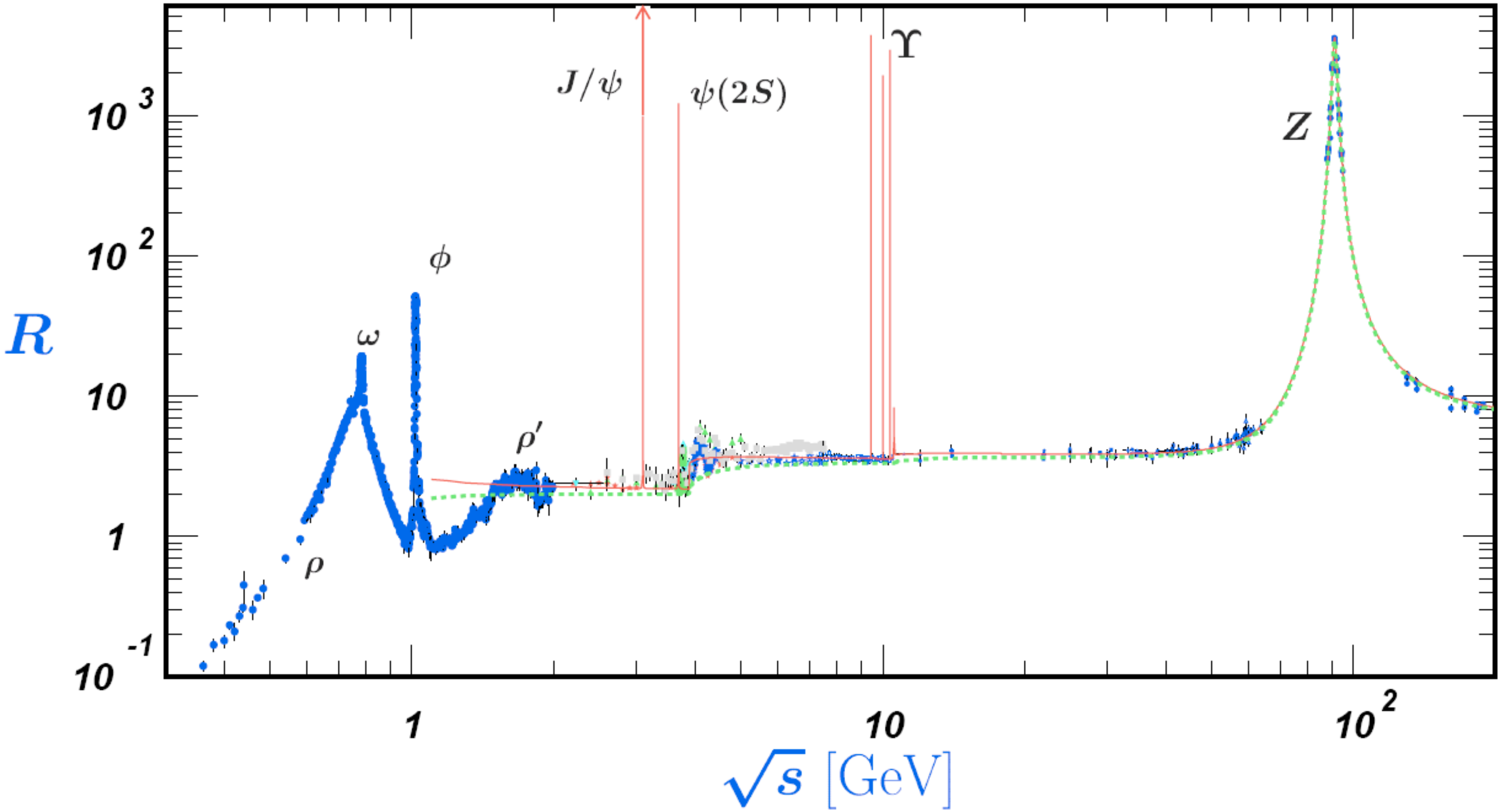
QCD is (non-)perturbative



Well approximated by $e^+e^- \to q\bar{q}$ (?)

= Quark–Hadron Duality
(Basic assumption in pQCD)

cartoon from Zeppenfeld's lecture


R
10^3
10^2
10
1
10^{-1}
1
10
10^2
$\sqrt{s}$ [GeV]
ρ
ω
ϕ
ρ'
J/ψ
$\psi(2S)$
Υ
Z

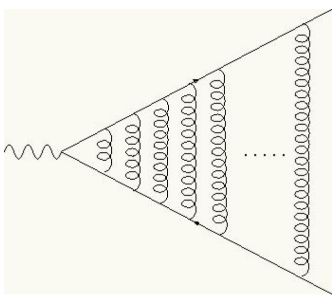
Quark-hadron duality

- Believed to be well satisfied, when
  - the process is sufficiently inclusive (How much? See below.)

- Working hypothesis in most of perturbative QCD analyses
  - How much tested?
  - How to estimate the error? Quantitatively??

Duality is badly violated…

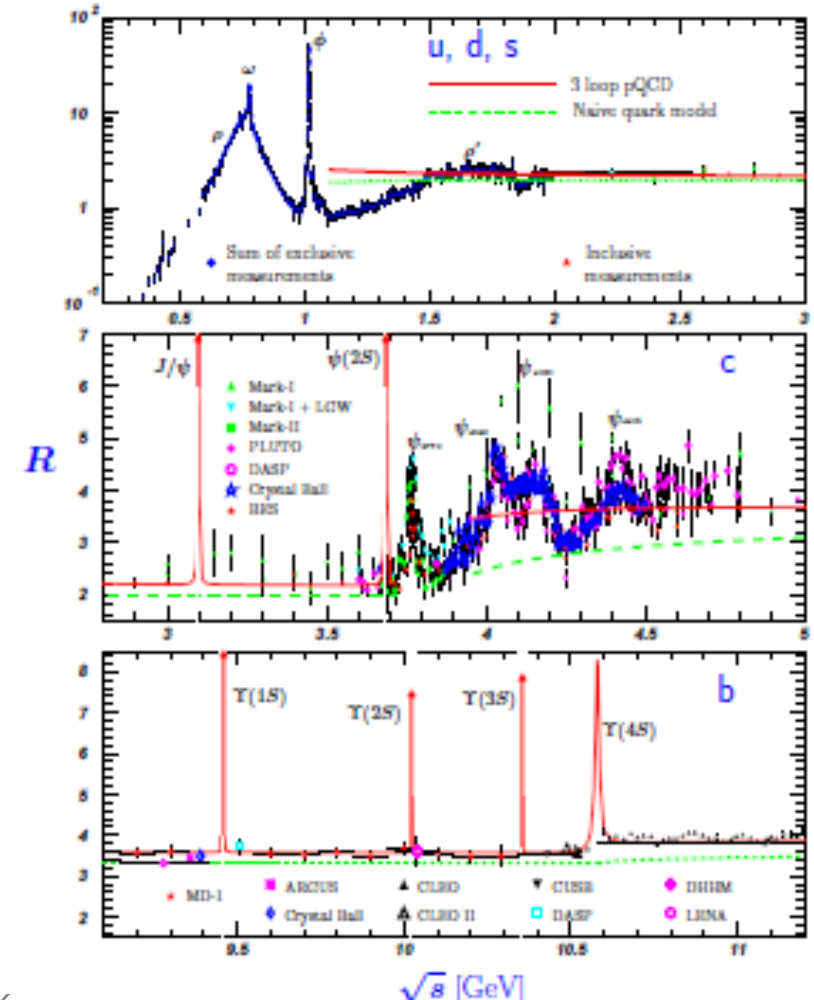
- A lot of resonances found in the e^+e^- collisions
  – Highly non-perturbative even for quarkonium:



$$\sim \left(\frac{\alpha_s}{v}\right)^n$$

(need to resum)

  – Even harder for light sectors



Duality at work

Poggio, Quinn, Weinberg, PRD13, 1958 (1976)

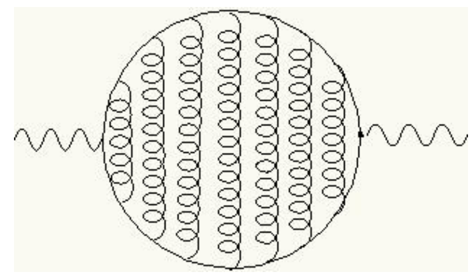
- Smearing
  - Consider a quantity smeared over some range.

$$\bar{R}(s,\Delta) \equiv \frac{\Delta}{\pi}\int_0^\infty ds' \frac{R(s')}{(s-s')^2+\Delta^2}$$

$$= \frac{1}{2\pi i}\int_0^\infty ds' R(s')\left(\frac{1}{s-s'+i\Delta} - \frac{1}{s-s'-i\Delta}\right)$$

$$= \frac{1}{2i}\left[\Pi(s+i\Delta) - \Pi(s-i\Delta)\right]$$

$$\mathrm{Im}\Pi(s) \propto R(s) = \frac{\sigma(e^+e^- \to q\bar{q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



  - One can avoid the threshold singularity.
  - Δ must be larger than Λ_{QCD} to avoid non-perturbative physics, but how much?

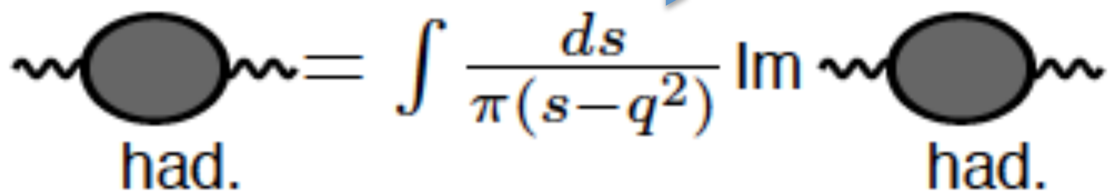
Still qualitative…

Quantitative solution

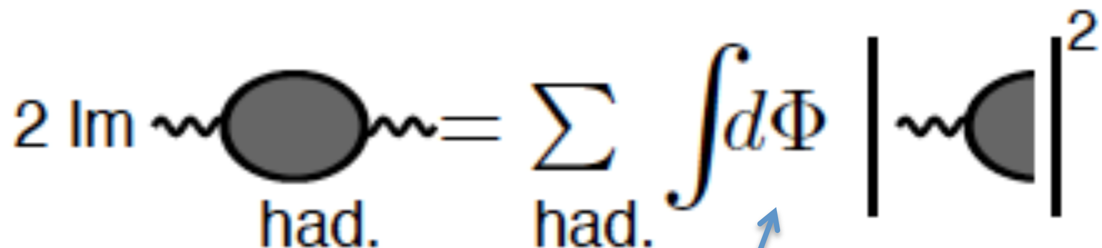
= smear according to Cauchy…

Weighted integral over momentum transfer

Dispersion relation:



Optical theorem:



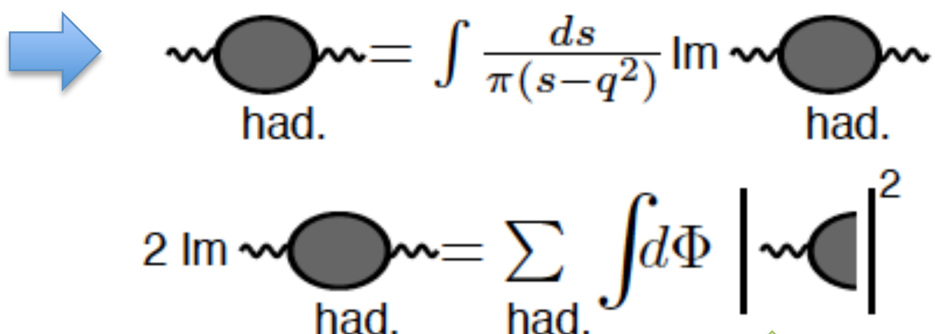
all possible final states

“QCD sum rule”

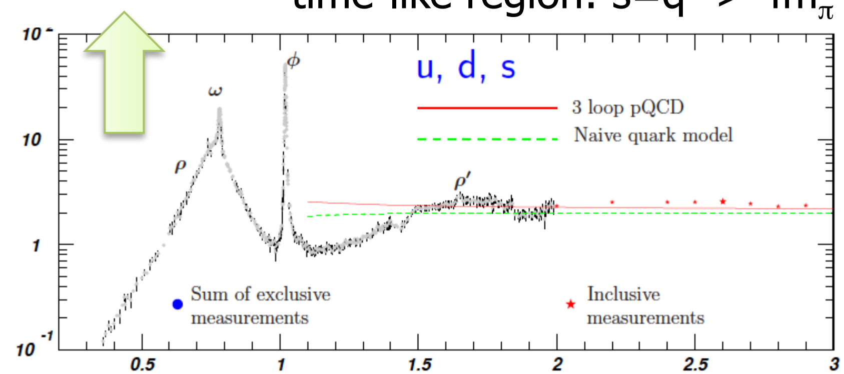
Shifman, Vainshtein, Zakharov, NPB147 385, 448 (1979)

$\Pi(Q^2)$: calculable by pQCD and OPE (+ Borel sum, etc)

space-like region: $Q^2 = -q^2 > 0$



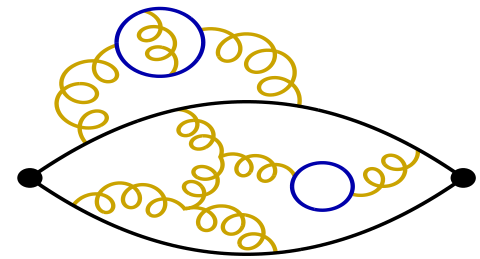
time-like region: $s = q^2 > 4m_\pi^2$



$\Pi(Q^2)$: Why not lattice?

Well, it's surely possible!

$$\Pi_{\mu\nu}(x) = \langle 0 | \not{T} \{ J_\mu(x) J_\nu(0) \} | 0 \rangle$$



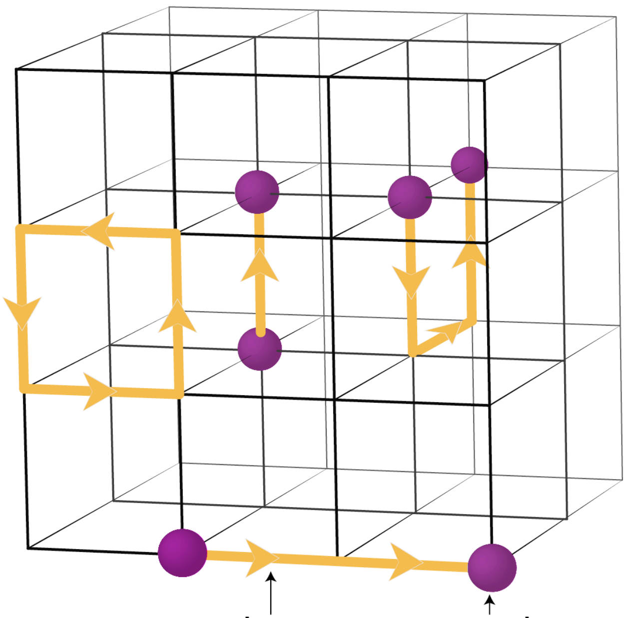
x and 0 are separated space-like

- Calculation on an Euclidean lattice naturally provides this.
- A bread-and-butter calculation.

- Input for hadronic vacuum polarization (HVP) for muon g-2.

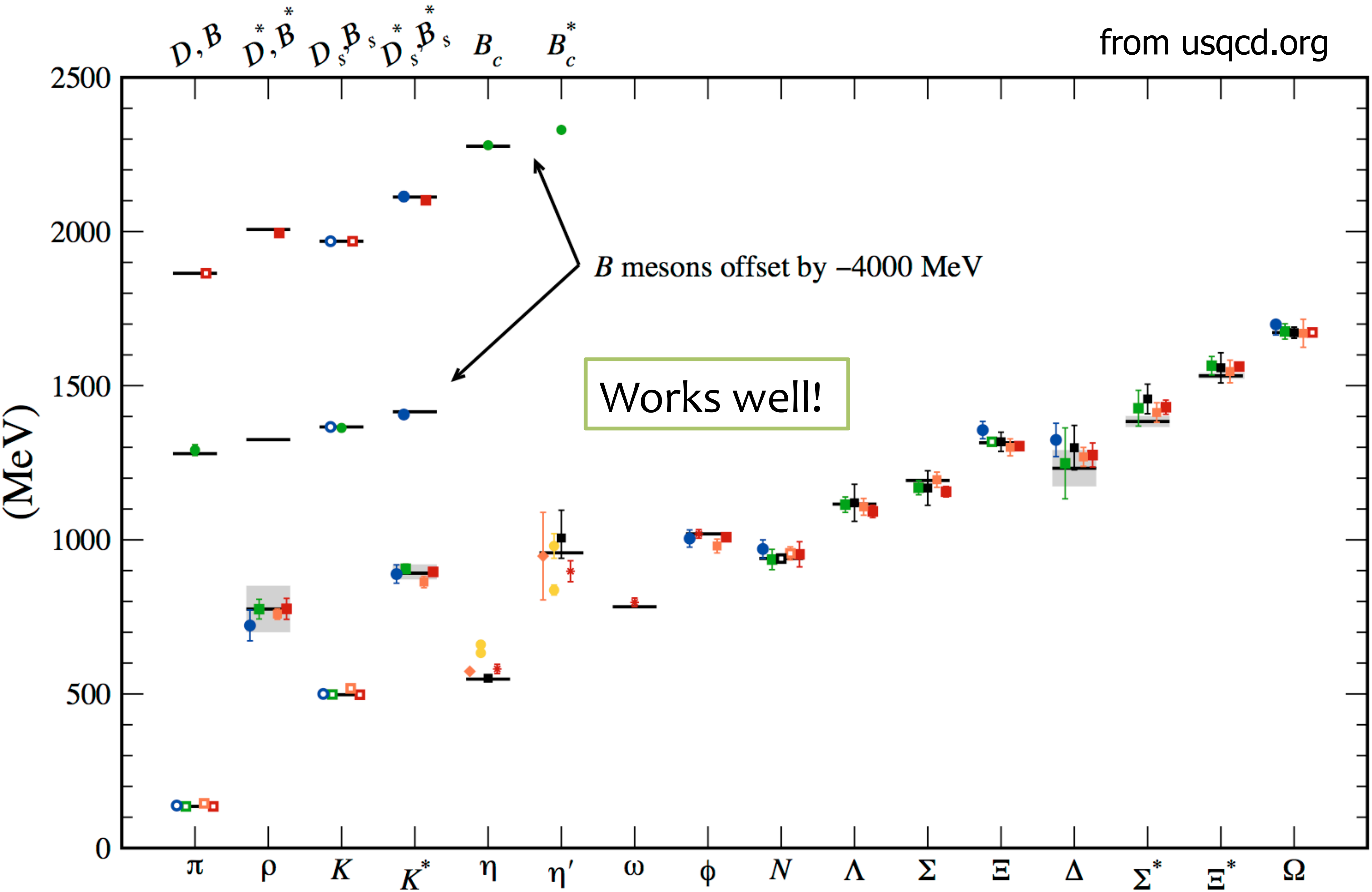
Euclidean Lattice QCD

- LQCD = ab initio calculation of QCD



- Define the quark and gluon fields on the **Euclidean** lattice.
- Perform the path integral numerically (Monte Carlo).

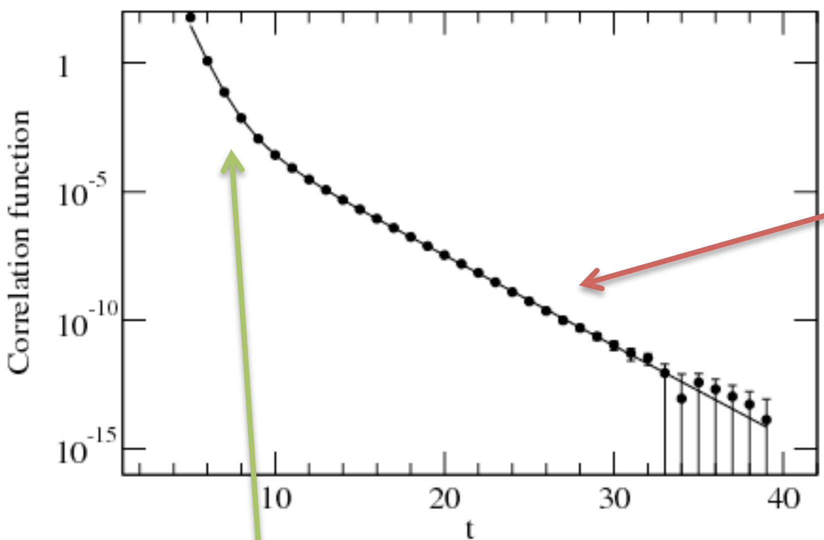
from usqcd.org



Euclidean correlator

- e^{-Et} instead of e^{-iEt}
  - Hadron correlator

$$\int d^3\mathbf{x}\langle\mathcal{O}(\mathbf{x},t)\mathcal{O}^\dagger(0)\rangle$$



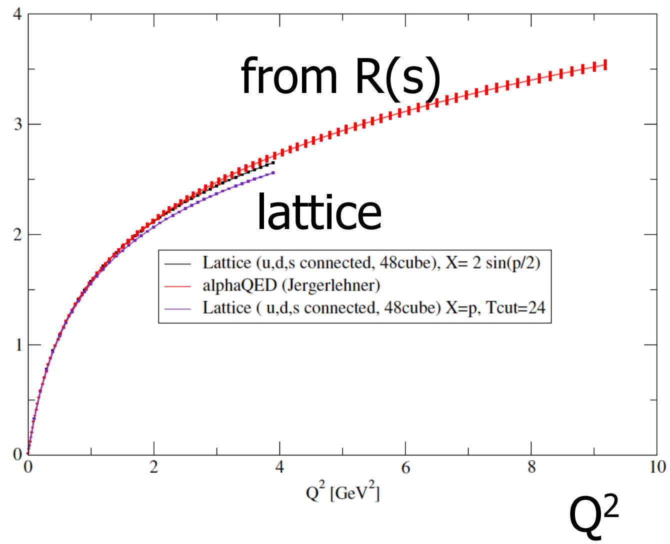
read off the exponential slope at long distances
→ hadron energy (or mass)

More physics info included in the short-distance region.

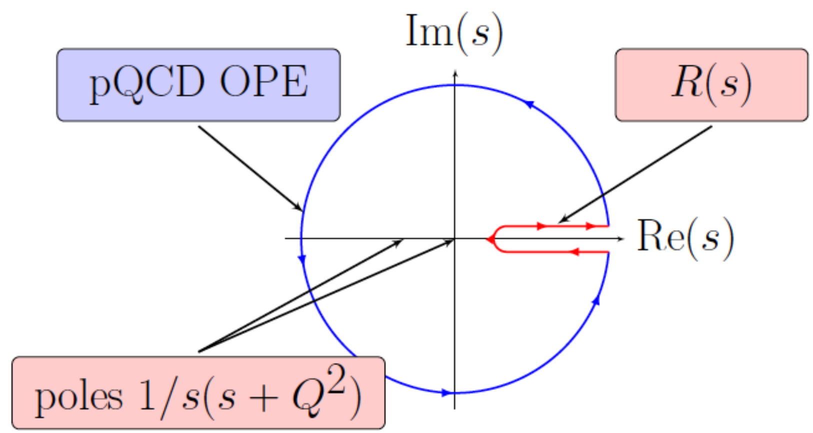
Fourier transform (in 4D)

- Produces the space-like $\Pi(Q^2)$:

RBC/UKQCD:
Izubuchi @ g-2 WS (2017)

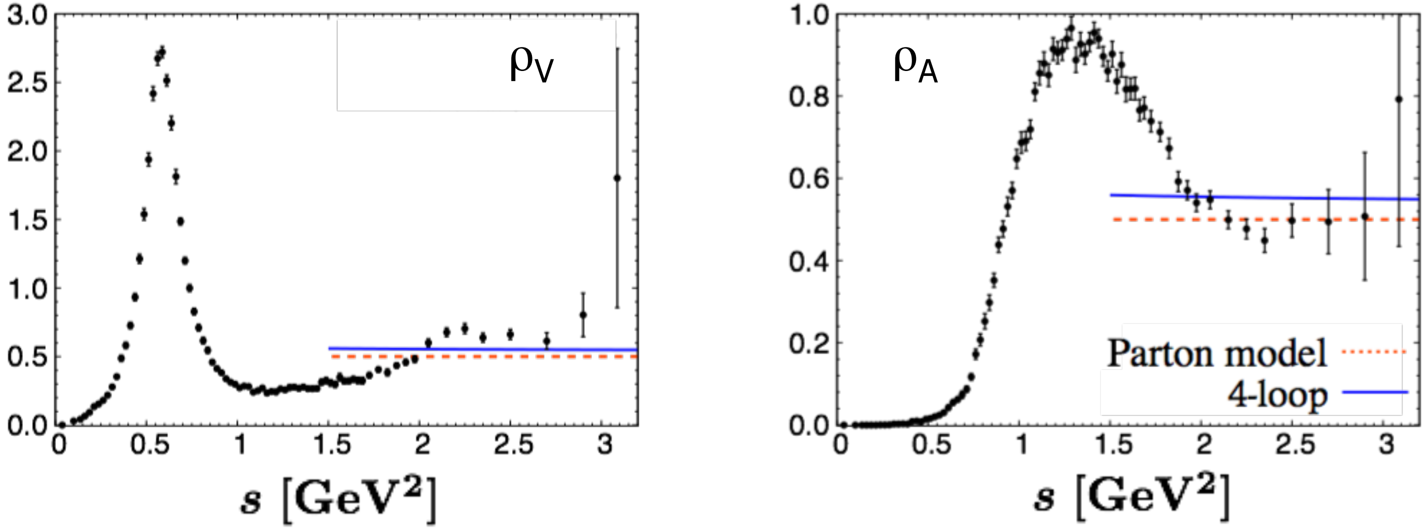


$$\hat{\Pi}(Q^2) = Q^2 \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$



More detailed analyses

- Spectral function $\rho(s) \propto \mathrm{Im}\Pi(s)$
  - Experimental data available for I=1 VV and AA from hadronic τ decays (ALEPH, 2013/14)



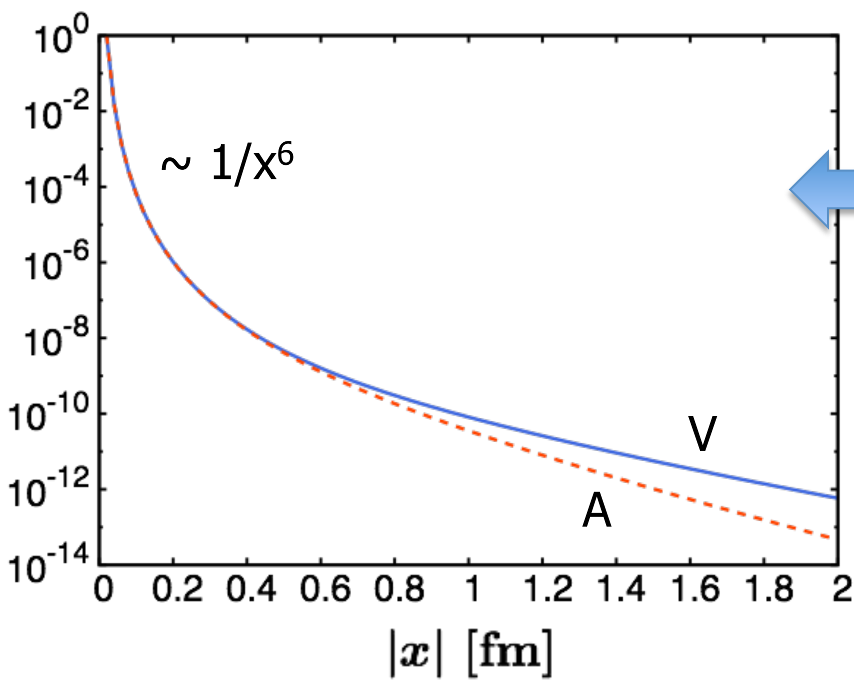
  - Resonances at low energy; perturbative at high energy

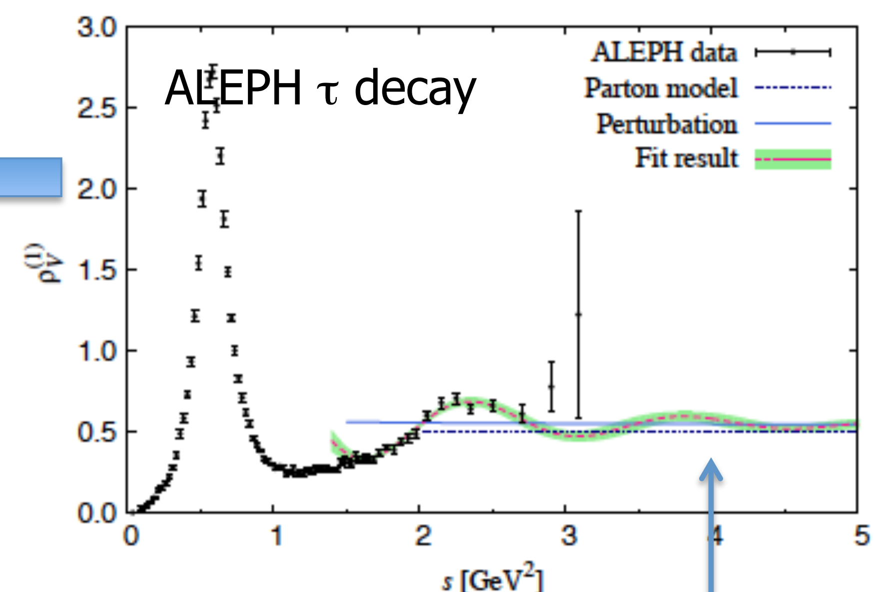
On the Euclidean space…

- "Fourier trans" from the ALEPH data for τ decay

$$\Pi(x) \propto \int_0^\infty ds\, s^{3/2} \rho(s) \frac{K_1(\sqrt{s}|x|)}{|x|}$$

Schafer, Shuryak, 2001





supplemented by pert at high q^2

On the Euclidean space…

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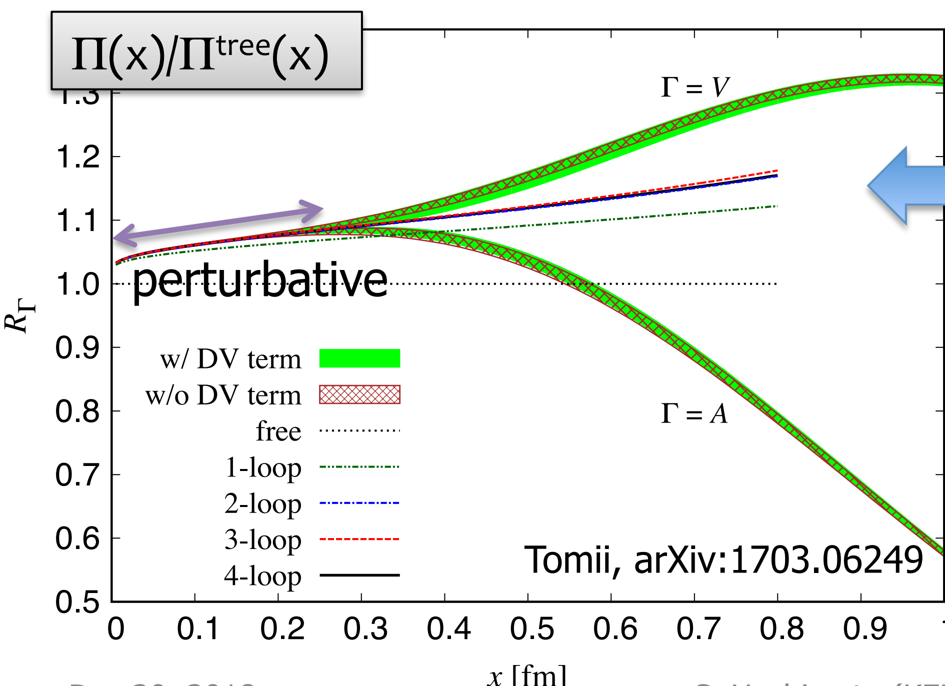
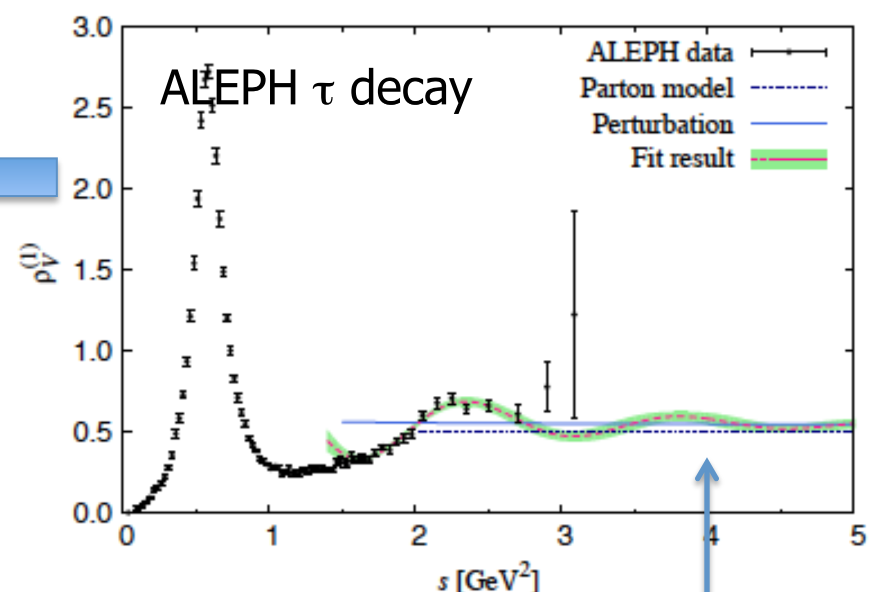


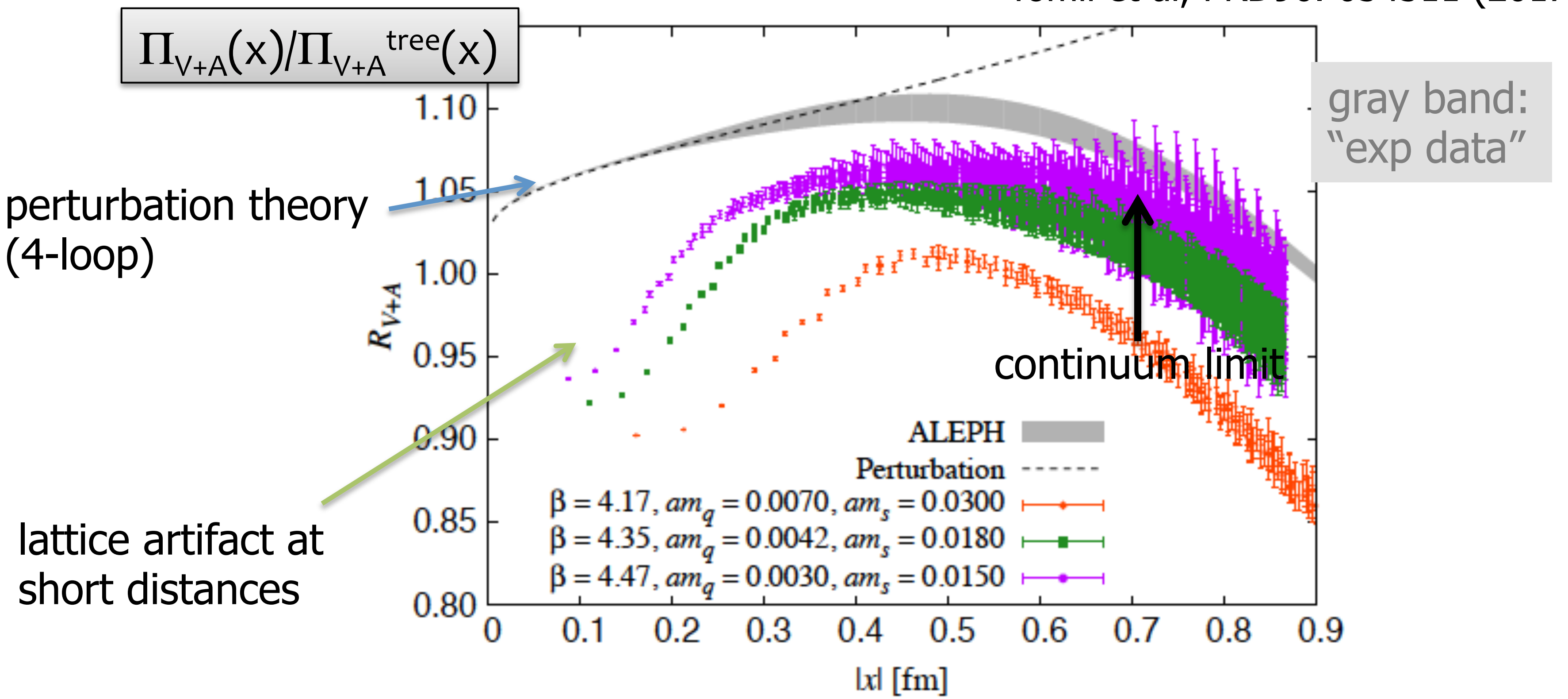


supplemented by pert at high q^2

Lattice ⇔ Experiment

ex) light-hadron VPF (in the coordinate space)

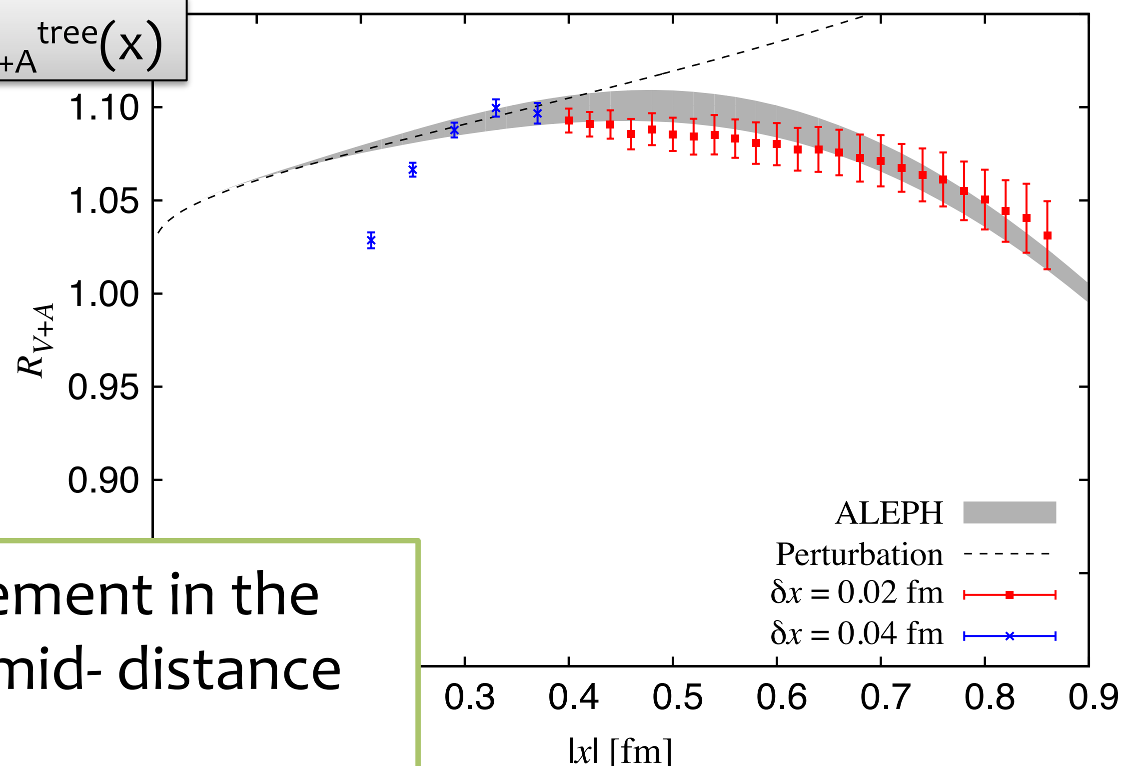
Tomii et al, PRD96. 054511 (2017)



Lattice ⇔ Experiment

Tomii et al, PRD96. 054511 (2017)

$\Pi_{V+A}(x)/\Pi_{V+A}^{tree}(x)$



Good agreement in the short- and mid- distance region.

Looks nice …

- No assumption is involved (so far)?
  - Other than, analyticity & optical theorem
  - Smearing (sum over final state) is achieved by the Cauchy integral. Still an exact relation.
  - Lattice friendly quantity (space-like).

= No issue of quark-hadron duality any more!
  - Caveat: sensitivity to individual resonances/cuts/etc is lost. (This is what the duality meant.)
  - Testing ground of the OPE analyses

OPE

Consider in the coordinate space: no extra divergences

$$\Pi(x) = \frac{c_0(\alpha_s)}{x^6} + \frac{c_2 m_q^2}{x^4} + \frac{c_{4,\bar{q}q} m_q \langle \bar{q}q \rangle + c_{4,G} \langle GG \rangle + \cdots}{x^2} + \cdots$$

Perturbative expansion at m=0; sometimes, known to α_s^4

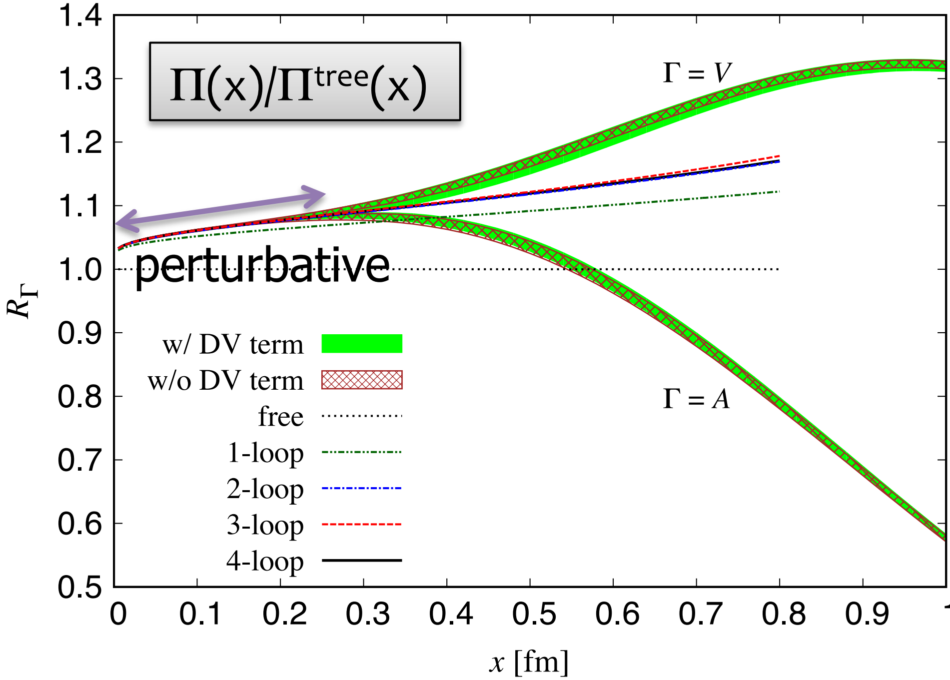
Finite m correction in PT; typically tiny

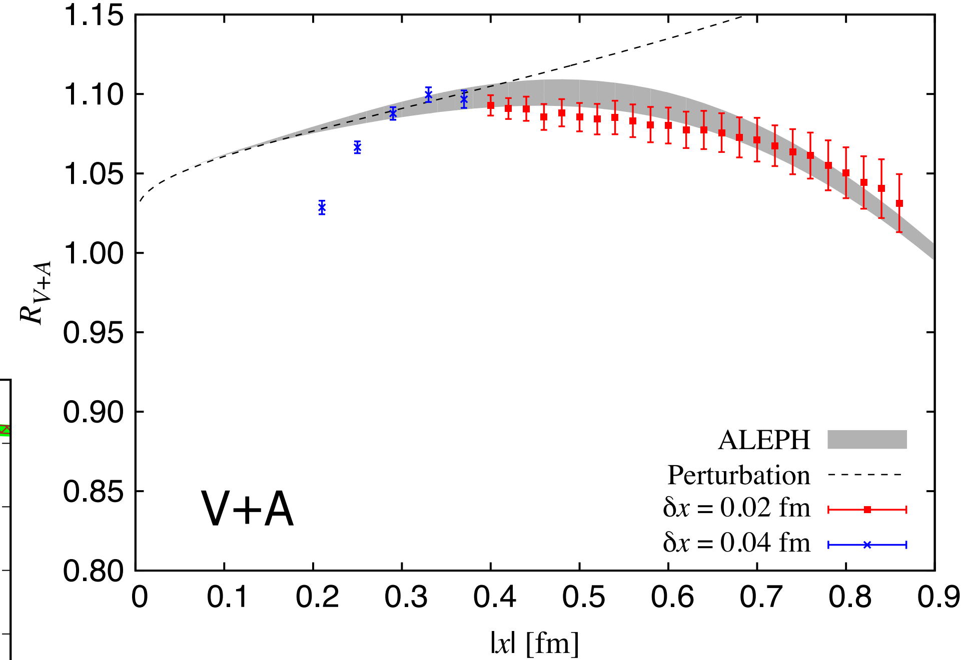
Non-perturbative corrections represented by condensates

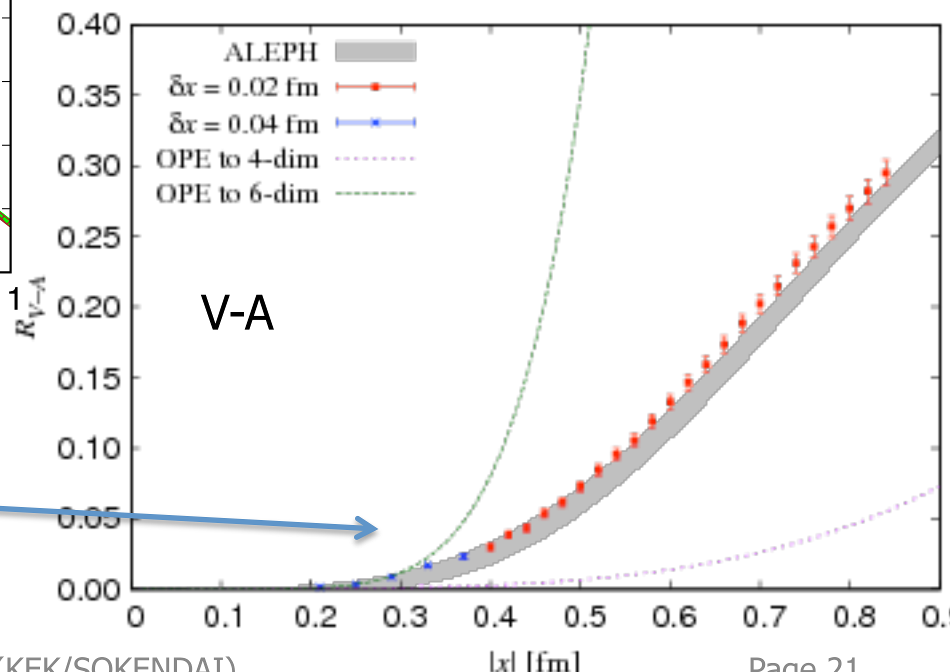
Consistent?
LHS: Directly calculable in LQCD.
RHS: perturbative and power expansions.

Tomii et al, PRD96. 054511 (2017)

Ratio to the tree-level:





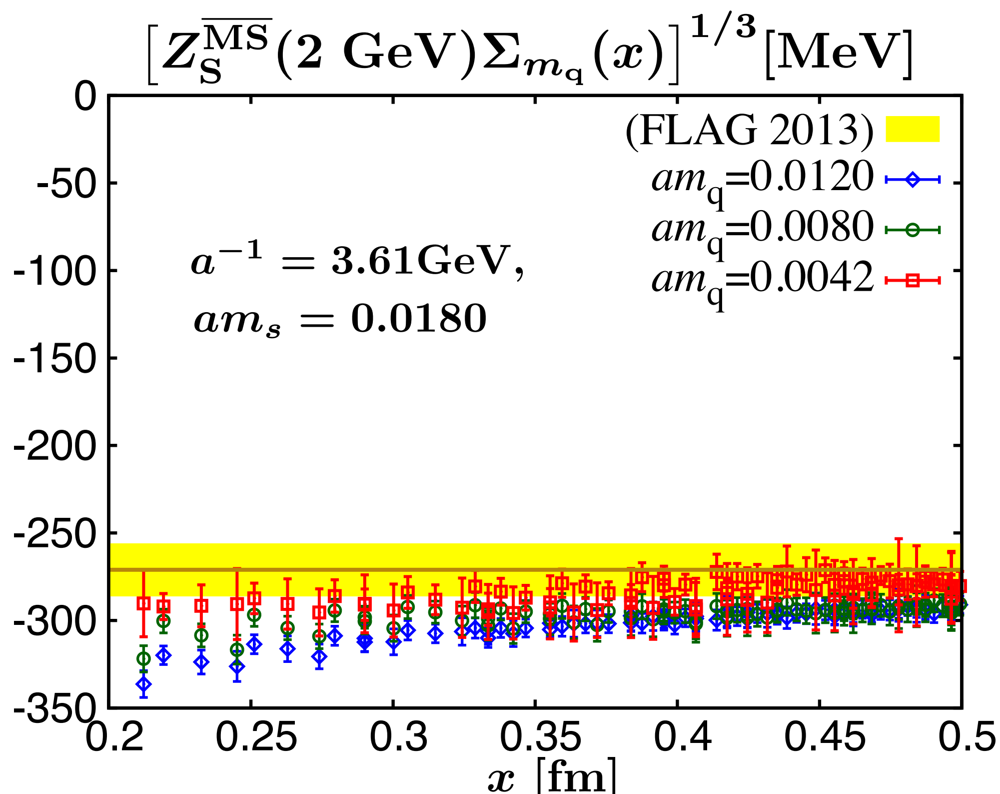


Slow convergence of OPE

Quark condensate term

Looking at the non-conserving part of the axial current

$$\Sigma_{m_q}(x) \equiv -\frac{\pi^2}{2m_q} x^2 x_\nu \partial_\mu \Pi_{A-V,\mu\nu}(x) = \langle \bar{q}q \rangle + O(m_q) \cdot O(x^{-2})$$



Consistent with the FLAG average <qbar q> = [–271(15) MeV]^3

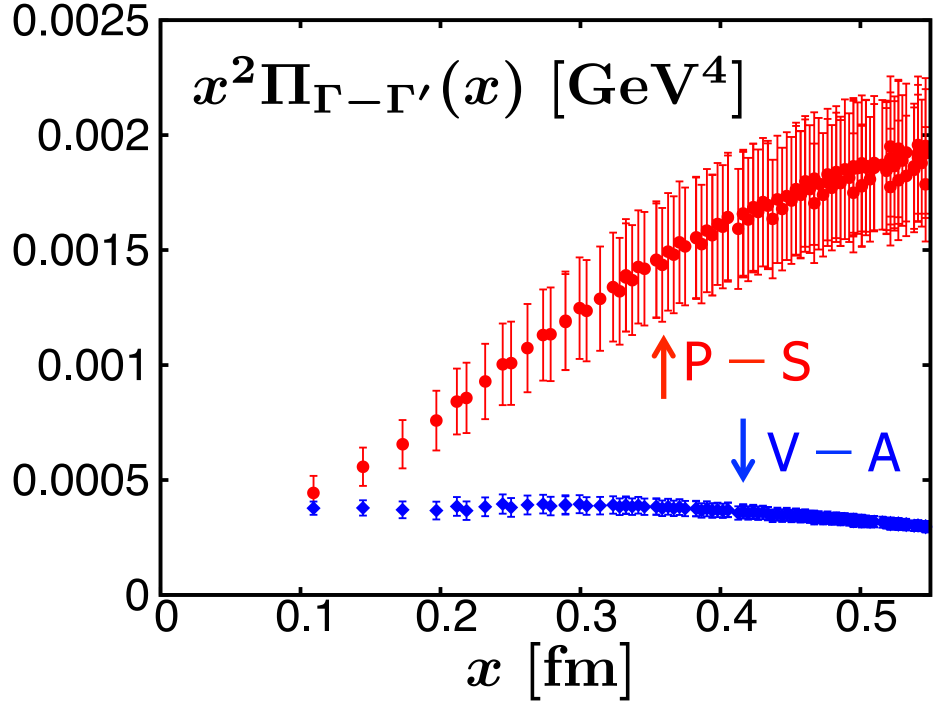
Or, even worse...

Non-perturbative effect too large for scalar-pseudoscalar

- OPE predicts the term of $m\langle\bar{q}q\rangle$

  [P-S] ~ 0.5 x [V-A]

- Lattice result: [P-S] > [V-A] and have a different x-dep

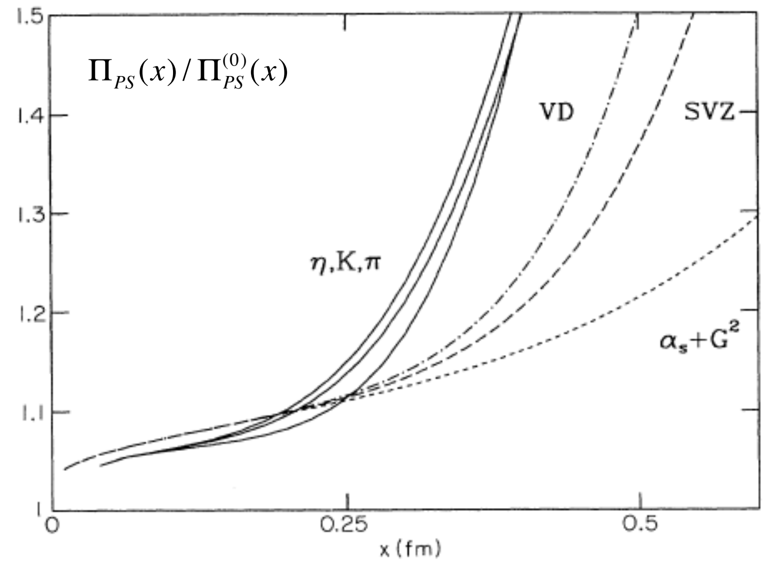


OPE failure?

Known for some time:

- Novikov, Shifman, Vainshtein, Zakharov, "*Are all hadrons alike?*", NPB191 (1981) 301.
  - Spin-0 correlation functions may deviate from OPE at shorter distances.
- Shuryak, ➔
- Chetyrkin, Narison, Zakharov, "*Short-distance tachyonic gluon mass and 1/Q*^{2} *corrections*," NPB550 (1999) 353.
- Narison, Zakharov, "*Hints on the power corrections from current correlators in x-space*," PLB522 (2001) 266.

Shuryak, RMP65, 1 (1993)



Lattice observations:

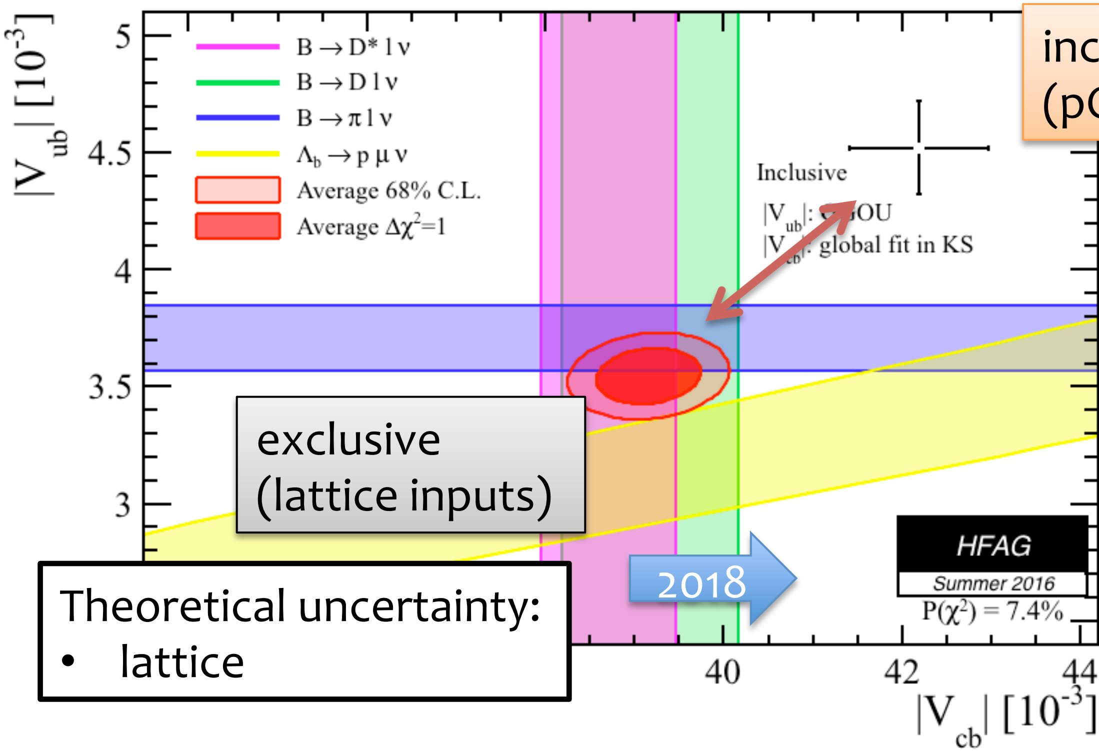
- Chu, Grandy Huang, Negele, PRD48 (1993) 3340.
- DeGrand, PRD64 (2001) 094508.

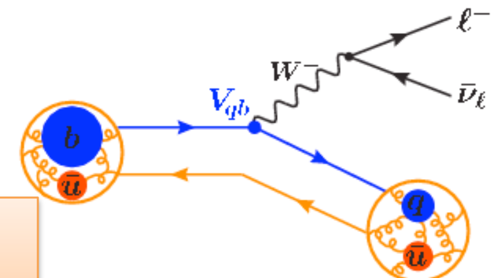
Summary (of Part I)

- OPE is a textbook subject of quantum field theory. But, yet to be tested.
  - ~~We~~ I don't understand when it works and when it doesn't.
  - Lattice will play a crucial role.
- Using lattice, one doesn't have to assume duality and rely on pQCD+OPE.

Why bothered with duality?

- Determination of $|V_{cb}|$, $|V_{ub}|$





inclusive
(pQCD + HQE)

exclusive
(lattice inputs)

Theoretical uncertainty:
- lattice

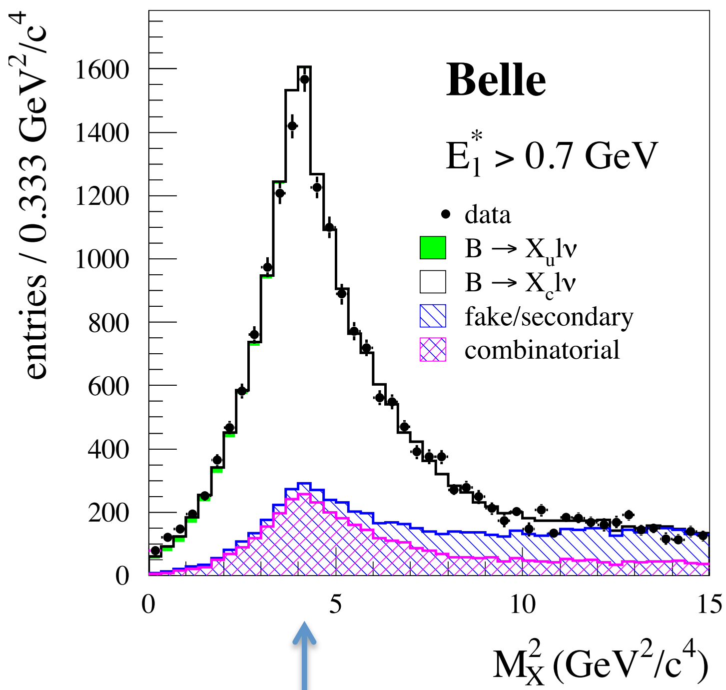
Theoretical uncertainty:
- perturbative expansion?
- heavy quark expansion?
- duality?

→ tested by analyzing various moments

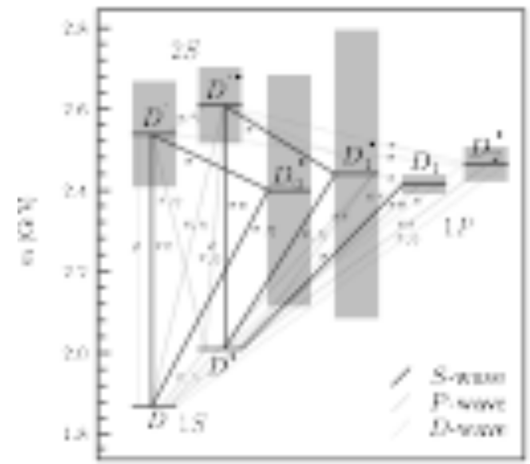
→ further test with lattice inputs?

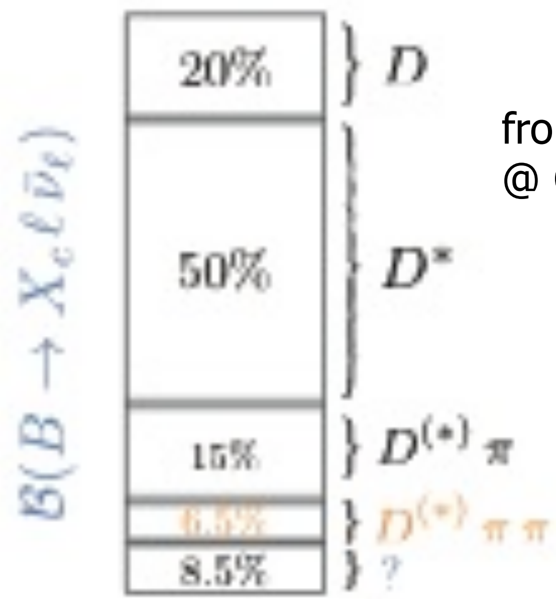
Inclusive?

M_X spectrum



Dominated by D and D* by about 70%.



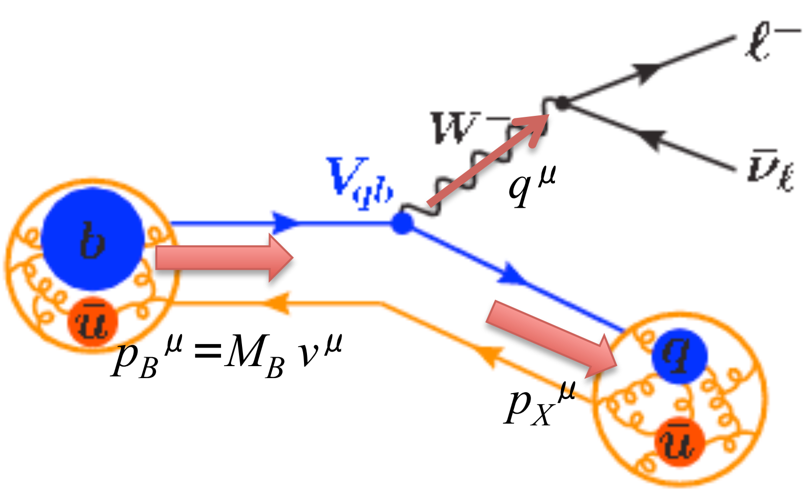


from Bernlohner @ CKM2014

Duality? What if 90% ??

Inclusive semi-leptonic B decays

– More complicated because of ...



2 kinematical variables:

- q^2 : lepton pair inv mass
- $v.q$: energy taken by leptons

Inclusive decays =

- $p_X{}^2 = m_X{}^2$ arbitrary

Summing all possible final states

- D, D^*, $D\pi$, $D\pi\pi$, ...

Decay amplitude is an analytic function of q^2 and $v.q$.

Partial decay rate:

$$d\Gamma \sim |V_{cb}|^2 l^{\mu\nu} W_{\mu\nu}$$

Structure function:

$$W_{\mu\nu} = \sum_X (2\pi)^3 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

sum over all final states

Optical theorem: $-\frac{1}{\pi}\mathrm{Im}T_i = W_i$

analytic function of q^2 and $v.q$

Forward scattering matrix element:

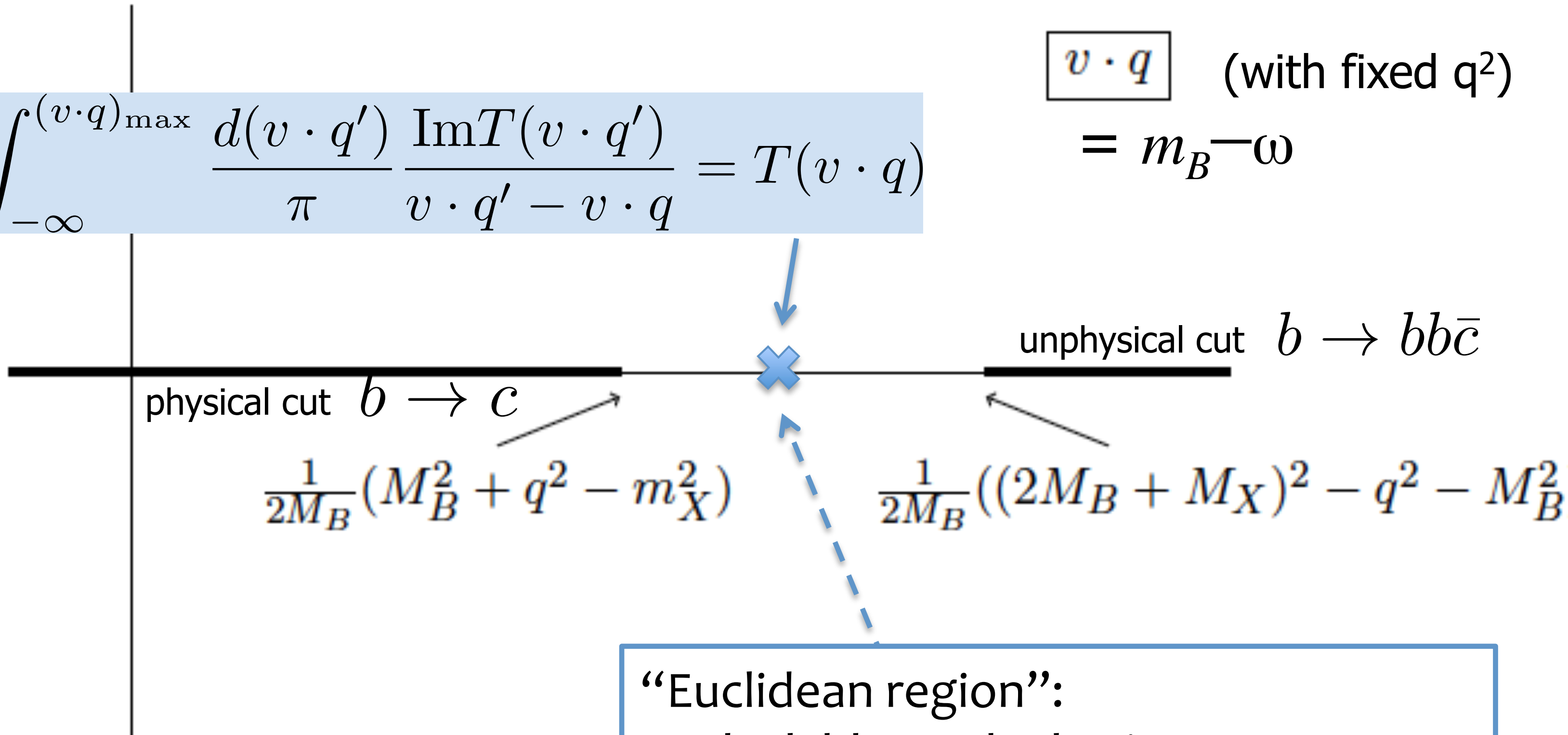
$$T_{\mu\nu} = i\int d^4x e^{-iqx} \frac{1}{2M_B} \langle B|T\{J_\mu^\dagger(x) J_\nu(0)\}|B\rangle$$

$$T_{\mu\nu} = i\int d^4x e^{-iqx}\frac{1}{2M_B}\langle B|T\{J^\dagger_\mu(x)J_\nu(0)\}|B\rangle$$

analytic function of q^2 and $v.q$

Analytic structure:

$$\int_{-\infty}^{(v\cdot q)_{\max}} \frac{d(v\cdot q')}{\pi}\frac{\mathrm{Im}T(v\cdot q')}{v\cdot q' - v\cdot q} = T(v\cdot q)$$

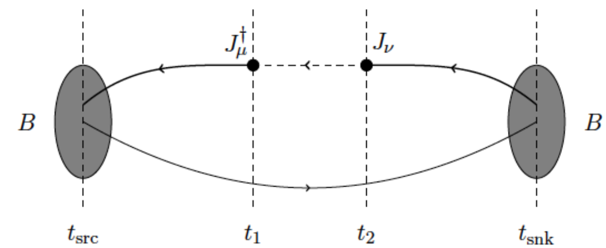


"Euclidean region":
calculable on the lattice
in the unphysical kinematical region

SH, arXiv:1703.01881

Lattice calculation: recipe

1. Four-point function:
   – calculate on the lattice



2. after taking appropriate ratios to cancel the external B meson source, we construct

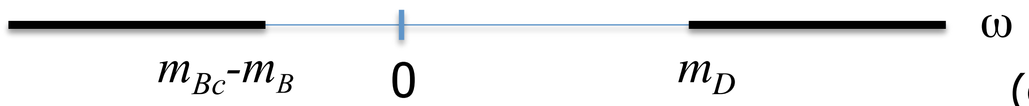
$$C_{\mu\nu}^{JJ}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})|J_\mu^\dagger(\mathbf{x},t) J_\nu(0)|B(\mathbf{0})\rangle$$

3. do the "Fourier transform" in the time direction

$$T_{\mu\nu}^{JJ}(\omega,\mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C_{\mu\nu}^{JJ}(t;\mathbf{q})$$

   – Corresponds to $T_{\mu\nu}(v\cdot q, q^2)$ at $p_X=(\omega,-\boldsymbol{q})$, $q=(m_B-\omega,\boldsymbol{q})$
   – Possible only when ω is lower than the lowest state energy.

“Fourier” (or Laplace) transform: $T^{JJ}_{\mu\nu}(\omega, \mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t; \mathbf{q})$



(energy given to charm)

Cauchy’s integral:

Compare at unphysical point

SH, PTEP 2017, 053B03

$$T(\omega) = \int d\omega' \frac{\frac{1}{\pi}\mathrm{Im}T(\omega')}{\omega' - \omega}$$

input from exp

OR

lattice calc

$$C(t) = \int d\omega' e^{-\omega' t} \left(-\frac{1}{\pi}\mathrm{Im}T(\omega') \right)$$

Compare with exp

Inverse problem: Backus-Gilbert method, etc

Hansen, Meyer, Robaina, PRD96, 094513 (2017)

Ensembles from JLQCD

- With Mobius domain-wall fermion (2012~)
  - 2+1 flavor (uds)
  - Mobius domain-wall fermion [with stout link]
    - residual mass < O(1 MeV)
  - lattice spacing : $1/a$ = 2.4, 3.6, 4.5 GeV
  - volume : L = 2.7 fm (32^3, 48^3, 64^3 lattices)
  - light quark mass : m_π = 230, 300, 400, 500 MeV
  - statistics : 50-200 measurements
- Valence sector
  - heavy (MDW) + strange (MDW)
  - tuned charm + (unphysical) bottom $m_b = (1.25)^2\, m_c$, $(1.25)^4\, m_c$

    (up to 3.4 GeV B_s meson)
  - on Oakforest-PACS with IROIRO++

β = 4.17, 1/a ~ 2.4 GeV, 32^3x64 (x12)

m_{ud}	m_π [MeV]	MD time
m_s = 0.030		
0.007	310	10,000
0.012	410	10,000
0.019	510	10,000
m_s = 0.040		
0.0035	230	10,000
0.0035 (48^3x96)	230	10,000
0.007	320	10,000 ←
0.012	410	10,000
0.019	510	10,000

β = 4.35, 1/a ~ 3.6 GeV, 48^3x96 (x8)

m_{ud}	m_π [MeV]	MD time
m_s = 0.018		
0.0042	300	10,000
0.0080	410	10,000
0.0120	500	10,000
m_s = 0.025		
0.0042	300	10,000 ←
0.080	410	10,000
0.0120	510	10,000

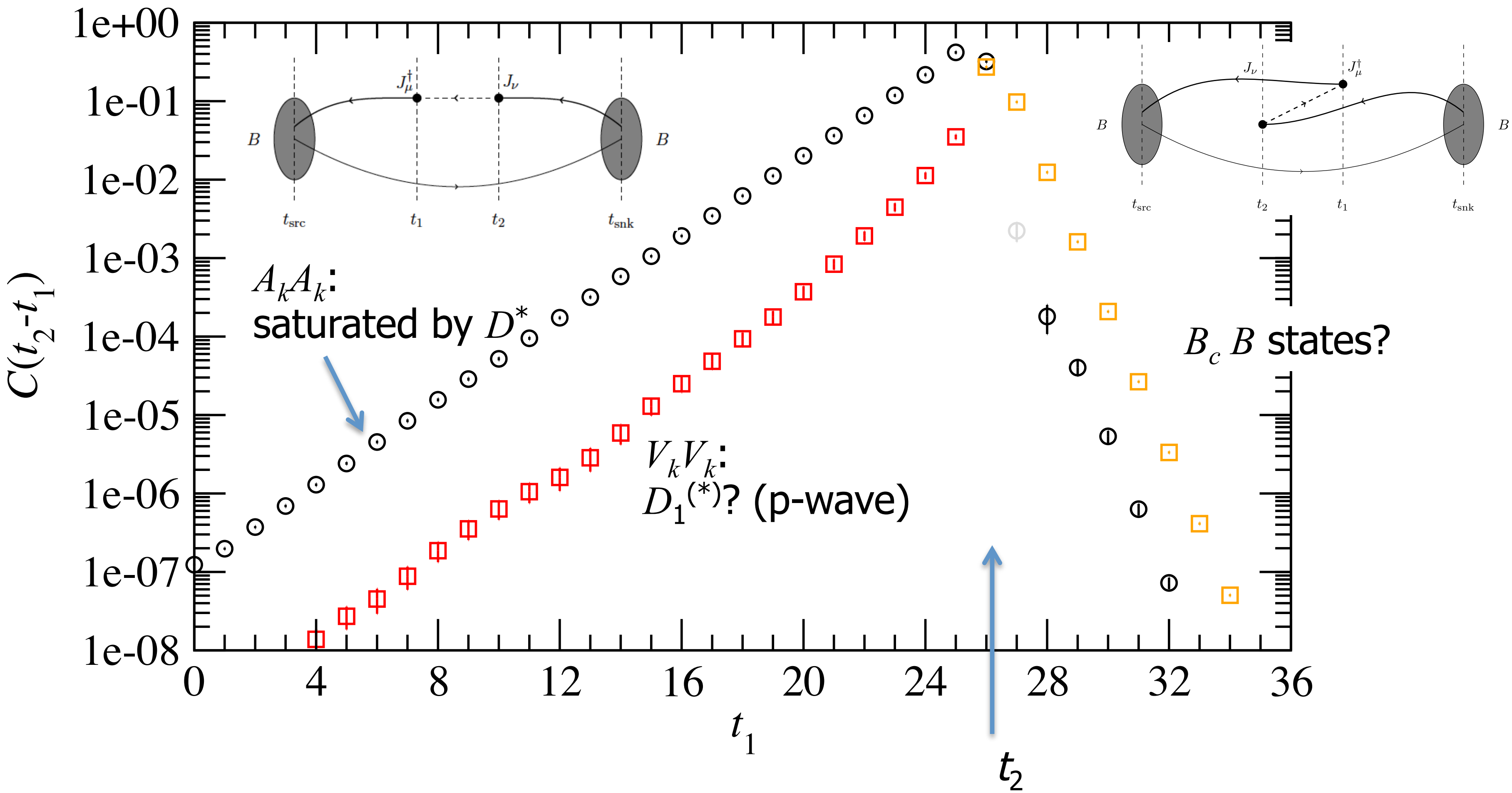
β = 4.47, 1/a ~ 4.6 GeV, 64^3x128 (x8)

0.0030	~ 300	10,000 ←

a glance at the lattice data:

$$C^{JJ}_{\mu\nu}(t;\mathbf{q}) = \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \frac{1}{2M_B} \langle B(\mathbf{0})|J^\dagger_\mu(\mathbf{x},t) J_\nu(0)|B(\mathbf{0})\rangle \qquad J_\mu = \bar{b}\gamma_\mu c \quad \text{or} \quad \bar{b}\gamma_\mu\gamma_5 c$$

$m_b=1.25^4 m_c$, zero recoil (**q=0**) $1/a$ = 3.6 GeV



"Fourier transform"

- Time direction should be analytically continued to go time-like, i.e. energy ω:

$$e^{ip_0 t} \to e^{\omega t} \quad : \quad T_{\mu\nu}^{JJ}(\omega, \mathrm{q}) = \int_0^\infty dt\, e^{\omega t} C_{\mu\nu}^{JJ}(t; \mathrm{q})$$

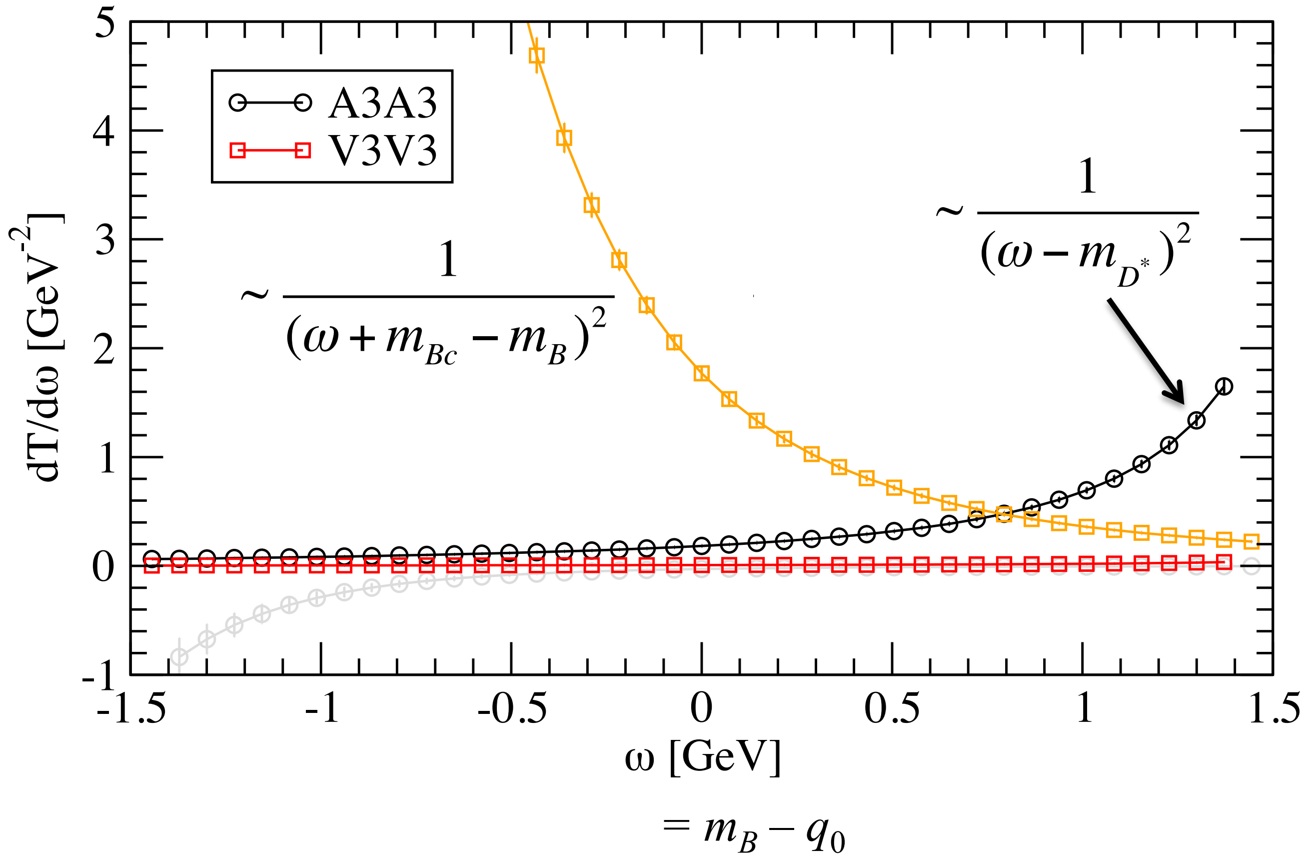
  - Can be understood by a Taylor expansion in p_0, and then reconstruct with $ip_0=\omega$.
  - Only below any singularity: pole, cut, …
  - Obviously,

$$e^{-mt} \xrightarrow{F.T.} \frac{1}{\omega - m}$$

$$T^{JJ}_{\mu\nu}(\omega,\mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t;\mathbf{q})$$

$m_b = 1.25^4 m_c$, zero recoil (**q**=**0**) $1/a$ = 3.6 GeV

derivative to avoid divergence (contact term)



Saturation by D$^{(*)}$?

Four-point function:

$$C_{\mu\nu}^{JJ}(t;0) = \frac{1}{2M_B}\sum_X \langle B(0)|J_\mu^\dagger|X(0)\rangle \frac{e^{-E_X t}}{2E_X}\langle X(0)|J_\nu|B(0)\rangle$$

Ground-state contribution:

$$\begin{aligned} \langle D(0)|V^0|B(0)\rangle &= 2\sqrt{M_B M_D}\, h_+(1), \\ \langle D^*(0)|A^k|B(0)\rangle &= 2\sqrt{M_B M_{D^*}}\, h_{A_1}(1)\, \varepsilon^{*k}. \end{aligned}$$

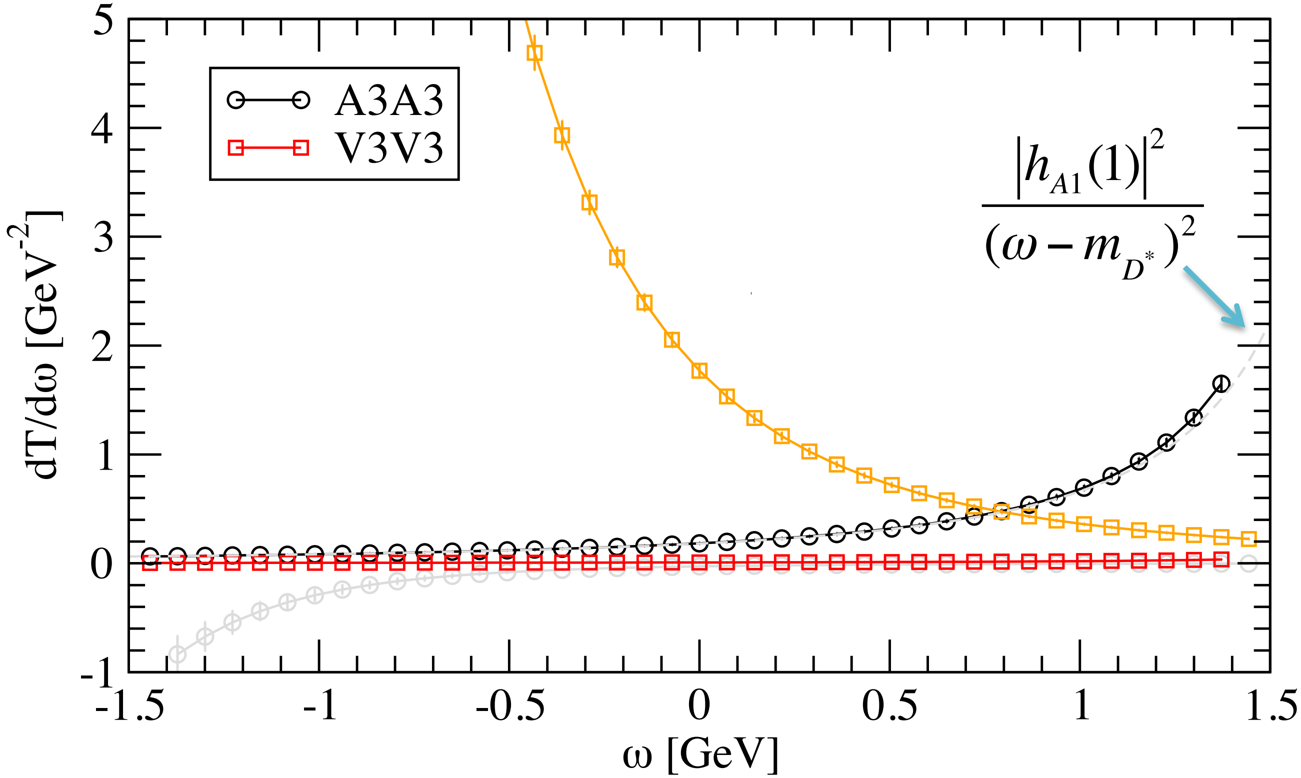
$$\begin{aligned} T_{00}^{VV}(\omega,0) &= \frac{|h_+(1)|^2}{M_D-\omega}, \\ T_{kk}^{AA}(\omega,0) &= \frac{|h_{A_1}(1)|^2}{M_{D^*}-\omega}. \end{aligned}$$

zero-recoil form factors h(1)
~ Isgur-Wise function ξ(1)=1

$$T_{\mu\nu}^{JJ}(\omega, \mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C_{\mu\nu}^{JJ}(t; \mathbf{q})$$

m_b=1.25$^4 m_c$, zero recoil (**q**=**0**)   1/a = 3.6 GeV

derivative to avoid divergence (contact term)



$= m_B - q_0$

Wrong parity channel?

$B \to D_1^{(*)}$ (at zero recoil): p-wave, 1^+ states

D_1^*: s_l=1/2, 2427 MeV (broad)

D_1 : s_l=3/2, 2421 MeV (narrow)

$$\langle D_1^* | \bar{c}\gamma_\mu b | B \rangle = \sqrt{m_{D_1^*} m_B}\, g_{V_1}(1)\, \varepsilon_\mu^*$$

$$\langle D_1 | \bar{c}\gamma_\mu b | B \rangle = \sqrt{m_{D_1} m_B}\, f_{V_1}(1)\, \varepsilon_\mu^*$$

$$\Rightarrow \quad \frac{dT_{kk}^{VV}}{d\omega} \sim \frac{\left|g_{V_1}\right|^2}{\left(m_{D_1^*} - \omega\right)^2} + \frac{\left|f_{V_1}\right|^2}{\left(m_{D_1} - \omega\right)^2}$$

Bernlochner, Ligeti, Robinson, arXiv:1711.03110

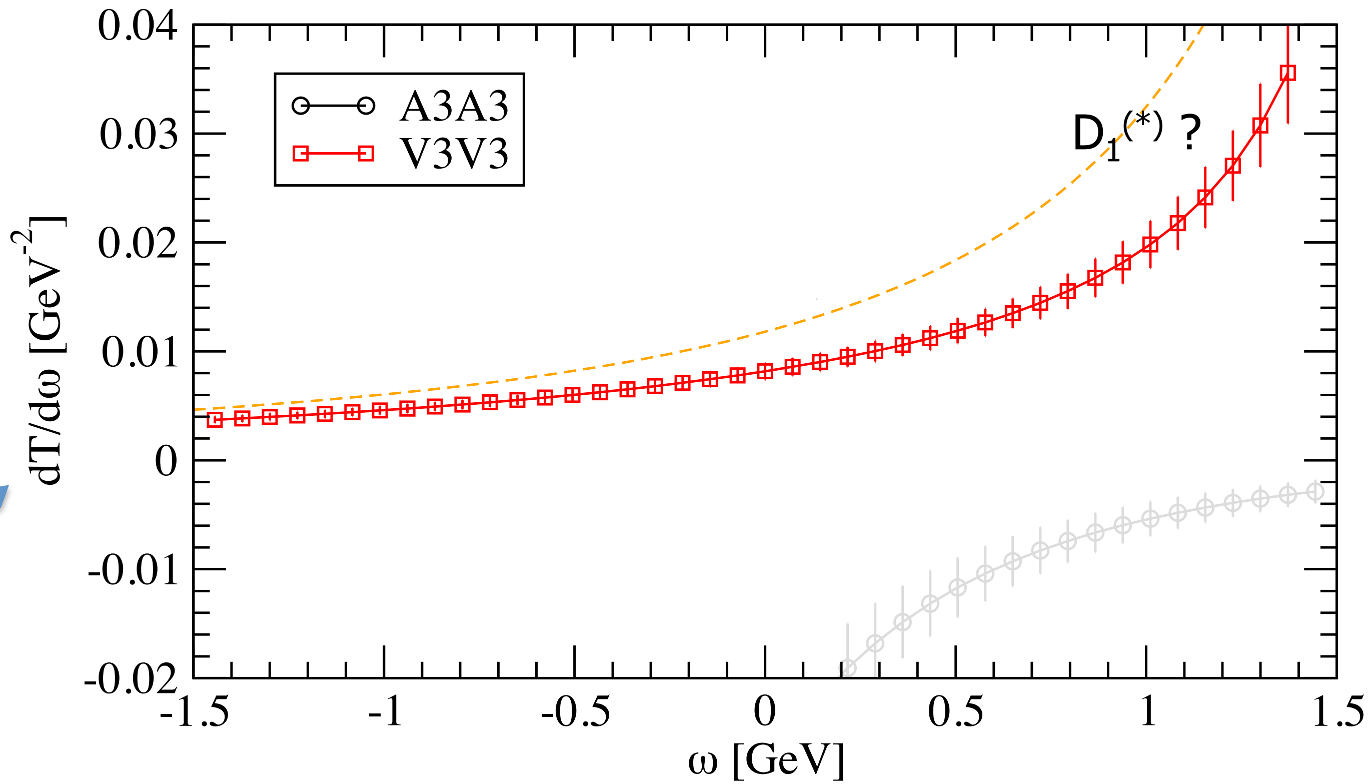
$$g_{V_1}(1) = \left(\varepsilon_c - 3\varepsilon_b\right)\left(\bar{\Lambda}^* - \bar{\Lambda}\right)\zeta(1)$$

$$f_{V_1}(1) = -\frac{8}{\sqrt{6}}\varepsilon_c\left(\bar{\Lambda}' - \bar{\Lambda}\right)\tau(1)$$

$$T^{JJ}_{\mu\nu}(\omega,\mathbf{q}) = \int_0^\infty dt\, e^{\omega t} C^{JJ}_{\mu\nu}(t;\mathbf{q})$$

m_b=1.25$^4 m_c$, zero recoil (**q**=**0**) $1/a$ = 3.6 GeV

derivative to avoid divergence (contact term)

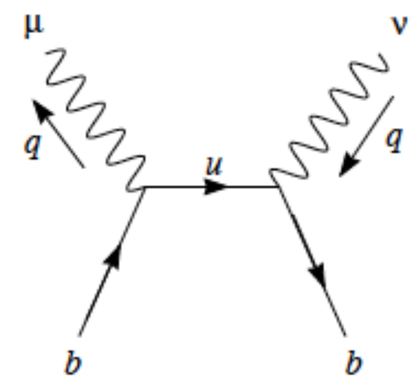


$= m_B - q_0$

Comparison with Continuum

(Based on discussions with P. Gambino)

- Heavy Quark Expansion (tree-level formulae):
  Blok, Koyrakh, Shifman, Vainshtein, PRD49, 3356 (1994).
  Manohar, Wise, PRD49, 1310 (1993).
  Falk, Ligeti, Neubert, Nir, PLB326, 145 (1994)
  Balk, Korner, Pirjol, Schilcher, ZP C64, 37 (1994).

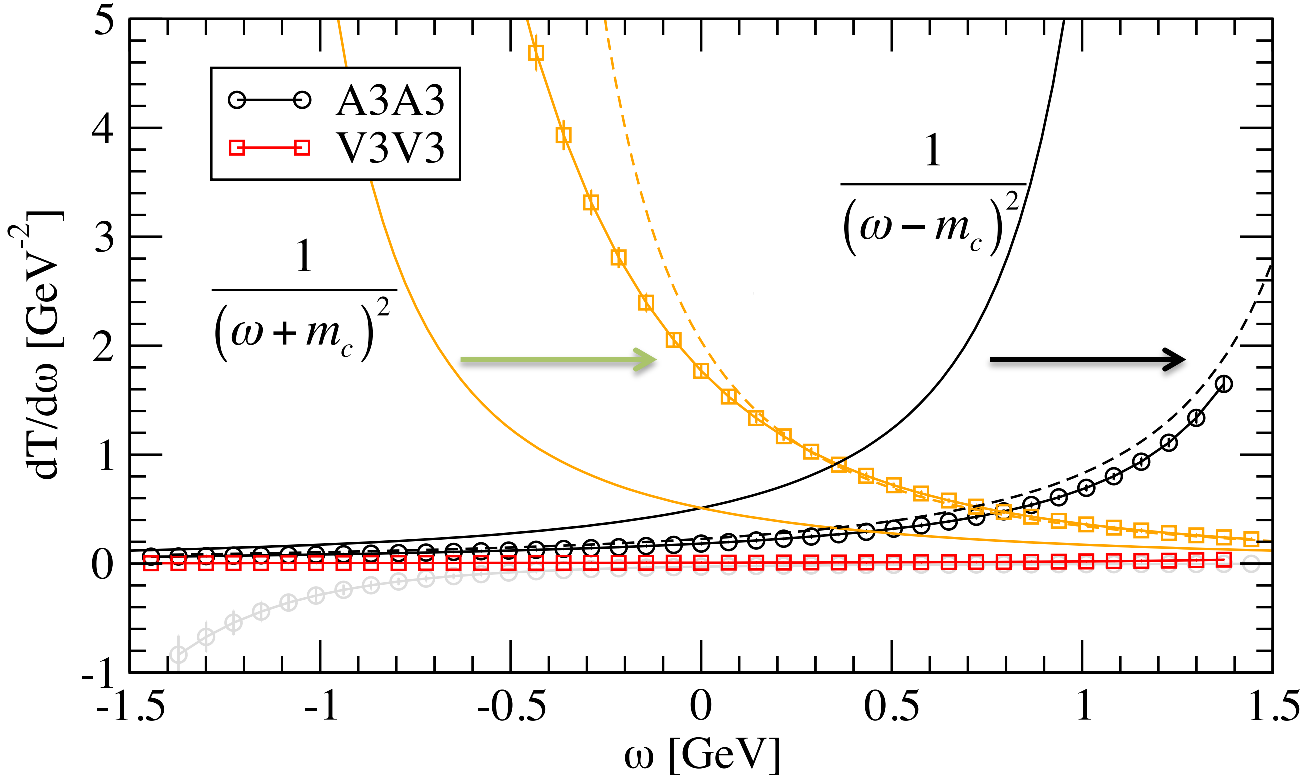


- Expand $\dfrac{1}{m_b \not{v} - \not{q} + \not{k} - m_c}$ in small *k*.

- Zero-recoil limit ($V_k V_K$ or $A_k A_k$ channel, leading order)

$$T_1^{VV} = -\frac{\omega - m_c}{\omega^2 - m_c^2}, \quad T_1^{AA} = -\frac{\omega + m_c}{\omega^2 - m_c^2} \qquad (\omega = m_B - q_0)$$

… pole at $\omega = -m_c$ ($V_k V_k$) or $\omega = m_c$ ($A_k A_k$)



Significant shift due to $m_c \rightarrow m_D$
or to $m_c \rightarrow m_{Bc}$-m_B

Non-perturbative effect

Significant shift due to $m_c \rightarrow m_D$
or to $m_c \rightarrow m_{Bc}$-m_B

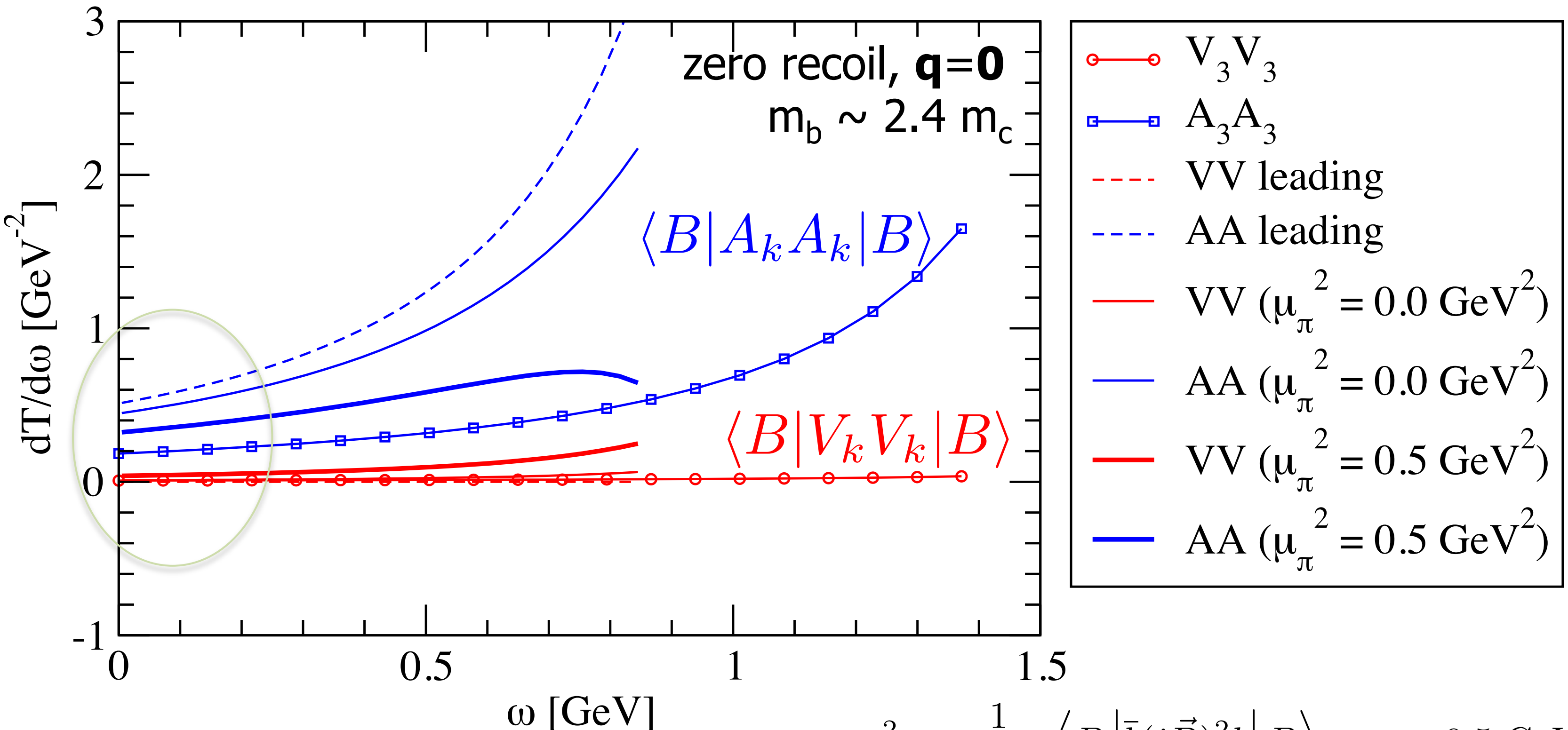
Understandable by the 1/m_b expansion?

- Probably not, because the effect should survive in the heavy quark limit.
- Then, what is missing in the heavy quark expansion?
- How much does perturbation theory show the sign of the non-perturbative effects?
- Or, m_b (in my calculation) is too close to m_c to induce really "inclusive" decays?

c.f. Boyd, Grinstein, Manohar (1996): consistency between exclusive and 1/m expansion was found in the SV limit m_b, $m_c >> m_b$-$m_c >> \Lambda_{QCD}$.

Comparison with Heavy Quark Expansion:

- Exp data not available in the form I can use in this analysis.
- HQE to order $1/m^2$



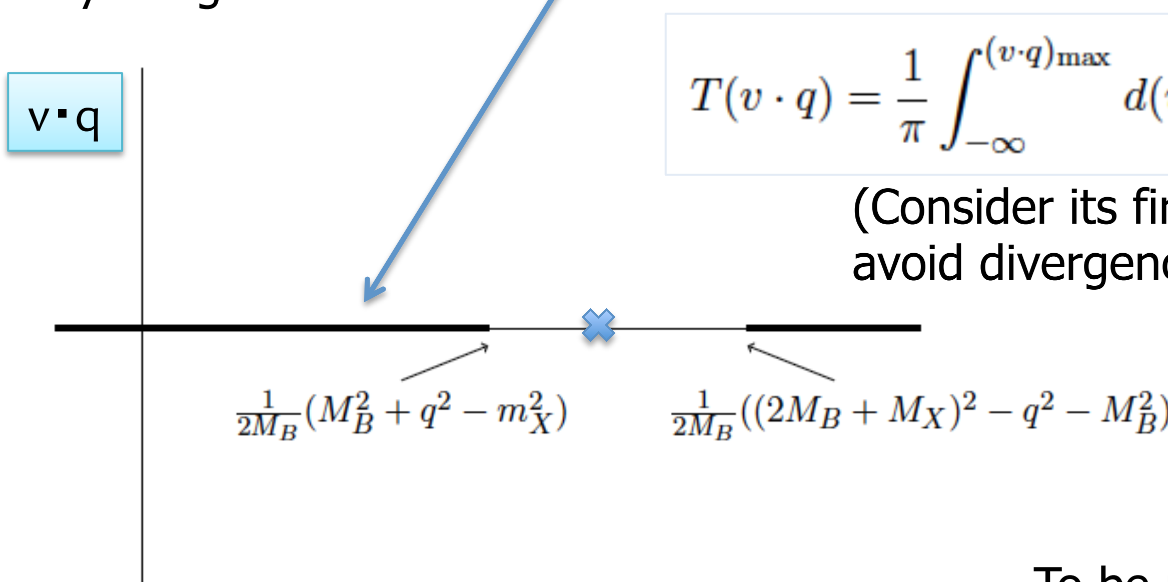
$$\mu_\pi^2 = \frac{1}{2M_B}\left\langle B\left|\bar{b}(i\vec{D})^2 b\right|B\right\rangle \qquad \sim 0.5\ \mathrm{GeV}^2,$$

$$\mu_G^2 = \frac{1}{2M_B}\left\langle B\left|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b\right|B\right\rangle = 0.37\ \mathrm{GeV}^2$$

Marginal agreement.
Need to include pert corrections.

Going to one-loop

- One-loop corrections:
  Trott, PRD70, 073003 (2004).
  Aquila, Gambino, Ridolfi, Uraltsev, NPB719, 77 (2005).
  - Available only for the Imaginary part (cut); Needed to perform the Cauchy integral.



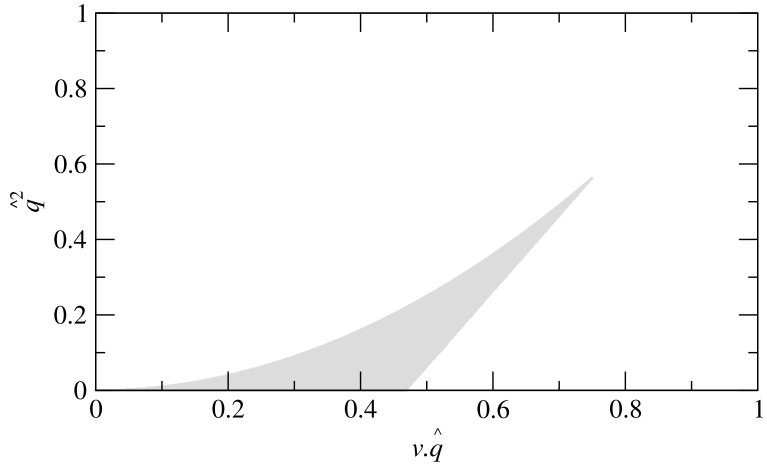
$$T(v\cdot q) = \frac{1}{\pi}\int_{-\infty}^{(v\cdot q)_{\max}} d(v\cdot q')\frac{\mathrm{Im}T(v\cdot q')}{v\cdot q' - v\cdot q}$$

(Consider its first derivative to avoid divergence.)

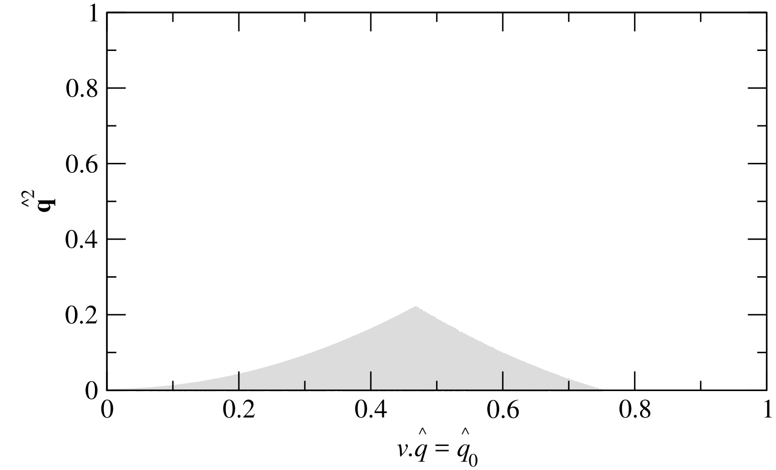
To be performed.

Then, to experiment

fixed q^2, or



fixed recoil mom



- Need 2D distribution of the experimental data.
- Unavailable region must be supplemented by perturbation theory.

Summary (of Part II)

- Inclusive structure functions calculable on the lattice, but at unphysical kinematics. May test the consistency of theoretical methods.
  - The region of ground state saturation (D or D^*) is consistent with expectation (from form factors). The wrong parity channel too.
  - May be compared with HQE. Non-perturbative effect needs to be understood.

- More applications
  - From B decays to nucleon structure.