Extending precision tests of the standard model using Lattice QCD

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December 1, 2016

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RBC and UKQCD Collaborations

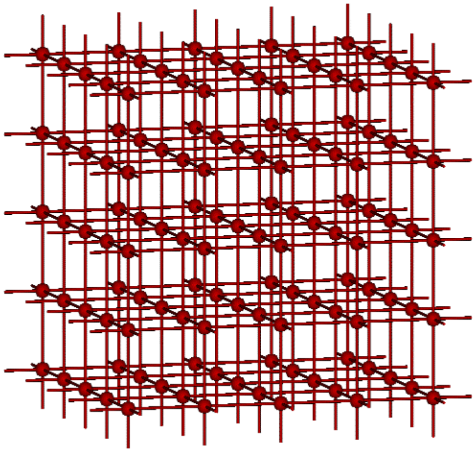
Outline

- Lattice QCD: methods and status
- Five precision tests of the standard model:
  1) $K \rightarrow \pi\,\pi$ decay and direct CP: ε'
  2) $K_L - K_S$ mass difference
  3) Long distance contribution to ε_K
  4) Long distance contribution to rare kaon decay: $K^+ \rightarrow \pi^+ \nu \bar{\nu}$
  5) Muon g-2

State-of-the-art Lattice QCD

Lattice QCD

- Introduce a space-time lattice.
- Evaluate the Euclidean Feynman path integral
  - Study $e^{-H_{QCD}\,t}$
  - Precise non-perturbative formulation
  - Capable of numerical evaluation



$$\sum_n \langle n | e^{-H(T-t)} \mathcal{O} e^{-Ht} | n \rangle = \int d[U_\mu(n)] e^{-\mathcal{A}[U]} \det(D+m) \mathcal{O}[U](t)$$

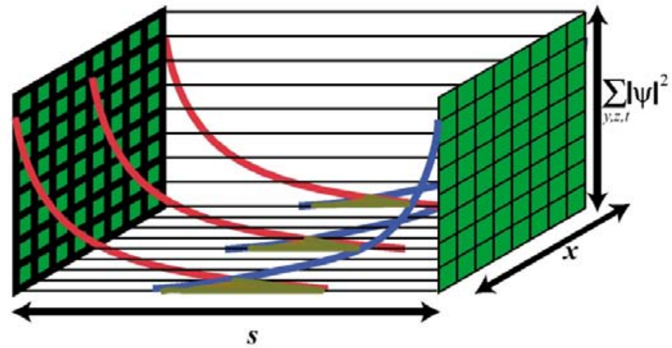
- Evaluate using Monte Carlo importance sampling with hybrid, molecular dynamics/Langevin evolution.
- Integration volume of 1 billion dimensions.

Lattice QCD – 2016

- Physical quark masses (ChPT not needed)
- Chiral quarks (doubling problem solved)
- Large physical volumes: $(6\ \mathrm{fm})^3$
- Small lattice spacing: $1/a = 2.4$ GeV
  - $(\Lambda_{\mathrm{QCD}}\, a)^2$ effects < 1% ☺
  - $(m_{\mathrm{charm}}\, a)^2$ effects ~ 20% ☹

Elaborate methods required

- Use 5-D, domain wall lattice fermions – physical quarks bound to 4D boundaries



  - Measurements on 64^3 x 128 ensembles sustain > 1 Pflops on 32 BG/Q racks at ANL.
  - Compute 2000 lowest Dirac eigenvectors to speed up Dirac operator inversion.
  - KNL chip has 68 cores, each with 4 thread and two 512-bit wide, pipelined FPUs.
  - Broad collaboration and substantial funding needed.

RBC Collaboration

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  - Chulwoo Jung
  - Taku Izubuchi
  - Christoph Lehner
  - Meifeng Lin
  - Amarjit Soni
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  - Ava Khamseh
  - Antonin Portelli
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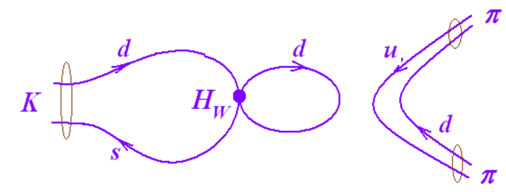
Precision tests of the Standard Model

Direct CP violation $K \rightarrow \pi\,\pi$

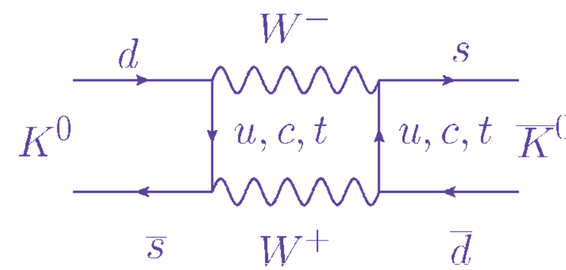
$|\varepsilon'| = 3.70(53) \times 10^{-6}$



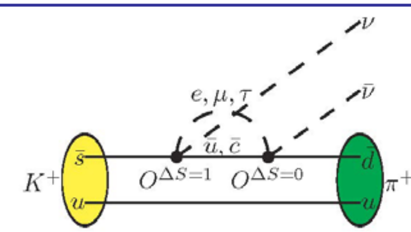
Indirect CP violation $K \rightarrow \pi\,\pi$

$|\varepsilon| = 0.002228\ (11)$

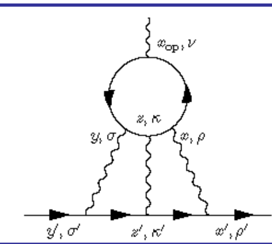
$m_{K_L} - m_{K_S} = 3.19(41)(96) \times 10^{-12}$ MeV



$K^+ \rightarrow \pi^+ \nu\, \bar{\nu}$, BR=$1.73\ ^{+1.15}_{-1.05} \times 10^{-10}$



$g_\mu - 2 = 0.00116592089(63)$



$K \to \pi\pi$ Decay

$K \to \pi\,\pi$ and CP violation

- Final $\pi\pi$ states can have I = 0 or 2.

$$\langle \pi\pi(I=2)|H_w|K^0\rangle = A_2 e^{i\delta_2} \qquad \Delta I = 3/2$$

$$\langle \pi\pi(I=0)|H_w|K^0\rangle = A_0 e^{i\delta_0} \qquad \Delta I = 1/2$$

- CP symmetry requires A_0 and A_2 be real.
- Direct CP violation in this decay is characterized by:

$$\epsilon' = \frac{i\,e^{\delta_2-\delta_0}}{\sqrt{2}}\left|\frac{A_2}{A_0}\right|\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}-\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

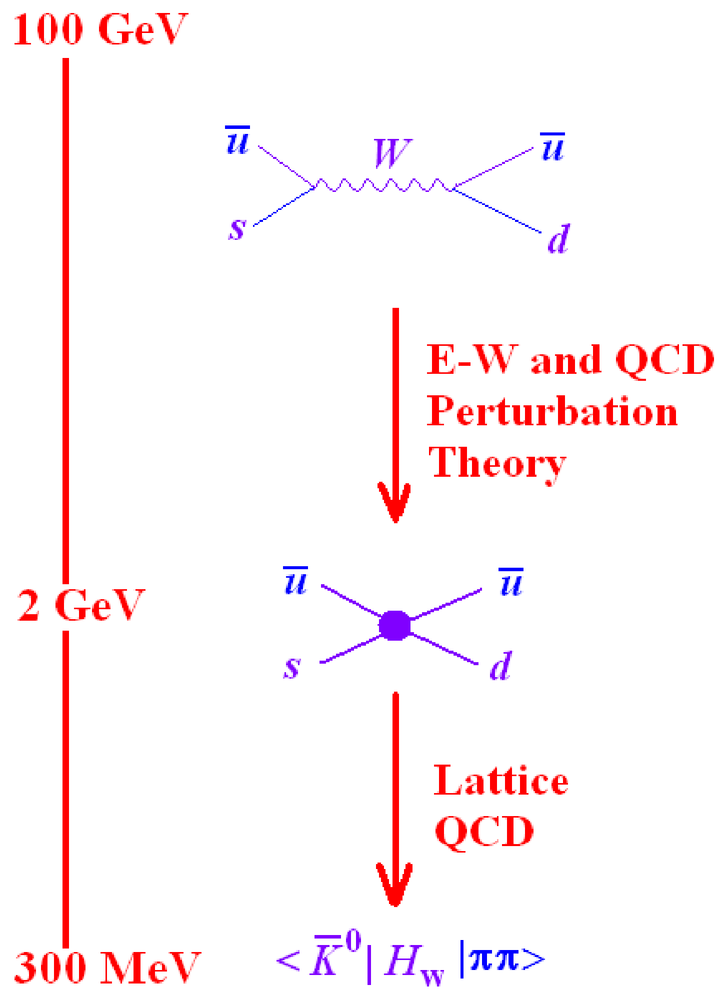
Direct CP violation

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

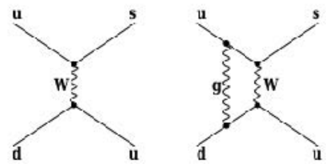
$$\mathcal{H}^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \Big[z_i(\mu) + \tau y_i(\mu) \Big] Q_i \right\}$$

- $\tau = -\frac{V_{td} V_{ts}^*}{V_{ud} V_{us}^*} = (1.543 + 0.635 i) \times 10^{-3}$
- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Local four quark operators

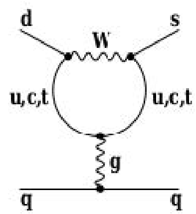
- **Current-current operators**



$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$

$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**



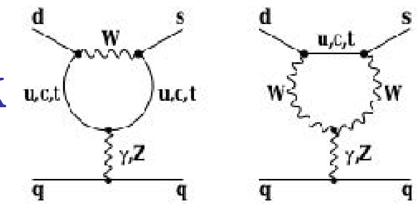
$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}$$

$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$

$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}$$

$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

- **Electro-Weak Penguins**



$$Q_7 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V+A}$$

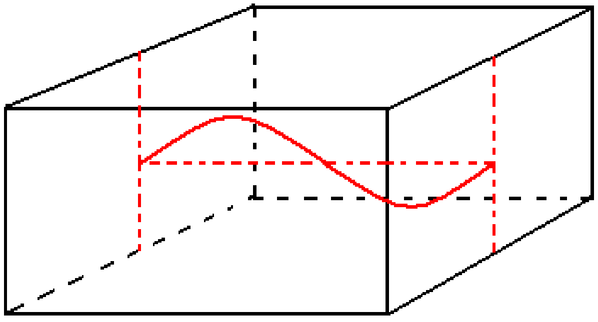
$$Q_8 \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V+A}$$

$$Q_9 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V-A}$$

$$Q_{10} \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V-A}$$

Physical $\pi\pi$ states
Lellouch-Luscher

- Euclidean $e^{-H_{QCD}t}$ projects onto $|\pi\pi(\vec{p}=0)\rangle$
- Exploit finite-volume quantization.



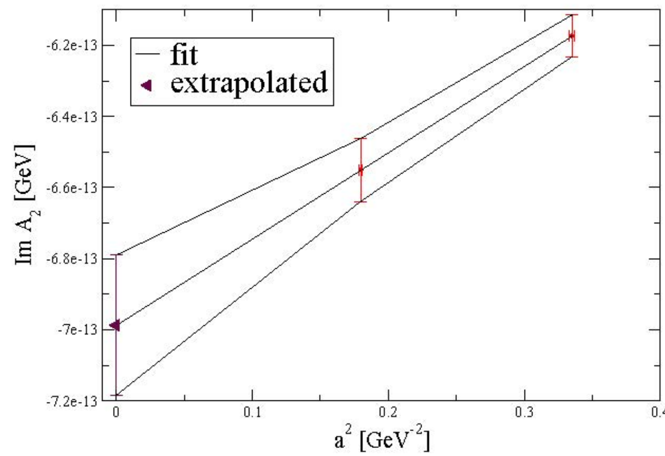
- Boundary conditions give ground state with physical $\vec{p}$
  - $\Delta I = 3/2$: impose anti-periodic BC on *d* quark
  - $\Delta I = 1/2$: impose G-parity BC
- Correctly include π - π interactions, including normalization

Calculation of A_2

ΔI = 3/2 – Continuum Results

(M. Lightman, E. Goode T. Janowski)

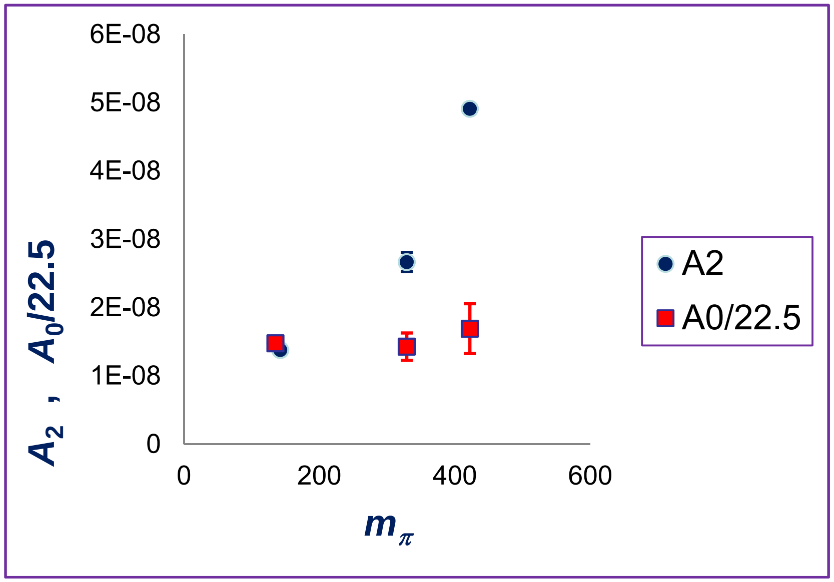
- Use two large ensembles to remove a^2 error (m_p=135 MeV, L=5.4 fm)
  - 48^3 x 96, 1/a=1.73 GeV
  - 64^3 x 128, 1/a=2.28 GeV



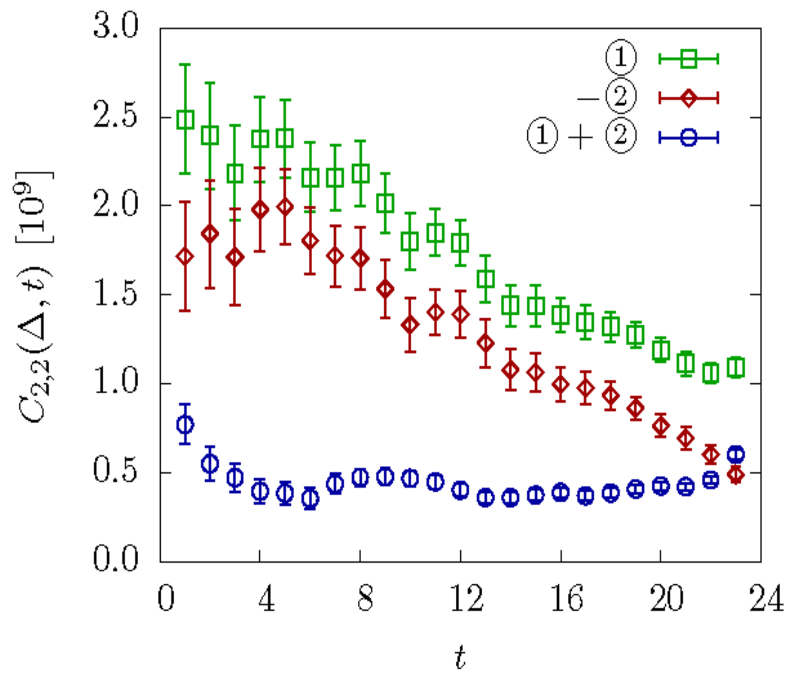
- Continuum results:
  - $\mathrm{Re}(A_2)$ = 1.50(0.04_{stat}) $(0.14)_{\mathrm{syst}}\times 10^{-8}$ GeV
  - $\mathrm{Im}(A_2)$ = - 6.99$(0.20)_{\mathrm{stat}}$ $(0.84)_{\mathrm{syst}}\times 10^{-13}$ GeV
- Experiment: $\mathrm{Re}(A_2)$ = 1.479(4) 10^{-8} GeV
- $E_{\pi\pi} \rightarrow \delta_2 = -11.6(2.5)(1.2)^{\circ}$
- Phys.Rev. **D91**, 074502 (2015)

$\Delta I = 1/2$ Rule
(Qiu Liu)

Compare A_2 and $A_0/22.5$



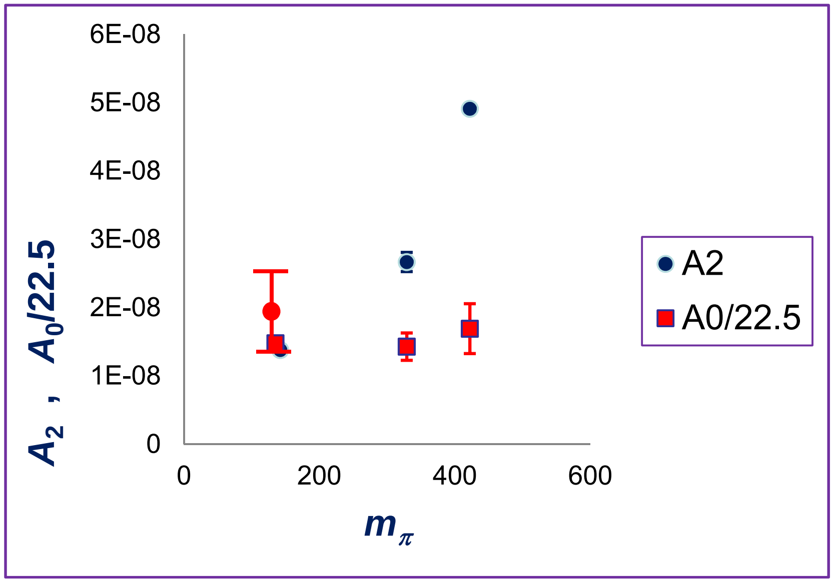
Cancellation in A_2



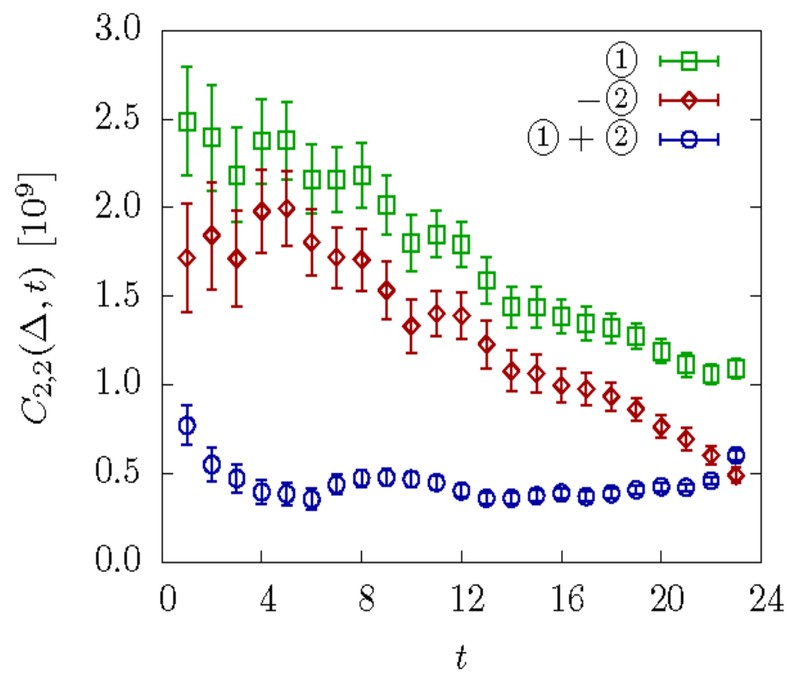
- 50 year puzzle resolved!
- A dynamical QCD effect – no more explanation needed?

ΔI = 1/2 Rule
(Qiu Liu)

Compare A_2 and A_0 /22.5



Cancellation in A_2



- 50 year puzzle resolved!
- A dynamical QCD effect – no more explanation needed?

Calculation of A_0 and ε'

Overview of calculation

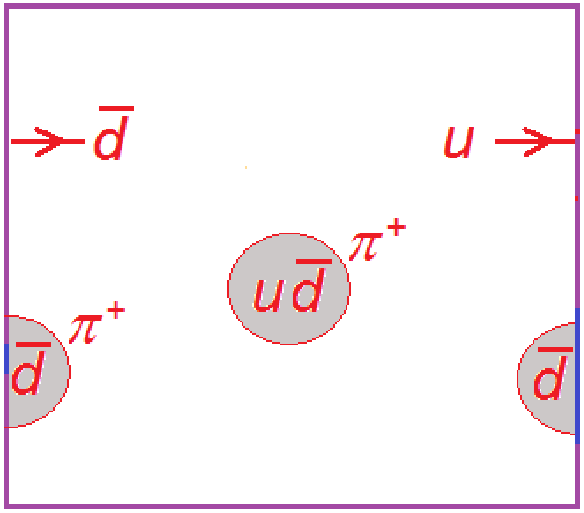
(Chris Kelly, Daiqian Zhang)

- Use 32^3 x 64 ensemble
  - $1/a$ = 1.3784(68) GeV, L = 4.53 fm.
  - 216 configurations separated by 4 time units
  - 900 low modes for all-to-all propagators
  - Solve for $\pi\pi$ and kaon sources on each of 64 time slices

$\Delta I = ½\ K \to \pi\pi$ – above threshold

(Chris Kelly & Daiqian Zhang)

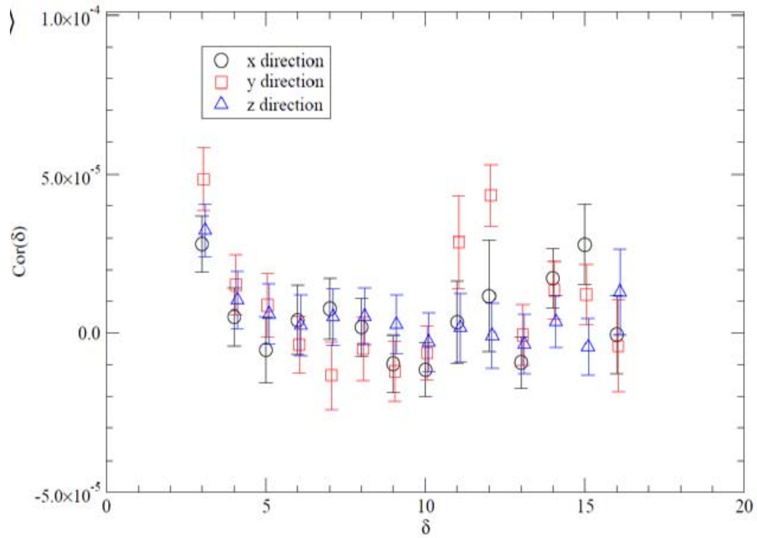
- Use **G-parity** BC to obtain $p_\pi = 205$ MeV (Changhoan Kim, hep-lat/0210003)
  - $G = C e^{i\pi I_y}$
  - Non-trivial: $\begin{pmatrix} u \\ d \end{pmatrix} \to \begin{pmatrix} \bar{d} \\ -\bar{u} \end{pmatrix}$
  - Extra $I = 1/2$, **s′** quark adds $e^{-m_K L}$ error.
  - Tests: f_K and B_K correct within errors.



Overview of calculation

- Achieve essentially physical kinematics:
  - M_π = 143.1(2.0)
  - M_K = 490.6(2.2) MeV
  - $E_{\pi\pi}$ = 498(11) MeV
  - m_{res} = 0.001842(7)

- Error in ensemble generation
  (*u* and *d* quark forces computed from the same random numbers after shift by 12 in y-direction)



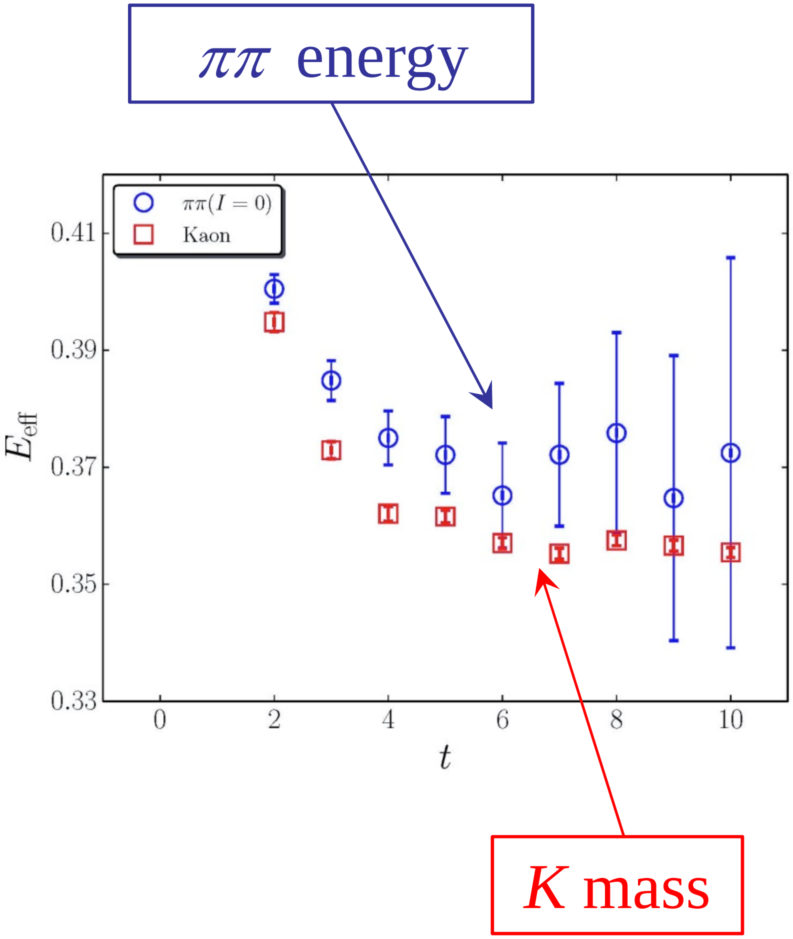
Average plaquette

Correct ensemble 0.512239(3)(7)

Incorrect ensemble 0.512239(6)

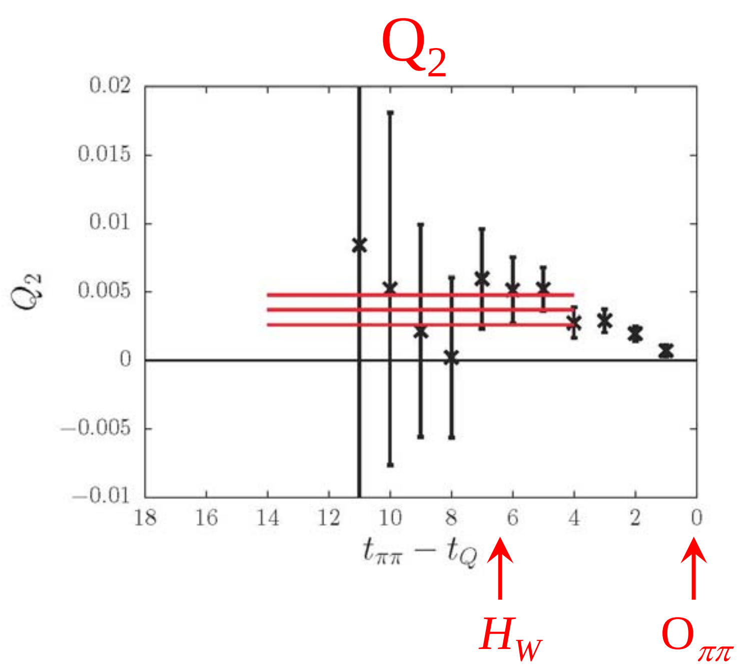
I = 0, $\pi\pi - \pi\pi$ correlator

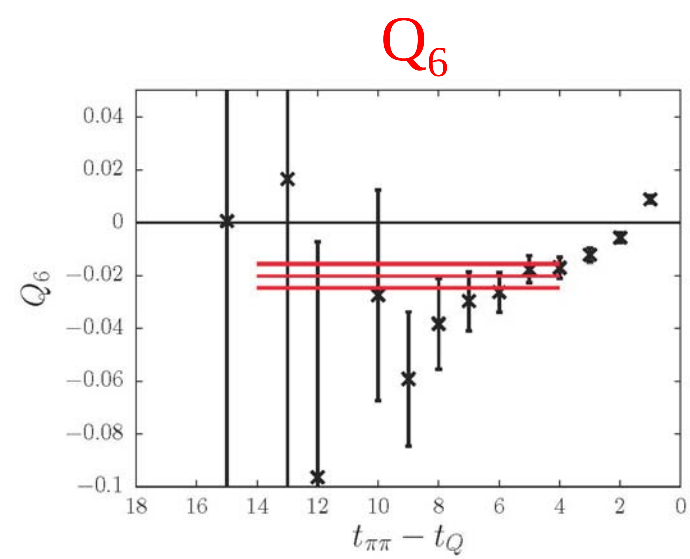
- Determine normalization of $\pi\pi$ interpolating operator
- Determine energy of finite volume, I = 0, $\pi\pi$ state: $E_{\pi\pi}$ = 498(11) MeV
- Determine I = 0 $\pi\pi$ phase shift: δ_0 = 23.8(4.9)(2.2)^o
- Phenomenological result: δ_0 = 38.0(1.3)^o [G. Colangelo]



$\Delta I = 1/2$ $K \to \pi\pi$ matrix elements

- Vary time separation between H_W and $\pi\pi$ operator.
- Show data for all $K - H_W$ separations $t_Q - t_K \geq 6$ and $t_{\pi\pi} - t_K = 10, 12, 14, 16$ and 18.
- Fit correlators with $t_{\pi\pi} - t_Q \geq 4$
- Obtain consistent results for $t_{\pi\pi} - t_Q \geq 3$ or 5





Results

- Determine the complex ΔI=1/2 amplitude A_0
  - Re(A_0) = (4.66 ± 1.00$_{stat}$ ± 1.26 $_{sys}$) x 10^{-7} GeV
  - Expt: (3.3201 ± 0.0018) x 10^{-7} GeV
  - Im(A_0) = (-1.90 ± 1.23$_{stat}$ ± 1.08$_{sys}$) x 10^{-11} GeV
- Calculate Re(ε'/ε):
- Re(ε'/ε) = (1.38 ± 5.15$_{stat}$ ± 4.59$_{sys}$) x 10^{-4}
  - Expt.: (16.6 ± 2.3) x 10^{-4}
  - 2.1 σ difference

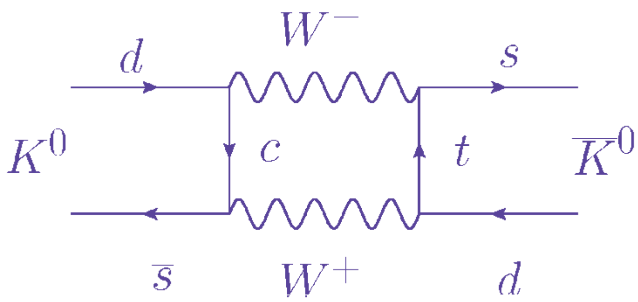
$K^0 - \overline{K^0}$ mixing

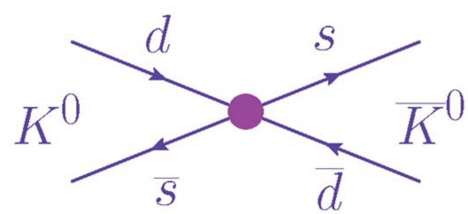
$(\Delta M_K$ & $\varepsilon_K)$

$K^0 - \overline{K^0}$ Mixing

- CP violating: $p \sim m_t$

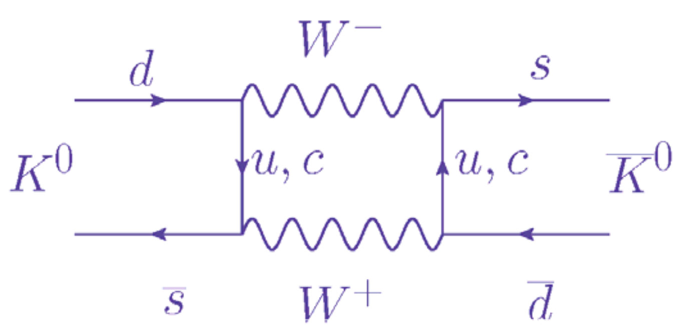
$$\epsilon_K = \frac{i}{2}\left\{\frac{\mathrm{Im}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\,\Gamma_{0\bar{0}}}{\mathrm{Re}\,M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\,\Gamma_{0\bar{0}}}\right\} + i\,\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}$$

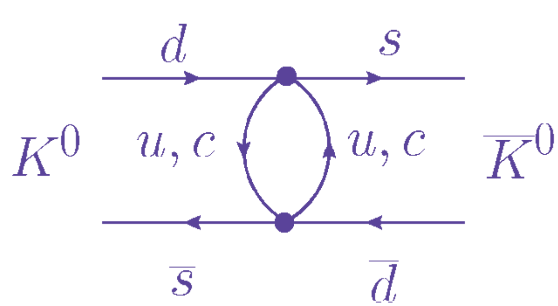




- CP conserving: $p \leq m_c$

$$m_{K_S} - m_{K_L} = 2\mathrm{Re}\{M_{0\bar{0}}\}$$





$K^0 - \overline{K^0}$ Mixing

- ΔS=1 weak decay allows K^0 and $\overline{K^0}$ to decay to the same $\pi - \pi$ state
- Resulting mixing described by Wigner-Weisskopf:

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \left\{ \begin{pmatrix} M_{00} & M_{0\bar{0}} \\ M_{\bar{0}0} & M_{\overline{00}} \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_{00} & \Gamma_{0\bar{0}} \\ \Gamma_{\bar{0}0} & \Gamma_{\overline{00}} \end{pmatrix} \right\} \begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

where

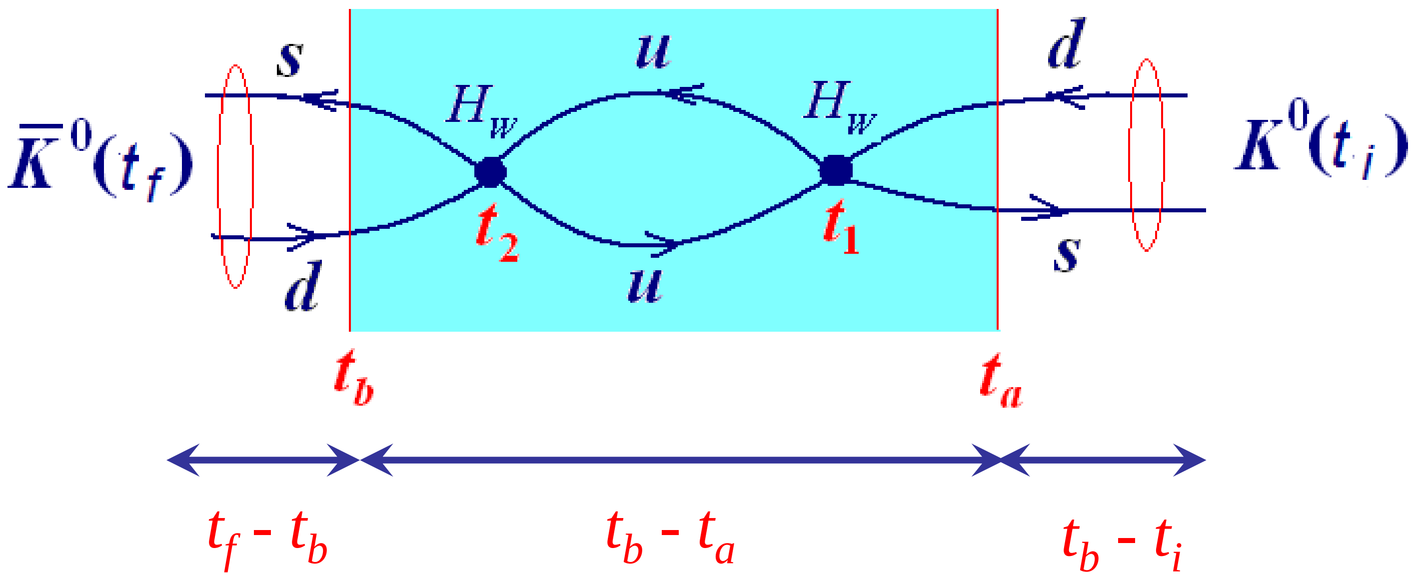
$$\Gamma_{ij} = 2\pi \sum_{\alpha} \int_{2m_\pi}^{\infty} dE \langle i | H_W | \alpha(E) \rangle \langle \alpha(E) | H_W | j \rangle \delta(E - m_K)$$

$$M_{ij} = \sum_{\alpha} \mathcal{P} \int_{2m_\pi}^{\infty} dE \frac{\langle i | H_W | \alpha(E) \rangle \langle \alpha(E) | H_W | j \rangle}{m_K - E}$$

Lattice Version

- Evaluate standard, Euclidean, 2nd order $\overline{K}^0 - K^0$ amplitude:

$$\mathcal{A} = \langle 0|T\left(K^0(t_f)\frac{1}{2}\int_{t_a}^{t_b} dt_2 \int_{t_a}^{t_b} dt_1 H_W(t_2) H_W(t_1) K^{0\,\prime}(t_i)\right)|0\rangle$$



Interpret Lattice Result

$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left(\underset{1.}{-(t_b - t_a)} \underset{2.}{- \frac{1}{M_K - E_n}} \underset{3.}{+ \frac{e^{(M_K - E_n)(t_b - t_a)}}{M_K - E_n}} \right)$$

1. $\Delta m_K^{\,\mathrm{FV}}$
2. Uninteresting constant
3. Growing or decreasing exponential: states with $E_n < m_K$ must be removed!

- Finite volume correction:

$$M_{K_L} - M_{K_S} = 2 \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} - 2 \frac{d(\phi + \delta_0)}{dk}\bigg|_{m_K} |\langle n_0 | H_W | K^0 \rangle|^2 \cot(\phi + \delta_0)\big|_{M_K}$$

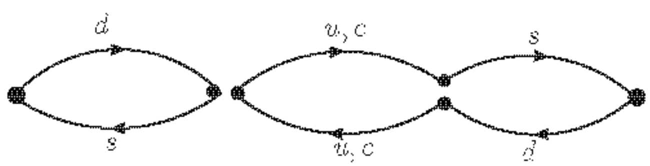
(N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda, arXiv:1504.01170)

$K_L - K_S$ mass difference

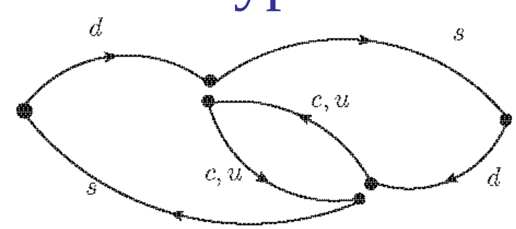
$K_L - K_S$ mass difference

- M_{K_L}-M_{K_S} = 3.483(6) x 10^{-12} MeV: sensitive to 1000 TeV scale physics.
- Perturbative result integrates out charm and shows poor convergence (Brod and Gorbahn)
- We must include charm quark (GIM)

Type 1



Type 2



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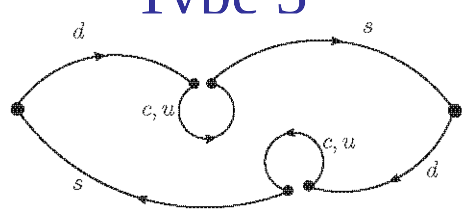

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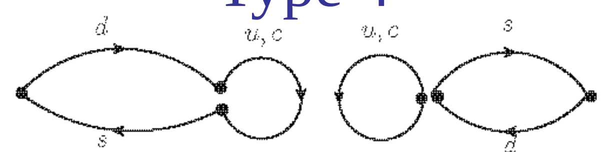
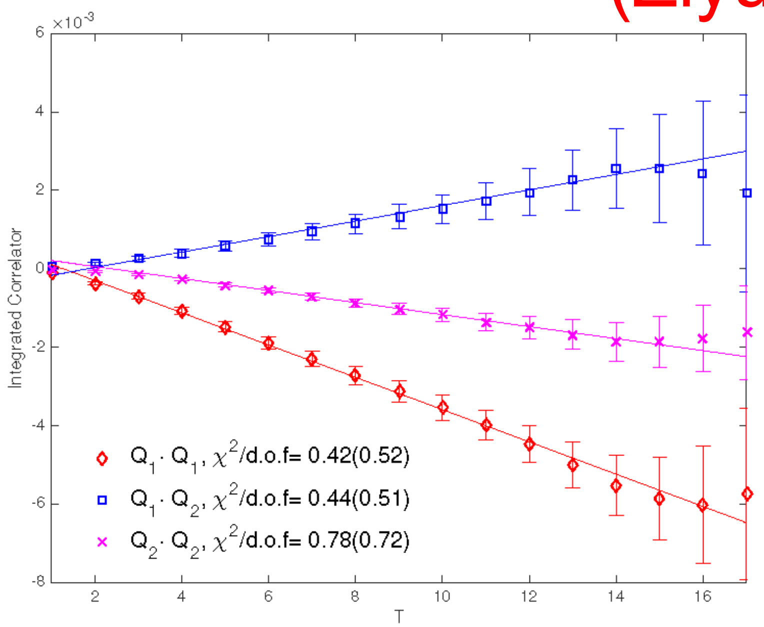


ΔM_K Present Results
(Ziyuan Bai)



	ΔM_K x 10^{+12} MeV
Types 1-4	3.26(63)
Types 1-2	4.19(15)
η	0
π	0.27(14)
$\pi\pi$, $I=0$	-0.097(49)
$\pi\pi$, $I=2$	-6.56(6) x 10^{-4}
D_{FV}	0.029(19)
Expt.	3.483(6)

- m_c = 750 MeV, M_π = 170 MeV
- Disconnected contribution small
- $\pi\pi$ contribution $\sim$2% and FV correction ~0.5%
- New 64^3x128, $1/a$=2.38 GeV, m_c=1.2 GeV, M_π = 140 MeV
  60 configs: ΔM_K = 4.0(2.4) x 10^{-12} MeV (60 → 200?)

Long distance part of ε_K

$K^0 - \overline{K}^0$ mixing: Indirect CP Violation

- CP violation leads to K_L and K_S states which are not CP eigenstates:

$$K_S = \frac{K_+ + \overline{\epsilon} K_-}{\sqrt{1 + |\overline{\epsilon}|^2}} \qquad K_L = \frac{K_- + \overline{\epsilon} K_+}{\sqrt{1 + |\overline{\epsilon}|^2}} \qquad \overline{\epsilon} = \frac{i}{2}\left\{\frac{\mathrm{Im} M_{0\bar{0}} - \frac{i}{2}\mathrm{Im}\Gamma_{0\bar{0}}}{\mathrm{Re} M_{0\bar{0}} - \frac{i}{2}\mathrm{Re}\Gamma_{0\bar{0}}}\right\}$$

- Here $\overline{\varepsilon}$ is closely related to $\epsilon_K = \overline{\epsilon} + i\,\frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}$
- Where $|\varepsilon_K|$ = (2.228 ±0.011) x 10^{-3}
  - Short distance prediction [W.Lee, 1611.04261]:
    $|\varepsilon_K|$ = 1.69 ± 0.17
  - Long distance estimate [Buras, et al. 1002.3612] : results in 6% reduction

ΔS = 1, four-flavor operators

(Ziyuan Bai)

- Choose appropriate $N_f = 4$ effective Hamiltonian:

$$H_W^{\Delta S=1;\Delta C=\pm 1,0} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{q,q'=u,c} V_{q's}^* V_{qd} \sum_{i=1}^{2} C_i Q_i^{q'q} + V_{ts}^* V_{td} \sum_{i=3}^{6} C_i Q_i \right\}$$

$$\begin{aligned} Q_1^{q'q} &= (\bar{s}_i q'_j)_{V-A} (\bar{q}_j d_i)_{V-A} \\ Q_2^{q'q} &= (\bar{s}_i q'_i)_{V-A} (\bar{q}_j d_j)_{V-A} \end{aligned}$$

current x current

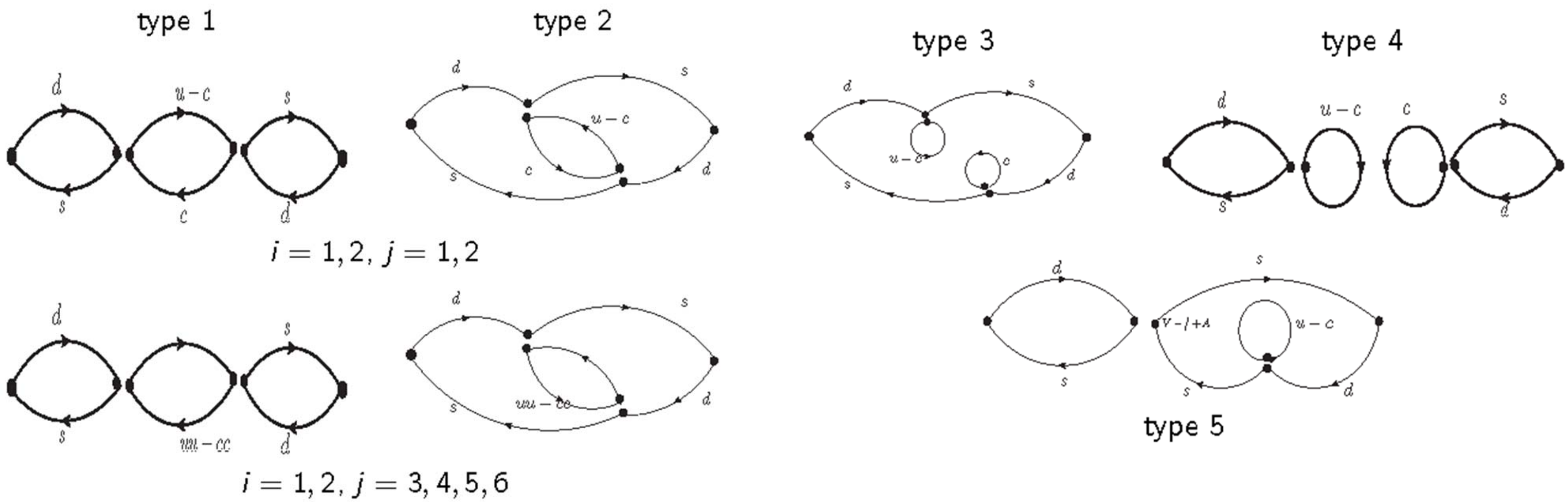
$$\begin{aligned} Q_3 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V-A} \\ Q_4 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V-A} \\ Q_5 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V+A} \\ Q_6 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V+A} \end{aligned}$$

QCD penguin

Diagrams for $\lambda_t \lambda_u$ contribution to ε_K

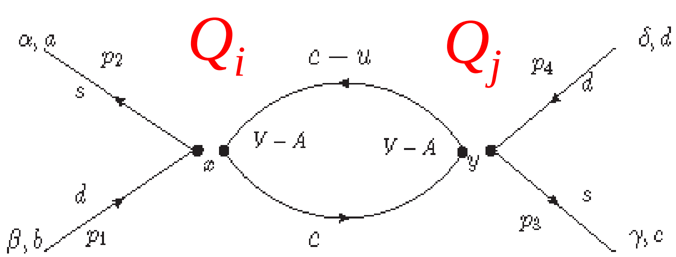
(Ziyuan Bai)

- Identify five types of diagrams

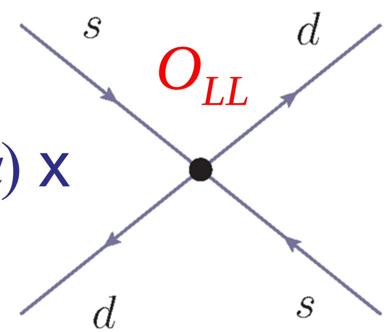


New ΔS = 2 counter term

(Ziyuan Bai)



$-X_{ij}(\mu)$ x



- Subtract $X_{ij}(\mu)\ (\bar{s}\gamma^{\nu}(1-\gamma^5)d)\ (\bar{s}\gamma^{\nu}(1-\gamma^5)d)$ to make off-shell Greens function vanish at $p_i^2 = \mu_{RI}{}^2$
- Define infrared-safe Rome-Southampton normalization for bi-local operator.

Progress toward long-distance part of ε_K

(Ziyuan Bai)

- Compute NLO (one-loop) conversion from bilocal RI to MS
- Preliminary

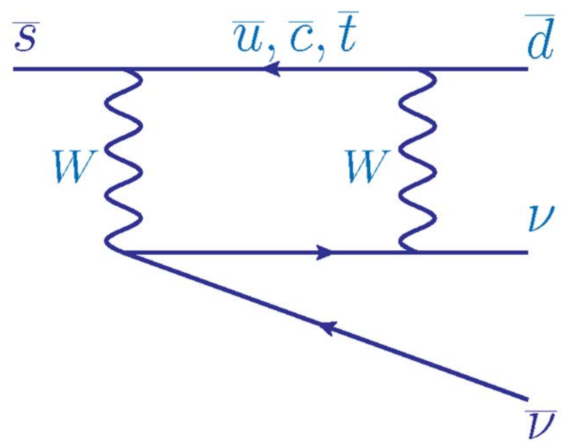
μ_{RI}	$\mathrm{Im}\, M_{\bar{0}0}{}^{ut,RI}$	$\mathrm{Im}\, M_{\bar{0}0}{}^{ut,RI\to\overline{MS}}$	$\mathrm{Im}\, M_{\bar{0}0}{}^{ut,ld\, corr}$	contribution to ε_K
1.54	-1.30(69)	0.352	-0.95(69)	$0.186(135)\times 10^{-3}$
1.92	-1.49(69)	0.476	-1.01(69)	$0.199(135)\times 10^{-3}$
2.11	-1.58(69)	0.537	-1.04(69)	$0.205(135)\times 10^{-3}$
2.31	-1.65(69)	0.599	-1.05(69)	$0.206(135)\times 10^{-3}$
2.56	-1.73(69)	0.674	-1.06(69)	$0.207(135)\times 10^{-3}$

- $|\varepsilon_K|$ = 2.228(11) x 10^{-3} expt.

Rare Kaon Decays

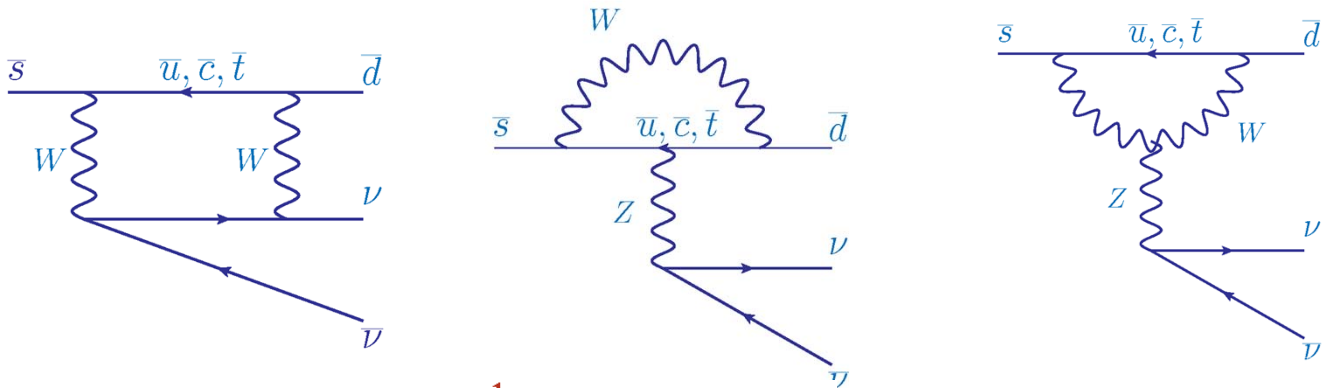
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$
(Xu Feng)



- Flavor changing neutral current
  - Allowed in the Standard Model only in second order
  - Short distance dominated
- Target of NA62 at CERN
  - 100 events in 2-3 years
  - Test Standard Model prediction at 10% level
  - Use lattice for long distance part: 5% effect ?

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ in the Standard Model

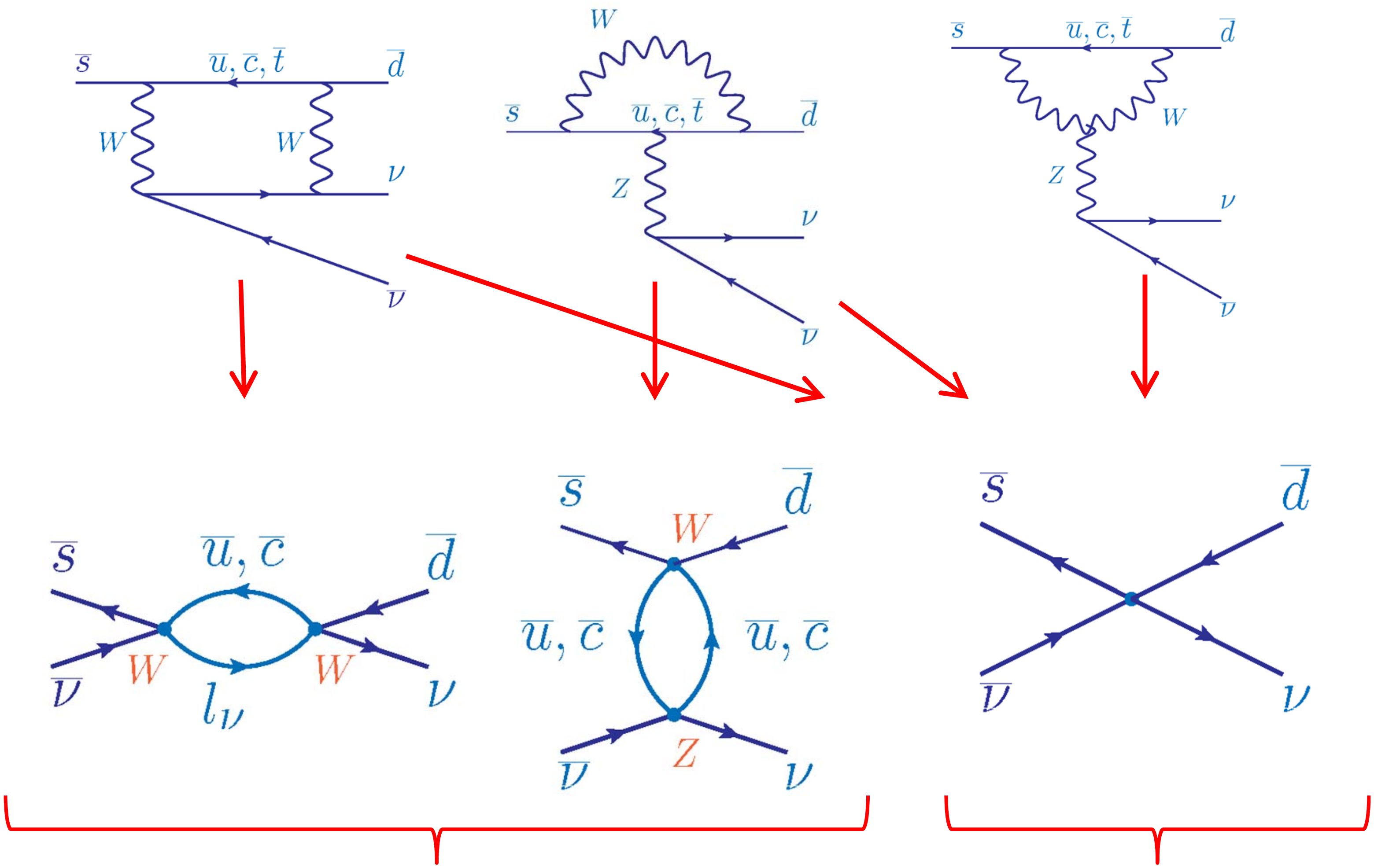


- Factors of $\frac{1}{M_W^4}$ or $\frac{1}{M_W^2 M_Z^2}$ force the largest contribution to come from short distance

Pert. Th.
- Top quark contribution largest.
- GIM implies charm-up $\sim \frac{m_c^2 - m_u^2}{M_W^4} \ln(M_W^2/m_c^2)$

Lattice
- Long distance part $\sim \frac{m_c^2}{M_W^4}$

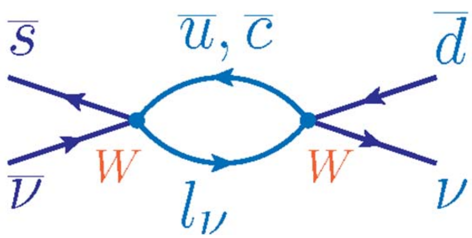
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$ at long distance

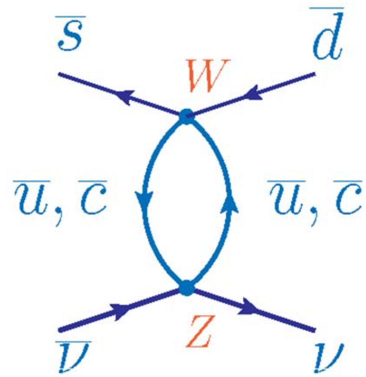


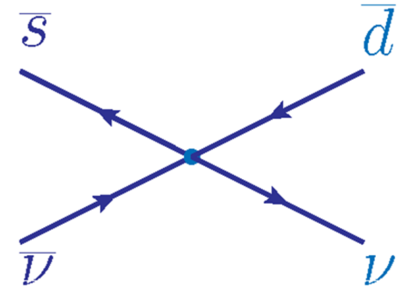
Use lattice QCD

Known $\langle \pi^+ | V_\mu | K^+ \rangle$

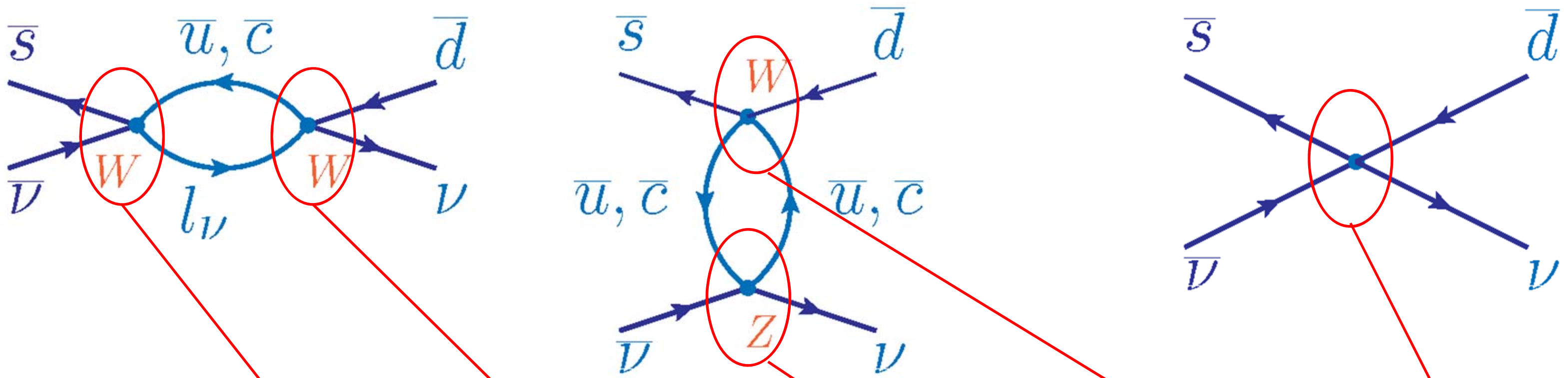
H_{eff} for $K^+ \to \pi^+ \nu \bar{\nu}$





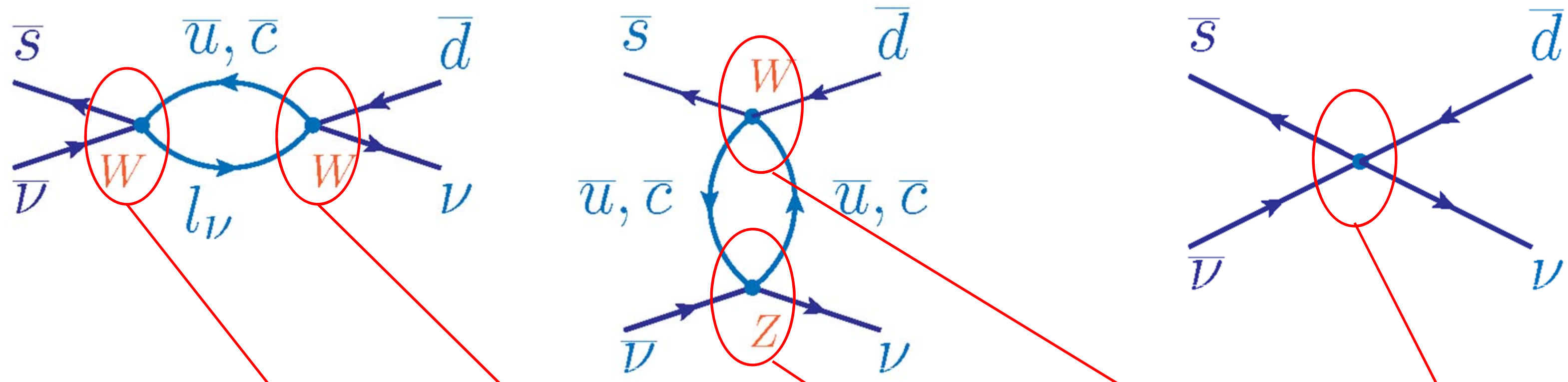


H_{eff} for $K^+ \to \pi^+ \nu \bar{\nu}$



$$\mathcal{H}_{\text{eff}} = +\frac{G_F}{\sqrt{2}} \left\{ \sum_{\substack{q=u,c \\ \ell=e,\mu,\tau}} \left(V_{qs}^* O_{q\ell}^{\Delta S=1} + V_{qd} O_{q\ell}^{\Delta S=0} \right) + \sum_{\ell=e,\mu,\tau} O_\ell^Z + \sum_{q=u,c} \lambda_q O_q^W \right\} + O_0$$

H_{eff} for $K^+ \to \pi^+ \nu \bar{\nu}$



$$\mathcal{H}_{\text{eff}} = +\frac{G_F}{\sqrt{2}} \left\{ \sum_{\substack{q=u,c \\ \ell=e,\mu,\tau}} \left(V_{qs}^* O_{q\ell}^{\Delta S=1} + V_{qd} O_{q\ell}^{\Delta S=0} \right) + \sum_{\ell=e,\mu,\tau} O_\ell^Z + \sum_{q=u,c} \lambda_q O_q^W \right\} + O_0$$

$$O^{\Delta S=1} = C_{\Delta S=1} (\bar{s}q)_{V-A} (\bar{\ell}\nu_\ell)_{V-A}$$

$$O_q^W = C_1 (\bar{s}_a q_b)_{V-A} (\bar{q}_b d_a)_{V-A} + C_2 (\bar{s}_a q_a)_{V-A} (\bar{q}_b d_b)_{V-A}$$

$$O^{\Delta S=0} = C_{\Delta S=0} (\bar{q}d)_{V-A} (\bar{\ell}\nu_\ell)_{V-A}$$

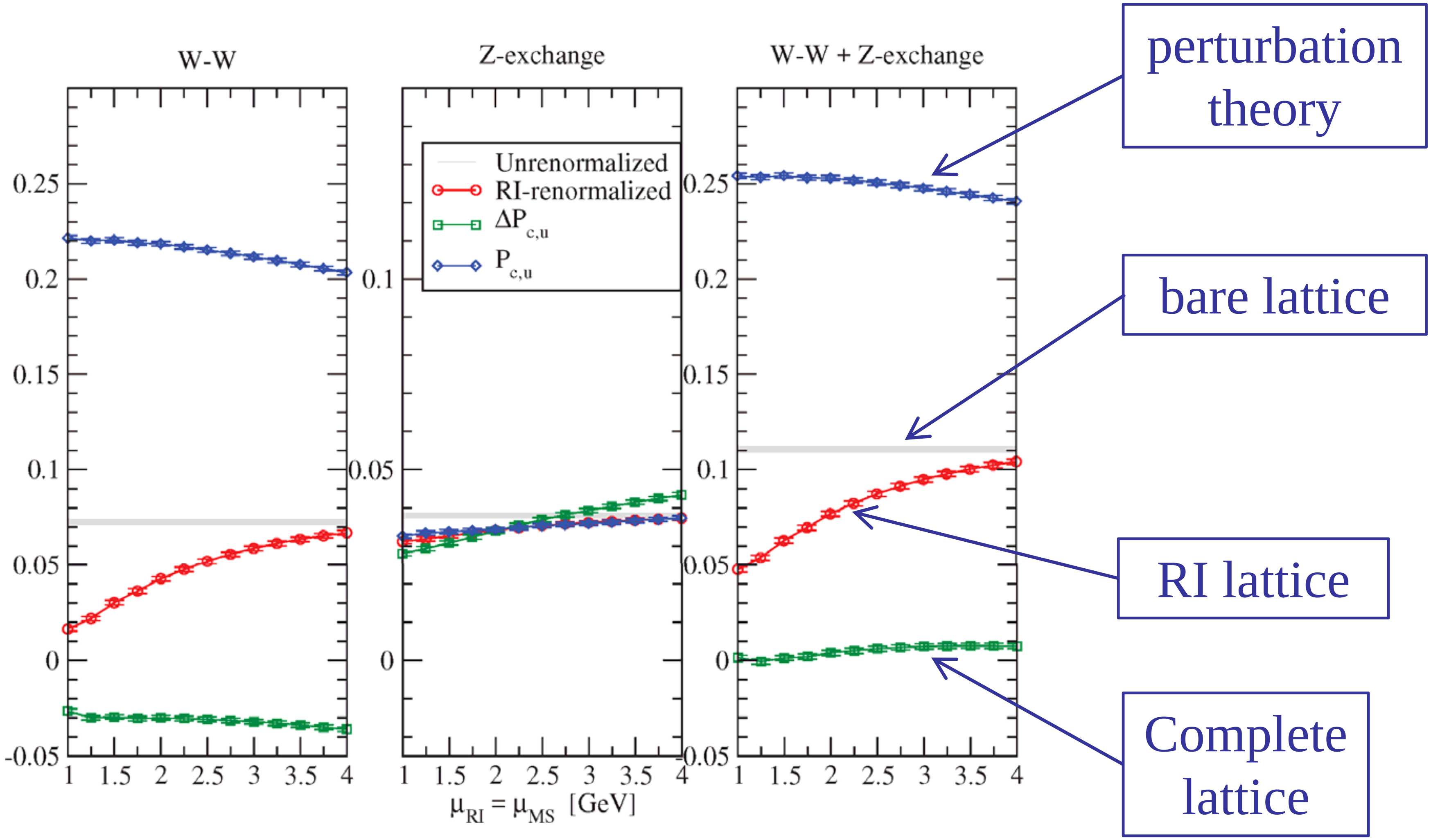
$$O_0 = C_0 \sum_{\ell=e,\mu,\tau} (\bar{s}d)_{V-A} (\bar{\nu}_\ell \nu_\ell)_{V-A}$$

$$O_\ell^Z = C_Z \sum_{q=u,c,d,s} \left(T_3^q \bar{q} \gamma_\mu (1-\gamma_5) q - Q_{\text{em},q} \sin^2\theta_W \bar{q} \gamma_\mu q \right) \overline{\nu_\ell} \gamma_\mu (1-\gamma_5) \ell$$

Exploratory Lattice Calculation

- 16^3 x 32, RBC-UKQCD ensemble
  - 2+1 flavor DWF, $1/a$ = 1.73 GeV
  - M_π = 420 MeV, M_K = 540 MeV,
  - $m_c(2\text{ GeV})^{\overline{\text{MS}}}$ = 863 GeV
- Calculate all diagrams
- 800 configurations
- Low-mode deflation with 100 modes
- Place sources on 32 time slices
- Treat internal lepton as an overlap fermion moving in an ∞ time extent.

Preliminary results: charm



Overview

- Decay rate is short distance dominated:

$$\mathrm{Br} = \kappa_+(1+\Delta_{\mathrm{EM}})\left[\left(\underbrace{\frac{\mathrm{Im}\lambda_t}{\lambda^4}X(x_t)}_{0.270\,\mathrm{x}1.481}\right)^2 + \left(\underbrace{\frac{\mathrm{Re}\lambda_c}{\lambda}P_c}_{-0.974\,\mathrm{x}\,0.365} + \underbrace{\frac{\mathrm{Re}\lambda_t}{\lambda^5}X(x_t)}_{-0.533\,\mathrm{x}1.481}\right)^2\right]$$

- Charm contribution is less than top but is significant (removing charm lowers BR by 50%).
- Result for P_c :
  - Perturbation theory [Buras, et al.,1503.02693]: $P_c = 0.365(12)$
  - LD correction [Isidori, et al., hep-ph/0503107]: $\delta P_{cu} = 0.04(2)$
    (estimate of non-perturbative and $(\Lambda_{\mathrm{QCD}}/m_c)^2$ effects)
  - Exploratory lattice result: $\Delta P_{c,u} = 0.0040(13)(32)$
    (replace PT estimate of bilocal matrix element with result of lattice evaluation)

g - 2 for the muon

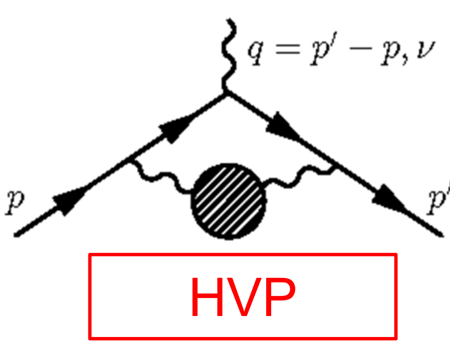
g - 2 for the muon

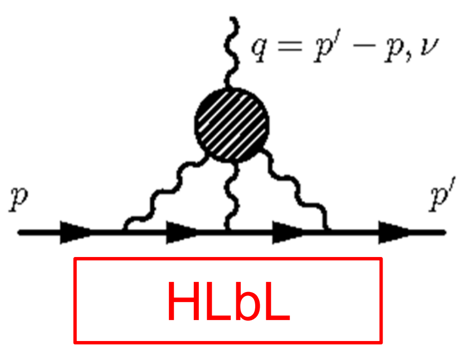
- Because of the larger muon mass, a_{m} = (g_{m} - 2)/2 is sensitive to new physics at high energy

$$\langle \mu(p',s') | J_\nu(0) | \mu(p,s) \rangle = -e\overline{u}(p',s,) \left(F_1(q^2)\gamma_\nu + i\frac{F_2(q^2)}{4m_\mu}\{\gamma_\nu,\gamma_\rho\}q^\rho \right) u(p,s)$$

$$a_\mu = F_2(0) = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2) + \ldots$$

- Tantalizing 3σ difference between the standard model prediction and experiment where σ ~ ½ ppm
- However, QCD enters at order a^2 :





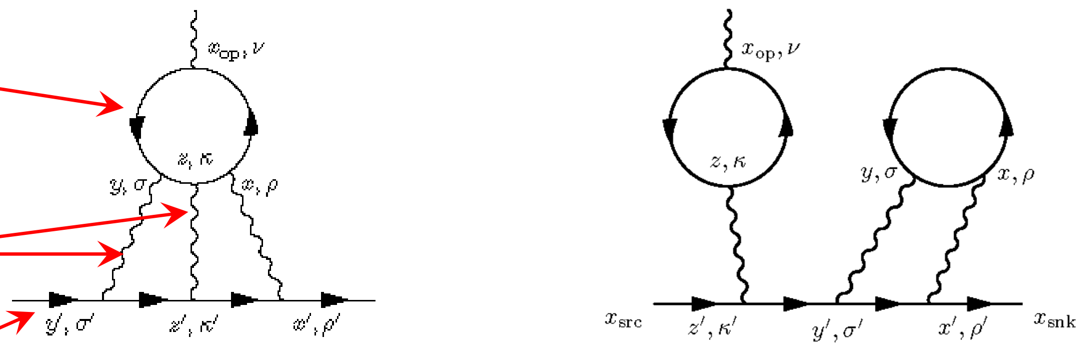
Muon anomalous magnetic moment

Current status: $(g_m\text{-}2)/2$ x

SM Contribution	Value ± Error	Ref
QED (incl. 5-loops)	116584718.951 ± 0.080	[3]
HVP LO	6949 ± 43	[4]
HVP NLO	-98.4 ± 0.7	[4, 5]
HVP NNLO	12.4 ± 0.1	[5]
HLbL	105 ± 26	[6]
Weak (incl. 2-loops)	153.6 ± 1.0	[7]
SM Total (0.51 ppm)	116591840 ± 59	[3]
Experiment (0.54 ppm)	116592089 ± 63	[2]
Difference (Exp − SM)	249 ± 87	[3]

Use lattice QCD to calculate HLbL
(Luchang Jin)

- Compute connected and leading disconnected parts.



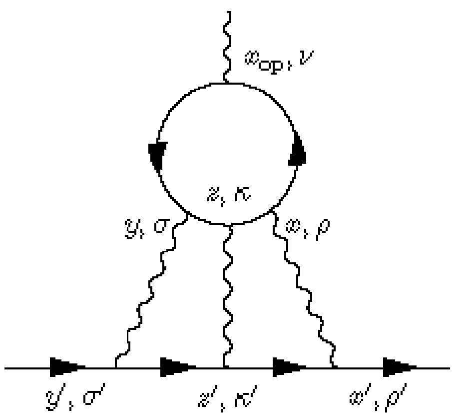
- Important challenge for lattice QCD
  - Treat E&M through an expansion in α_{EM}
  - Massless photon introduces new problems
- Perform three-loop calculation by summing stochastically over x and y

Use lattice QCD to calculate HLbL

- With quarks localized by fixed x and y, we can compute a_m directly from the moment [Blum, et al., 1510.07100]: $\vec{\mu} = \frac{1}{2}\int d^3r \left(\vec{r} \times \vec{j}(\vec{r})\right)$

$r = x - y$

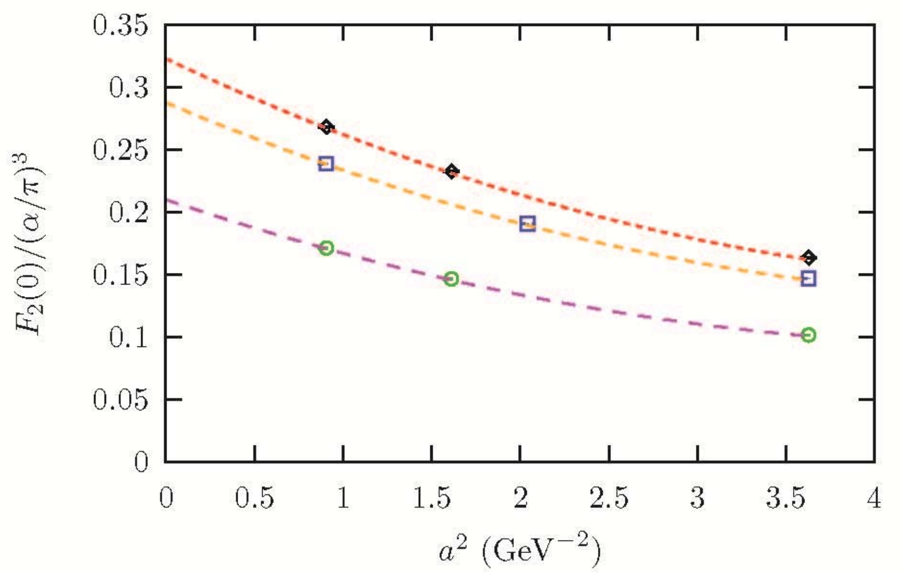
$$\frac{F_2(0)}{2m_\mu}\overline{u}(\vec{0},s')\vec{\Sigma}u(\vec{0},s) = \frac{1}{2}\sum_{r,z,x_{\text{op}}} \vec{x}_{\text{op}} \times i\overline{u}(\vec{0},s')\vec{\mathcal{F}}\left(\frac{r}{2},-\frac{r}{2},z,x_{\text{op}}\right)u(\vec{0},s)$$



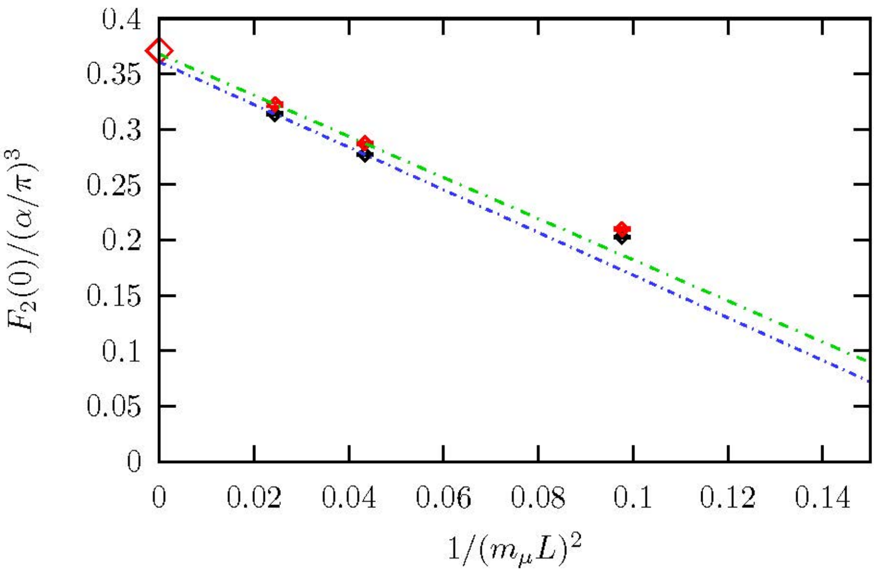
- No need to extrapolate to $q^2 = 0$.
- Test by replace the quark by a muon loop:

Test for muon loop

Limit of vanishing lattice spacing



Infinite volume limit



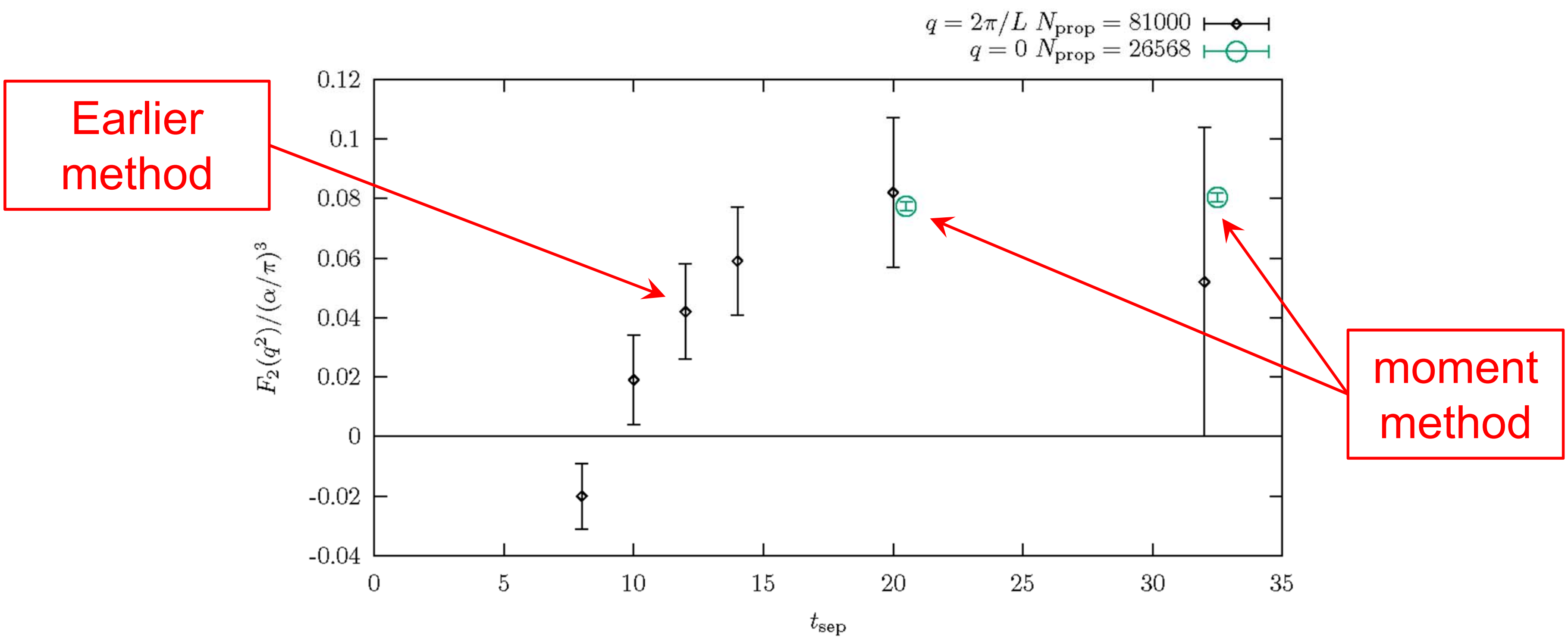
$$[F_2(0)]_{\rm quad}/(\alpha/\pi)^3 = 0.3679(42) - 1.86(11)/(m_\mu L)^2,$$

$$[F_2(0)]_{\rm lin}/(\alpha/\pi)^3 = 0.3608(30) - 1.92(8)/(m_\mu L)^2,$$

$$[F_2(0)]_{\rm PT}/(\alpha/\pi)^3 = 0.3710052921,$$

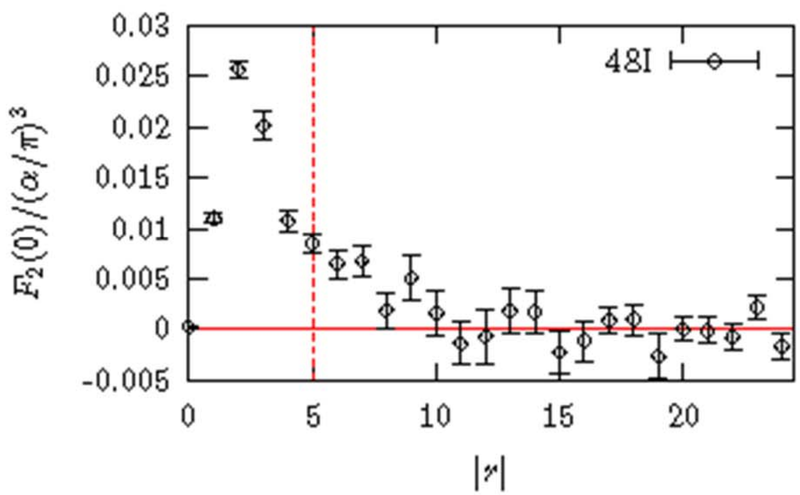
Muon anomalous magnetic moment

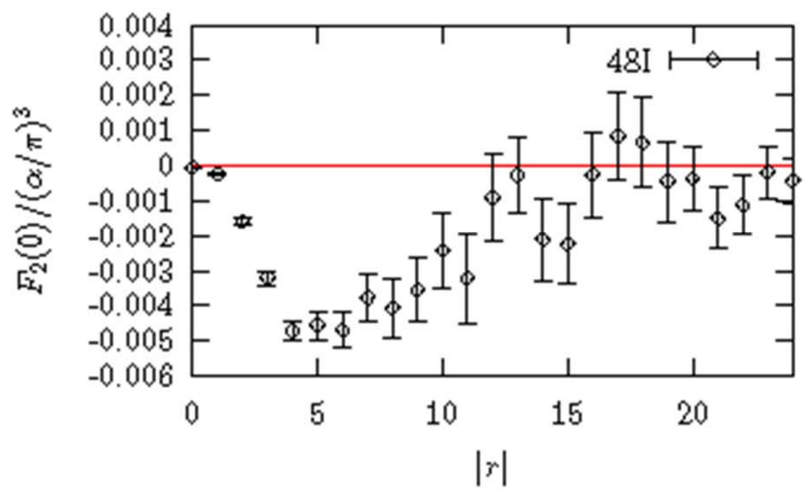
- Large improvement over early lattice gauge treatment of both gluons and photons [Blum et al,1407.2923]:



Result for HLbL

- 48^3 x 96 and physical pion mass result:





- Expect sizable finite volume and discretization errors:

$$a_\mu^{\mathrm{cHLbL}} = (11.60 \pm 0.96) \times 10^{-10}$$
$$a_\mu^{\mathrm{dHLbL}} = (-6.25 \pm 0.80) \times 10^{-10}$$
$$a_\mu^{\mathrm{HLbL}} = (5.35 \pm 1.35) \times 10^{-10}$$

arXiv:1610.04603

Outlook

- Lattice QCD is now capable of 1st-principles calculation of:
  - $K \rightarrow \pi\pi$, $\Delta I = 3/2$ and $1/2$, ε'/ε
  - $M_{K_L} - M_{K_S}$ and long distance. contribution to ε
  - Long distance parts of $K \rightarrow \pi l \bar{l}$, $K \rightarrow \pi \nu\bar{\nu}$
  - Hadronic light-by-light scattering part of g_m-2
- First realistic calculation of ΔM_K underway
- Must wait for next generation of computers to accurately include charm