Recent Progress in Hadronic Parity Violation

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   - Soft-Pion Theorem for $\Delta I=1$ PV
   - PQChPT and Disconnected Diagrams
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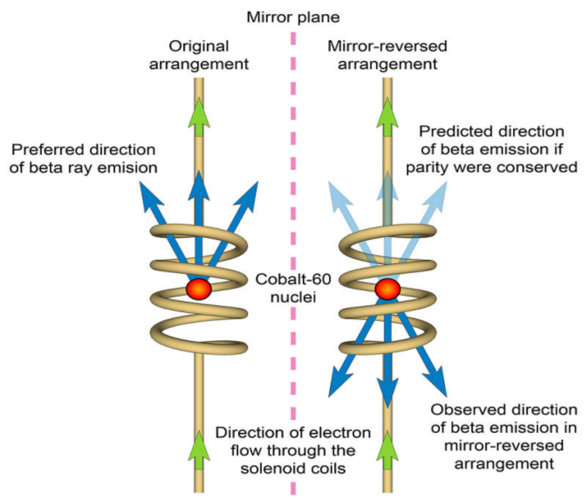
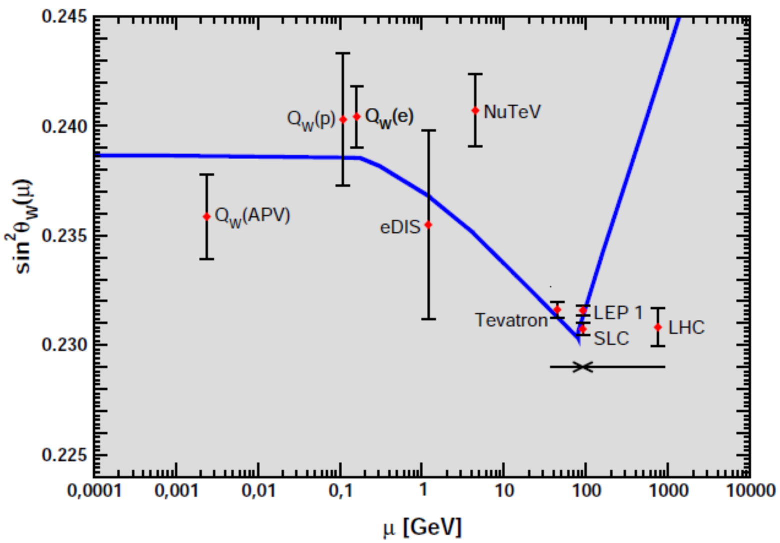
Motivations

Motivations

- There is a long history in the study **of Parity (P)-violation** in fundamental interactions
- P-violation in **charged weak current**: beta decay experiment of Co^{60} (Wu et al, 1957) provided hints for the V-A interaction

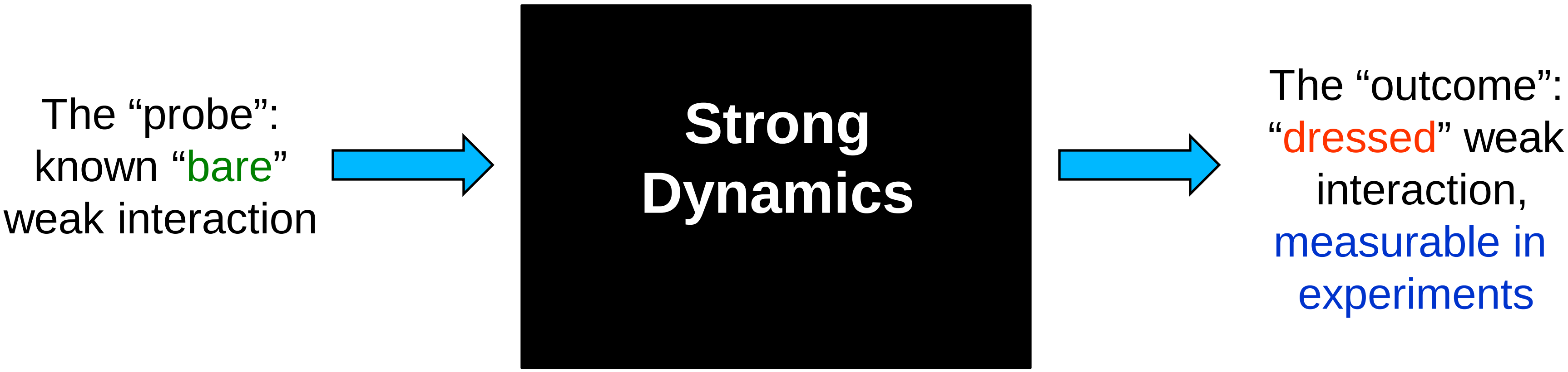




- P-violation in **neutral weak-current**: flavor-diagonal PV-interactions. E.g. PVES and PVDIS. Important in determining the weak mixing angle.
- Probing P-violation in **non-leptonic processes** is more difficult due to the dominance of strong interaction.

What do we want to learn?

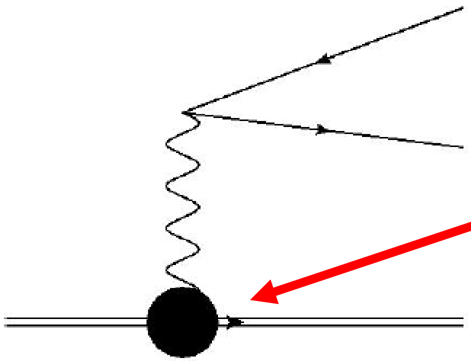
- Studies of hadronic **Weak** interaction helps improving our understandings of the black box of **Strong** dynamics!



- Furthermore, it allows for studies of strong dynamics that are difficult to probe in pure QCD processes themselves, such as strongly-dressed **axial** interactions.

Motivations

- Along this line of thought, we are more interested in "**UNPROTECTED**" hadronic weak processes.
- What does "protected" mean? E.g. beta decay of 0^+ nuclei:



The hadronic matrix element:

$$V_{ud}\left\langle 0^+ \left| \bar{d}\gamma^0(1-\gamma_5)u \right| 0^+ \right\rangle = 2\sqrt{2}V_{ud}m$$

is "**protected**" by isospin symmetry!

- This is good for extracting weak parameters, but **not very useful in probing strong dynamics**.
- Therefore, we wish to studies weak hadronic processes that are **NOT** protected by well-known symmetries of QCD!

Motivations

- Weak interaction is too weak at low energy!

$$\frac{A_{\text{weak}}}{A_{\text{strong}}} \sim G_F m_\pi^2 \sim \frac{m_\pi^2}{m_W^2} \sim 10^{-7}$$

- Usually overshadowed by pure strong interaction; exceptions are observables that **violate discrete symmetries**: e.g. **hadronic P-violations** ,which are also **unprotected** processes in general!
- Examples of low-energy hadronic P-violation experiments:
  - Longitudinal Analyzing Power (LAP)
  - Gamma-ray asymmetry
  - Gamma-ray circular polarization
  - ….
- Fundamental challenge for **theorists**: are we able to precisely **reproduce experimental results from SM**?

Theories and Experiments

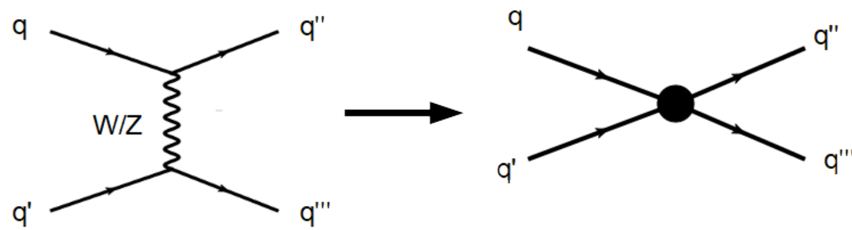
Theories

- Electroweak interaction in quark sector:

$$\mathcal{L}^{q}_{\rm ew} = -eA_\mu J^\mu_\gamma - \frac{g}{\sqrt{2}}\left\{W^+_\mu J^\mu_W + W^-_\mu J^{\mu\dagger}_W\right\} - \frac{g}{\cos\theta_W} Z_\mu J^\mu_Z$$

(CKM matrix included in the charged weak current)

- Heavy boson integrated out, in replacement of four-quark operators:



$$\mathcal{L}^{q}_{ew} \to \sum_i C^i_{4q}(\mu)\hat{O}^i_{4q}(\mu)$$

- Possible isospin channels: $\boxed{\Delta I=0,1,2}$
- The Wilson coefficients are (more or less) known
- At lower energy the effective degrees of freedom become hadrons. Effective field theories (EFTs) are used.

Theories

Parameters Known? | **Energy Scale**

YES — **SM EW Lagrangian**

m_W

YES (……) — **Four-Quark Operators**

Λ_χ

NO/ From Experiment

Desplanques et al., Ann. Phys. 124 (1980) 449
Meson-Exchange Theory (DDH)

Zhu et al., Nucl.Phys.A 748 (2005) 435
Pion-full EFT

m_π

NO/ From Experiment — **Pion-less EFT/ NR nuclear potential**

Theories

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Zhu et al., Nucl.Phys.A 748 (2005) 435
Pion-full EFT

m_π

NO/ From Experiment — **Pion-less EFT/ NR nuclear potential**

Equivalent at very low energy. Five independent parameters in terms of **Danilov S-P partial waves**:

$$\boxed{\Lambda_0^{{}^1S_0-{}^3P_0},\ \Lambda_0^{{}^3S_1-{}^1P_1},\ \Lambda_1^{{}^1S_0-{}^3P_0},\ \Lambda_1^{{}^3S_1-{}^3P_1},\ \Lambda_2^{{}^1S_0-{}^3P_0}}$$

Danilov, Phys.Lett,18 (1965)40, PLB 35 (1971)579

Theories

Classic Model: the DDH meson-exchange theory

$$\mathcal{H}_{\rm wk} = \frac{h^1_{\pi NN}}{\sqrt{2}} \bar{N}(\tau \times \pi)_3 N$$
$$- \bar{N}\left(h^0_\rho \tau \cdot \rho^\mu + h^1_\rho \rho^\mu_3 + \frac{h^2_\rho}{2\sqrt{6}}(3\tau_3\rho^\mu_3 - \tau \cdot \rho^\mu) \right) \gamma_\mu \gamma_5 N$$
$$- \bar{N}(h^0_\omega \omega^\mu + h^1_\omega \tau_3 \omega^\mu) \gamma_\mu \gamma_5 N + h'^1_\rho \bar{N}(\tau \times \rho^\mu)_3 \frac{\sigma_{\mu\nu} k^\nu}{2m_N} \gamma_5 N.$$

Meson exchanged: π, ρ, ω

- Special roles of **ΔI=1** hadronic parity-violation:

  a) Primarily probing the **neutral weak current**

  $$J_W \sim \bar{u}_L \gamma^\mu d_L : \Delta I = 1 \,(\mathrm{V}_{\mathrm{CKM}} = 1 \,\mathrm{limit})$$
  $$J_W^+ \otimes J_W \to (\Delta I = 0) \oplus (\Delta I = 2)$$

  b) The only **pion-exchange interaction** in DDH model. Thus, for a very long time it was expected to dominate the long-range nuclear parity violation.

Theories

Recent **large-N_c analysis** suggests a **suppression** of ΔI=1 PV:

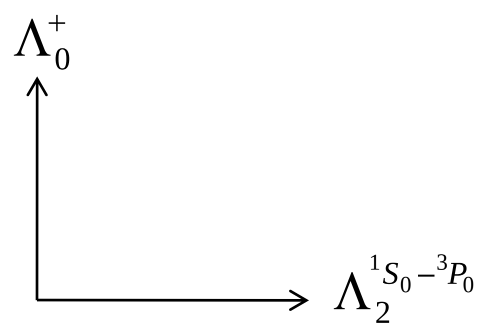
The "Rosetta Stone" (one version of EFT)

Coeff	DDH	Girlanda	Large N_c
$\Lambda_0^+ \equiv \frac{3}{4}\Lambda_0^{^3S_1-^1P_1} + \frac{1}{4}\Lambda_0^{^1S_0-^3P_0}$	$-g_\rho h_\rho^0(\frac{1}{2}+\frac{5}{2}\chi_\rho) - g_\omega h_\omega^0(\frac{1}{2}\text{-}\frac{1}{2}\chi_\omega)$	$2\mathcal{G}_1 + \tilde{\mathcal{G}}_1$	$\sim N_c$
$\Lambda_0^- \equiv \frac{1}{4}\Lambda_0^{^3S_1-^1P_1} - \frac{3}{4}\Lambda_0^{^1S_0-^3P_0}$	$g_\omega h_\omega^0(\frac{3}{2}+\chi_\omega) + \frac{3}{2}g_\rho h_\rho^0$	$-\mathcal{G}_1 - 2\tilde{\mathcal{G}}_1$	$\sim 1/N_c$
$\Lambda_1^{^1S_0-^3P_0}$	$-g_\rho h_\rho^1(2+\chi_\rho) - g_\omega h_\omega^1(2+\chi_\omega)$	$\mathcal{G}_2$	$\sim \sin^2\theta_w$
$\Lambda_1^{^3S_1-^3P_1}$	$\frac{1}{\sqrt{2}}g_{\pi NN}h_\pi^1\left(\frac{m_\rho}{m_\pi}\right)^2 + g_\rho(h_\rho^1 - h_\rho^{1\prime}) - g_\omega h_\omega^1$	$2\mathcal{G}_6$	$\sim \sin^2\theta_w$
$\Lambda_2^{^1S_0-^3P_0}$	$-g_\rho h_\rho^2(2+\chi_\rho)$	$-2\sqrt{6}\mathcal{G}_5$	$\sim N_c$

Gardner, Haxton and Holstein, Ann.Rev.Nucl.Part.Sci 67 (2017) 1917

"Hadronic parity violation: a new paradigm"

Two LO LECs at large-N_c: ΔI=0,2



Experiments

(1) **Longitudinal Analyzing Power (LAP)** in scattering between longitudinally-polarized nucleon beam and unpolarized target:

$$\vec{N} + N' \to N + N'$$

$$\bar{A}_L(E,\theta_1,\theta_2) = \frac{\int_{\theta_1}^{\theta_2} d\Omega (d\sigma_+ - d\sigma_-)}{\int_{\theta_1}^{\theta_2} d\Omega (d\sigma_+ + d\sigma_-)}.$$

$\vec{p} + p:$ Eversheim et al., PLB 256, 11(1991)
Kistryn et al., PRL 58, 1616 (1987)
Nagle, AIP Conf. Proc., 51, AIP, New York, (1978) 224;
Berdoz et al., PRL 87 (2001) 272301 TRIUMF, 221MeV

$\vec{p} + {}^4\mathrm{He}:$ Lang et al., PRL 54, 170 (1985)

"A somewhat discouraging aspect of the effort to understand hadronic PNC has been the slow pace of (experimental) results since 1980..."

Haxton and Holstein, Prog.Part.Nucl.Phys 71 (2013) 185

Experiments

(2) **Gamma-ray asymmetry** in radiative capture of longitudinally-polarized nucleon or decay of polarized nucleus:

$$\overrightarrow{N} + N' \to A + \gamma \qquad\qquad \vec{A}' \to A + \gamma$$

$$A_\gamma(\theta) = \frac{d\sigma_+(\theta) - d\sigma_-(\theta)}{d\sigma_+(\theta) + d\sigma_-(\theta)} = a_\gamma \cos\theta,$$

$\vec{n} + p$: Cavaignac, Vignon and Wilson, PLB 67 (1977) 148;
Gericke et al., PRC 83 (2011) 015505

^{19}F : Adelberger et al., PRC 27 (1983) 2833
Elsener et al, Nucl.Phys.A 461 (1987) 579; PRL 52 (1984) 1476

"A somewhat discouraging aspect of the effort to understand hadronic PNC has been the slow pace of (experimental) results since 1980..."

Haxton and Holstein, Prog.Part.Nucl.Phys 71 (2013) 185

Experiments

(3) The **circular polarization P_γ** of emitted photon in radiative capture of unpolarized nucleon or nuclear transition:

$$N + N' \to A + \gamma \qquad\qquad A' \to A + \gamma$$

$n + p:$ Lobashov et al., Nucl.Phys. A 197 (1972) 241;
Knyaz'kov et al., Nucl. Phys. A 417 (1984) 209

$^{18}\mathrm{F}:$ Barnes et al., PRL 40 (1978) 840;
Bini et al., PRL 55 (1985) 795;
Ahrens et al., Nucl.Phys.A 390 (1982) 486;
Page et al., PRC 35 (1987) 1119.

* Another class of experiments involve the measurement of **nuclear anapole moments**, but their constraints on PV are less reliable due to complexities in many-body calculation.

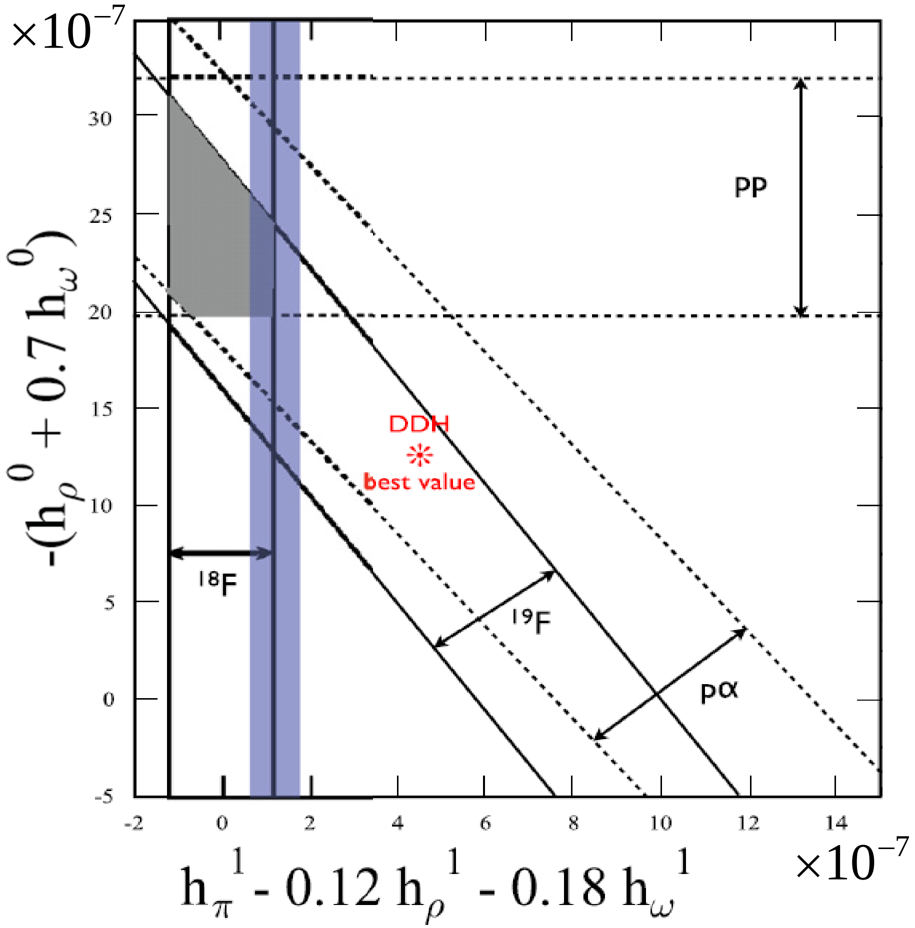
"A somewhat discouraging aspect of the effort to understand hadronic PNC has been the slow pace of (experimental) results since 1980..."

Haxton and Holstein, Prog.Part.Nucl.Phys 71 (2013) 185

Experiments

Probes of **ΔI=1 PV** have so far returned null results:

Haxton and Holstein, Prog.Part.Nucl.Phys.71(2013)185



Models	h_π^1
DDH Best Value [1]	4.6×10^{-7}
Quark Model [2]	1.3×10^{-7}
Quark Model [3]	2.7×10^{-7}
Chiral Soliton [4]	1.8×10^{-8}
Chiral Soliton [5]	2×10^{-8}
Chiral Soliton [6]	1.0×10^{-7}
QCD Sum Rules [7]	3×10^{-7}
QCD Sum Rules [8]	3.4×10^{-7}

[1] Desplanques, Donoghue and Holstein, Ann. Phys. 124, 449 (1980)
[2] Dubovik and Zenkin, Ann. Phys. 172,100 (1986)
[3] Feldman, Crawford, Dubach and Hostein, PRC, 43, 863 (1991)
[4] Kaiser and Meissner, Nucl. Phys. A 489, 671 (1988)
[5] Kaiser and Meissner. Nucl. Phys. A 499, 699 (1989)
[6] Meissner and Weigel, Phys.Lett.,B447,1(1999)
[7] Henley, Hwang and Kisslinger, Phys.Lett.B 440,449 (1998)
[8] Lobov, Phys.Atom.Nucl. 65,534 (2002)

All indicate **suppression** wrt NDA estimate:

$$h_\pi^1 \sim G_F F_\pi \Lambda_\chi \sim 10^{-6}$$

Experiments

Ongoing experimental efforts:

- **NPDGamma** $\vec{n}p \to d\gamma$ at the Spallation Neutron Source (SNS), Oak Ridge: sensitive to I=1 parity violation. Target: $A_\gamma \sim 10^{-8}$
- Proton asymmetry in $\vec{n} + {}^3\mathrm{He} \to {}^3\mathrm{H} + p$ at SNS: projected accuracy of $1.6*10^{-8}$.
- Neutron spin rotation in $\vec{n} + {}^4\mathrm{He}$ at the National Bureau of Standards and Technology (NIST)

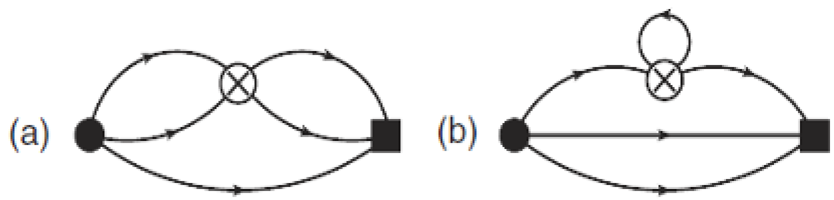
Experimental opportunities in China:

- **China Initial Accelerator-driven subcritical reactor research facility (CI-ADS)** and **High-flux heavy ion accelerator (HIAF)** are designed to provide high-intensity proton beam. Question: can the beam be made **polarized**?
  - If yes, then a TRIUMF-like experiment (measuring LAP) is possible;
  - If no, then detectors will be needed to measure the circular polarization of final products.

Lattice QCD

Lattice QCD: ΔI=1

- Lattice effort to evaluate h_π^1: Wasem, PRC 85, 022501 (2012)



Connected   Quark Loop

● : p state
■ : nπ state

$$h_\pi^1 = -\frac{i}{2m_N} \lim_{p_\pi \to 0} \langle n\pi^+ | \mathcal{L}_{\mathrm{PV}}^w(0) | p \rangle .$$

Signal/noise ratio too low!

$$h_{\pi NN}^{1,\mathrm{con}} = \left(1.099 \pm 0.505^{+0.058}_{-0.064}\right) \times 10^{-7},$$

Compatible with experiments and models

- Signal extracted only from **connected diagram**.
- Extra complications due to the **existence of π**:
  - Three-quark interpolating operator for Nπ state: small overlap
  - Energy insertion needed due to $E_{n\pi} > E_p$
  - Re-scattering effect of Nπ induces power-law finite-volume correction
- Unphysical pion mass $m_\pi = 389\,\mathrm{MeV}$
- Finite lattice spacing effect not addressed

Lattice QCD: ΔI=2

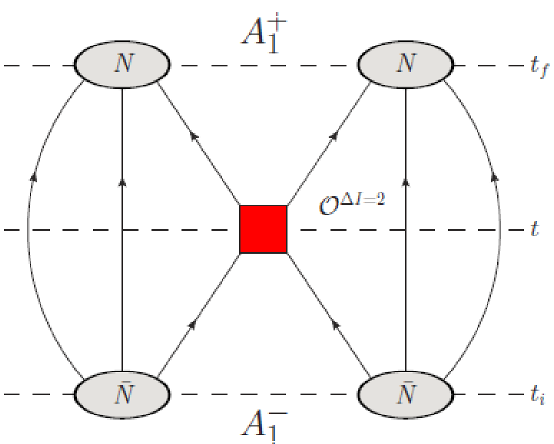
- Ongoing lattice calculation of ΔI=2 PV **NN-scattering amplitude**:

Kurth et al., PoS LATTICE2015 (2016) 329

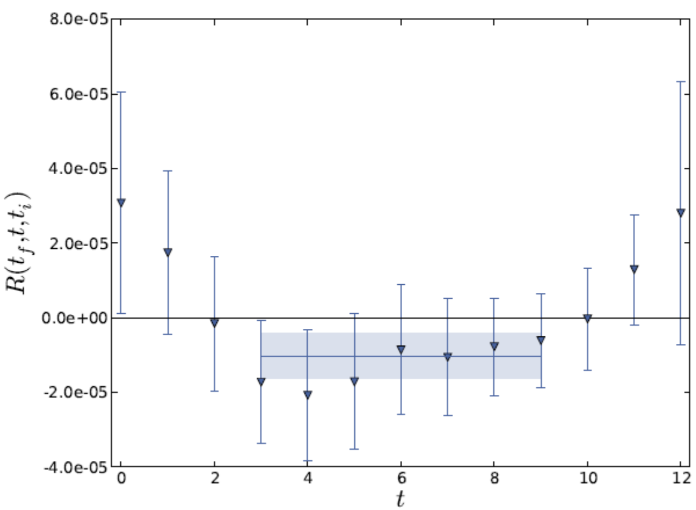
$$\mathscr{O}^{\Delta I=2}(t) = -\sum_{\mathbf{x}} \left[(\bar{q}\gamma_\mu\gamma_5\tau^3 q)(\bar{q}\gamma_\mu\tau^3 q) - \frac{1}{3}(\bar{q}\gamma_\mu\gamma_5\vec{\tau}q)(\bar{q}\gamma_\mu\vec{\tau}q) \right](\mathbf{x},t).$$

$$\Lambda_2^{{}^1S_0 - {}^3P_0} \propto \left\langle pp({}^1S_0)(t_f) \middle| O^{\Delta I=2}(t) \middle| pp({}^3P_0)(t_i) \right\rangle = -1.0(7)\times 10^{-5}$$

- Advantage: **no** disconnected (quark loop) diagrams



(one out of **2208** contractions!!!)



$m_\pi \sim 800$ MeV

- **Very preliminary**. Most lattice complexities are not addressed.

Some Recent Theoretical Developments

Soft-Pion Theorem for ΔI=1 PV

- Many complications in the study of h_π^1 stem from the existence of pion which makes the final state a two-body problem

$$h_\pi^1 = -\frac{i}{2m_N} \lim_{p_\pi \to 0} \langle n\pi^+ | \mathcal{L}_{\rm PV}^w(0) | p \rangle .$$

- Special role of pion in QCD: Goldstone boson resulting from SSB of SU(2) chiral symmetry!

$$\mathrm{SU(2)_V \times SU(2)_A \to SU(2)_V}$$

$$\mathrm{pNGB}:\{\pi^1, \pi^2, \pi^3\}$$

- Partially-Conserved Axial Current (PCAC) relation:

$$\lim_{p_\pi \to 0} \langle N' \pi^a | \hat{O} | N \rangle \approx \frac{i}{F_\pi} \langle N' | [\hat{O}, Q_A^a] | N \rangle$$

- Equivalent to leading-order Chiral Perturbation Theory (ChPT)

Soft-Pion Theorem for ΔI=1 PV

- Our alternative approach to study h^1_π:

X.Feng , F.K.Guo and CYS, arXiv:1711.09342[nucl-th]. Accepted by PRL.

- Starting point: instead of studying a P-odd N→N'π matrix element of four-quark operators, we may study a P-even N→N matrix element.

θ | Chiral Counterparts | θ'

$$\begin{aligned}
\theta_1 &= \bar{q}\gamma^\mu q\bar{q}\gamma_\mu\gamma_5\tau_3 q \\
\theta_2 &= \bar{q}_a\gamma^\mu q_b\bar{q}_b\gamma_\mu\gamma_5\tau_3 q_a \\
\theta_3 &= \bar{q}\gamma^\mu\gamma_5 q\bar{q}\gamma_\mu\tau_3 q \\
\theta_4 &= \bar{q}_a\gamma^\mu\gamma_5 q_b\bar{q}_b\gamma_\mu\tau_3 q_a \\
\theta_1^{(s)} &= \bar{s}\gamma^\mu s\bar{q}\gamma_\mu\gamma_5\tau_3 q \\
\theta_2^{(s)} &= \bar{s}_a\gamma^\mu s_b\bar{q}_b\gamma_\mu\gamma_5\tau_3 q_a \\
\theta_3^{(s)} &= \bar{s}\gamma^\mu\gamma_5 s\bar{q}\gamma_\mu\tau_3 q \\
\theta_4^{(s)} &= \bar{s}_a\gamma^\mu\gamma_5 s_b\bar{q}_b\gamma_\mu\tau_3 q_a .
\end{aligned}
\qquad \longleftrightarrow \qquad
\begin{aligned}
\theta'_1 &= \bar{q}\gamma^\mu q\bar{q}\gamma_\mu\tau_3 q \\
\theta'_2 &= \bar{q}_a\gamma^\mu q_b\bar{q}_b\gamma_\mu\tau_3 q_a \\
\theta'_3 &= \bar{q}\gamma^\mu\gamma_5 q\bar{q}\gamma_\mu\gamma_5\tau_3 q \\
\theta'_4 &= \bar{q}_a\gamma^\mu\gamma_5 q_b\bar{q}_b\gamma_\mu\gamma_5\tau_3 q_a \\
\theta_1^{(s)\prime} &= \bar{s}\gamma^\mu s\bar{q}\gamma_\mu\tau_3 q \\
\theta_2^{(s)\prime} &= \bar{s}_a\gamma^\mu s_b\bar{q}_b\gamma_\mu\tau_3 q_a \\
\theta_3^{(s)\prime} &= \bar{s}\gamma^\mu\gamma_5 s\bar{q}\gamma_\mu\gamma_5\tau_3 q \\
\theta_4^{(s)\prime} &= \bar{s}_a\gamma^\mu\gamma_5 s_b\bar{q}_b\gamma_\mu\gamma_5\tau_3 q_a .
\end{aligned}$$

- With PCAC + Wigner-Eckart Theorem:

$$\langle n\pi^+ | \hat{\theta}_i(0) | p\rangle \approx -\frac{\sqrt{2}i}{F_\pi} \langle p| \hat{\theta}'_i(0) | p\rangle = \frac{\sqrt{2}i}{F_\pi} \langle n| \hat{\theta}'_i(0) | n\rangle$$

Nucleon Mass Splitting (Feyman-Hellmann Theorem)

Soft-Pion Theorem for ΔI=1 PV

- Derivation from ChPT: combining P-even and P-odd components

$\tilde{\theta}_i = \theta_i + \theta_i'$ and $\tilde{\theta}_i^{(s)} = \theta_i^{(s)} + \theta_i^{(s)\prime}$

$$\boxed{\begin{aligned}
\tilde{\theta}_1 &= 2\bar{q}\gamma^\mu q\bar{q}_R\gamma_\mu\tau_3 q_R \\
\tilde{\theta}_2 &= 2\bar{q}_a\gamma^\mu q_b\bar{q}_{bR}\gamma_\mu\tau_3 q_{aR} \\
\tilde{\theta}_3 &= 2\bar{q}\gamma^\mu\gamma_5 q\bar{q}_R\gamma_\mu\tau_3 q_R \\
\tilde{\theta}_4 &= 2\bar{q}_a\gamma^\mu\gamma_5 q_b\bar{q}_{bR}\gamma_\mu\tau_3 q_{aR} \\
\tilde{\theta}_1^{(s)} &= 2\bar{s}\gamma^\mu s\bar{q}_R\gamma_\mu\tau_3 q_R \\
\tilde{\theta}_2^{(s)} &= 2\bar{s}_a\gamma^\mu s_b\bar{q}_{bR}\gamma_\mu\tau_3 q_{aR} \\
\tilde{\theta}_3^{(s)} &= 2\bar{s}\gamma^\mu\gamma_5 s\bar{q}_R\gamma_\mu\tau_3 q_R \\
\tilde{\theta}_4^{(s)} &= 2\bar{s}_a\gamma^\mu\gamma_5 s_b\bar{q}_{bR}\gamma_\mu\tau_3 q_{aR}.
\end{aligned}}$$

Implemented via a single Spurion:

$$SU(2)_R \times SU(2)_L : \tau_3 \to R u R^+$$

$$X_R = u^+\tau_3 u$$

Leading Chiral Lagrangian:

$$\mathcal{L}^w_{\mathrm{tot,LO}} = \alpha\bar{N}X_R N$$
$$= \underbrace{\boxed{\alpha\bar{N}\tau^3 N}}_{\text{P-even}} - \underbrace{\boxed{\frac{\sqrt{2}i}{F_0}\alpha(\bar{n}p\pi^- - \bar{p}n\pi^+)}}_{\text{P-odd}} + \dots$$

The Matching:

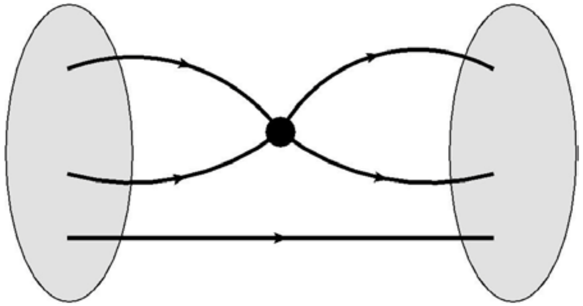
$$\boxed{F_\pi h_\pi^1 \approx -\frac{(\delta m_N)_{4q}}{\sqrt{2}}}$$

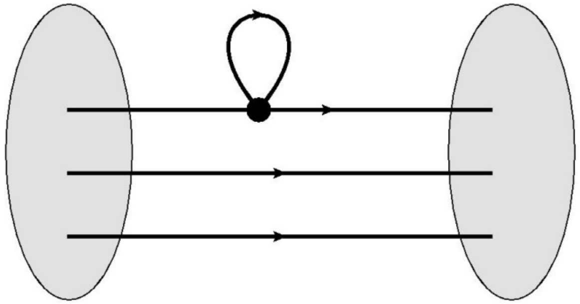
Protected against all QCD corrections up to NNLO!

$$\mathrm{Error} < \left(\frac{m_\pi}{\Lambda_\chi}\right)^2 \sim 0.01$$

Soft-Pion Theorem for ΔI=1 PV

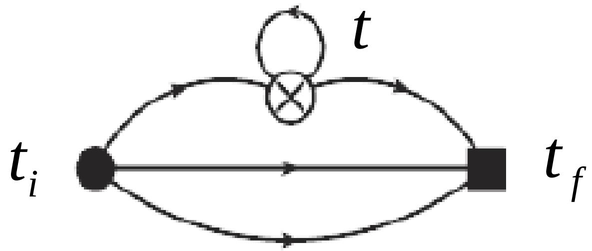
- Benefits our formalism:
  - Lattice:
    - **Avoid the power-law finite-volume effect** due to the pion-nucleon re-scattering
    - **Avoid the need of introducing extra LECs** due to the energy insertion to the p→nπ vertex
    - **Avoid calculating complicated contraction diagrams** involving the interaction between nucleon and pion
  - Other effective approaches:
    - Object of study becomes a simple **spectroscopic quantity**
    - For diagram-base approach (e.g. Dyson-Schwinger): **number of contraction diagrams is greatly reduced**





PQChPT and Disconnected Diagrams

- One major difficulty in lattice calculation is the evaluation of **DISCONNECTED DIAGRAMS** : diagrams involving propagators going from time t to time t
- They suffer from **unsuppressed noise,** leading to low signal-to-noise ratio at large Euclidean time separation



$$\text{Signal} \sim \exp[-E(t_f - t_i)]$$

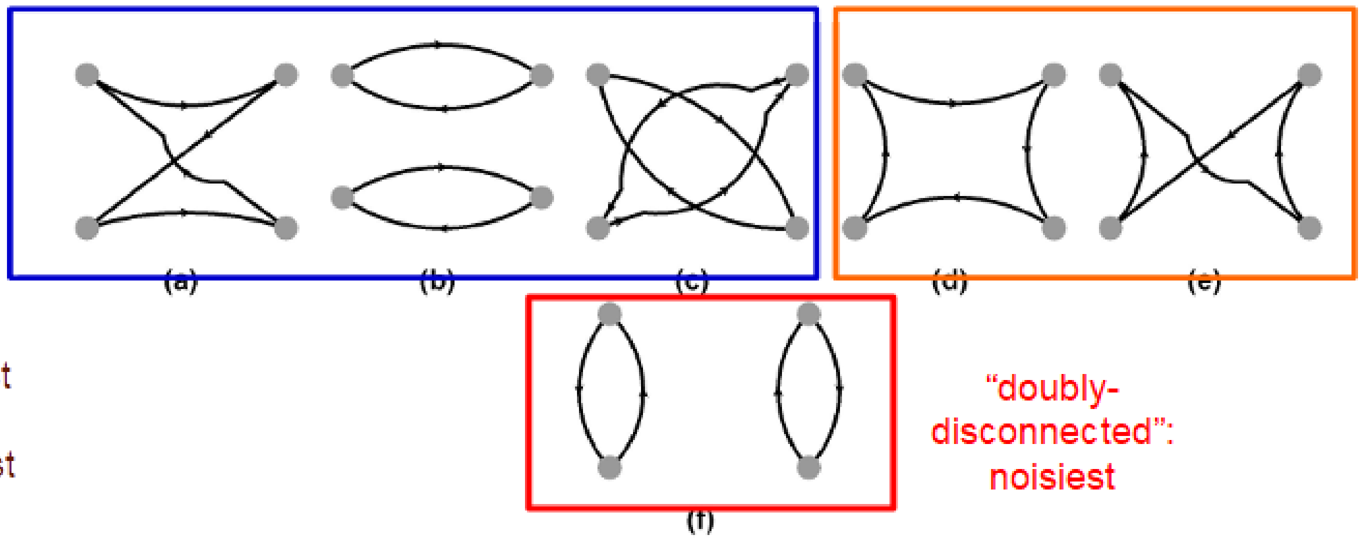
$$\text{Noise} \sim \exp[-E'(t-t)] \sim \text{const}$$

- Disconnected diagrams exist for the **ΔI=0,1** PV coefficients, making their lattice calculations extremely difficult!

PQChPT and Disconnected Diagrams

- Not a new problem as it happens as well in lattice simulation of **pion-pion scattering amplitude**, where **initial states share the same time coordinate** (so do **final states**):

"connected": cleanest



"singly-disconnected": more noisy

I=2: easiest
I=1: hard
I=0: hardest

"doubly-disconnected": noisiest

- EFT study of pion-pion scattering may therefore **provide insights!**
- Diagrams of definite contraction can be modeled in EFT by **increasing the quark flavor**
- **"Ghost" quarks** should be introduced simultaneously to keep the number of dynamical DOFs unchanged → Partially-Quenched ChPT (PQChPT)

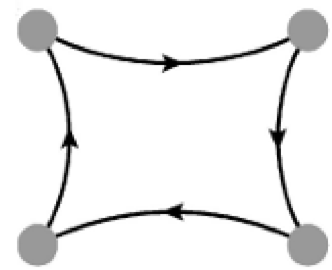
PQChPT and Disconnected Diagrams

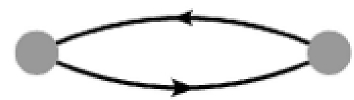
- **PQChPT** study of pion-pion scattering: F.K.Guo, N.Acharya, U.Meissner and CYS,Nucl.Phys. B922 (2017) 480-498

$$\mathrm{SU}(2) \rightarrow \mathrm{SU}(4\,|\,2)$$

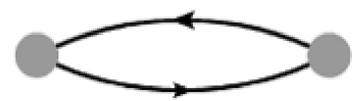
$$(u,d) \rightarrow (u,d,j,k,\tilde{j},\tilde{k})$$

$$T_1(s,t,u) \;=\; T_{(u\bar{d})(d\bar{j})\rightarrow(u\bar{k})(k\bar{j})}(s,t,u)$$





$$T_2(s,t,u) \;=\; T_{(u\bar{d})(j\bar{k})\rightarrow(u\bar{d})(j\bar{k})}(s,t,u)$$



PQChPT and Disconnected Diagrams

- **PQChPT** study of pion-pion scattering: F.K.Guo, N.Acharya, U.Meissner and CYS,Nucl.Phys. B922 (2017) 480-498

$$\mathrm{SU}(2) \to \mathrm{SU}(4\,|\,2)$$

$$(u, d) \to (u, d, j, k, \tilde{j}, \tilde{k})$$

Lagrangian at O(p^2):

$$\mathcal{L}^{(2)} = \frac{F_0^2}{4}\mathrm{Str}\left[(\partial_\mu U^\dagger)(\partial^\mu U)\right] + \frac{F_0^2 B_0}{2}\mathrm{Str}\left[MU^\dagger + UM^\dagger\right]$$

PQChPT and Disconnected Diagrams

- PQChPT study of pion-pion scattering: F.K.Guo, N.Acharya, U.Meissner and CYS,Nucl.Phys. B922 (2017) 480-498

$$\mathrm{SU(2)} \rightarrow \mathrm{SU}(4\,|\,2)$$

$$(u,d) \rightarrow (u,d,j,k,\tilde{j},\tilde{k})$$

Lagrangian at O(p^4):

$$\begin{aligned}
\mathcal{L}^{(4)} = {} & L_0^{\mathrm{PQ}}\mathrm{Str}\left[(\partial_\mu U^\dagger)(\partial_\nu U)(\partial^\mu U^\dagger)(\partial^\nu U)\right] + (L_1^{\mathrm{PQ}} - \frac{1}{2}L_0^{\mathrm{PQ}})\mathrm{Str}\left[(\partial_\mu U^\dagger)(\partial^\mu U)\right]\mathrm{Str}\left[(\partial_\nu U^\dagger)(\partial^\nu U)\right] \\
& +(L_2^{\mathrm{PQ}} - L_0^{\mathrm{PQ}})\mathrm{Str}\left[(\partial_\mu U^\dagger)(\partial_\nu U)\right]\mathrm{Str}\left[(\partial^\mu U^\dagger)(\partial^\nu U)\right] \\
& +(L_3^{\mathrm{PQ}} + 2L_0^{\mathrm{PQ}})\mathrm{Str}\left[(\partial_\mu U^\dagger)(\partial^\mu U)(\partial_\nu U^\dagger)(\partial^\nu U)\right] \\
& +2B_0 L_4^{\mathrm{PQ}}\mathrm{Str}\left[(\partial_\mu U^\dagger)(\partial^\mu U)\right]\mathrm{Str}\left[U^\dagger M + M^\dagger U\right] + 2B_0 L_5^{\mathrm{PQ}}\mathrm{Str}\left[(\partial_\mu U^\dagger)(\partial^\mu U)(U^\dagger M + M^\dagger U)\right] \\
& +4B_0^2 L_6^{\mathrm{PQ}}\left(\mathrm{Str}\left[U^\dagger M + M^\dagger U\right]\right)^2 + 4B_0^2 L_7^{\mathrm{PQ}}\left(\mathrm{Str}\left[U^\dagger M - M^\dagger U\right]\right)^2 \\
& +4B_0^2 L_8^{\mathrm{PQ}}\mathrm{Str}\left[MU^\dagger MU^\dagger + M^\dagger U M^\dagger U\right] .
\end{aligned}$$

- Contains Unphysical Low Energy Constants (LECs) that cannot be determined by experiment but can be fitted to lattice amplitude with definite contractions.

PQChPT and Disconnected Diagrams

- **PQChPT** study of pion-pion scattering: F.K.Guo, N.Acharya, U.Meissner and CYS,Nucl.Phys. B922 (2017) 480-498

$\mathrm{SU}(2) \to \mathrm{SU}(4|2)$

$$(u,d) \to (u,d,j,k,\tilde{j},\tilde{k})$$

$$\begin{aligned}
T_1(s,t,u) &= \frac{2m_\pi^2-u}{2F_\pi^2} + \left(\frac{3u-4m_\pi^2}{3F_\pi^2}\right)\mu_\pi + \left(\frac{2m_\pi^4-4m_\pi^2(s+t)+s(2s+t)}{12F_\pi^4}\right)J_{\pi\pi}^r(s) \\
&+ \left(\frac{2m_\pi^4-4m_\pi^2(s+t)+t(2t+s)}{12F_\pi^4}\right)J_{\pi\pi}^r(t) + \frac{4}{F_\pi^4}\left(s^2+t^2+u^2-4m_\pi^4\right)L_0^{\mathrm{PQ},r} \\
&+ \frac{2}{F_\pi^4}\left(4m_\pi^2 u+s^2+t^2-8m_\pi^4\right)L_3^{\mathrm{PQ},r} - \frac{4m_\pi^2 u}{F_\pi^4}L_5^{\mathrm{PQ},r} + \frac{16m_\pi^4}{F_\pi^4}L_8^{\mathrm{PQ},r} \\
&- \frac{m_\pi^4}{72\pi^2F_\pi^4} + \frac{m_\pi^2 u}{96\pi^2F_\pi^4} + \frac{2u^2-s^2-t^2}{576\pi^2F_\pi^4} \\
T_2(s,t,u) &= \frac{(s-2m_\pi)^2}{4F_\pi^4}J_{\pi\pi}^r(s) + \frac{(u-2m_\pi)^2}{4F_\pi^4}J_{\pi\pi}^r(u) + \left(\frac{2m_\pi^4+t^2}{4F_\pi^4}\right)J_{\pi\pi}^r(t) \\
&+ \frac{4}{F_\pi^4}\left(4m_\pi^4-s^2-t^2-u^2\right)L_0^{\mathrm{PQ},r} + \frac{2}{F_\pi^4}\left(t-2m_\pi^2\right)^2\left(l_1^r-2L_3^{\mathrm{PQ},r}\right) \\
&+ \frac{1}{F_\pi^4}\left(s^2+u^2+4m_\pi^2 t-8m_\pi^4\right)l_2^r + \frac{2m_\pi^2}{F_\pi^4}\left(t-2m_\pi^2\right)\left(l_4^r-4L_5^{\mathrm{PQ},r}\right) \\
&+ \frac{2m_\pi^4}{F_\pi^4}\left(l_3^r+l_4^r-8L_8^{\mathrm{PQ},r}\right)
\end{aligned}$$

l_i^r : Physical LECs

$L_0^{\mathrm{PQ},r}$: Unphysical LECs

- Same technique can be used to study disconnected diagrams in $h_{\pi NN}^{(1)}$

PV Couplings: Current Situation and Planned Improvements

	I=0 coupling	I=1 coupling	I=2 coupling
Current Situation	N/A	Preliminary $h_\pi^{1,\text{con}}$	Preliminary $\Lambda_2^{{}^1S_0-{}^3P_0}$
Main Difficulties	• Disconnected diagram	• Disconnected diagram • Complications due to external π • Unphysical pion mass • Finite lattice spacing	• (Extremely) unphysical pion mass • Finite lattice spacing
Possible Immediate Improvement	• PQChPT for disconnected diagram	Done • Reformulate as mass splitting calculation • PQChPT for disconnected diagram and chiral extrapolation On the way	• Chiral extrapolation

Personal thought: the fastest improved result may come from ΔI=1, followed by ΔI=2. More thoughts may be needed for ΔI=0.

Brief Summary

1. Nuclear PV is a good test ground for our current understanding of both strong and weak dynamics at low energy
2. Early studies focused on $\Delta I=1$ PV channel; Recent analysis incline more towards $\Delta I=0,2$
3. One main theoretical challenge is the prediction of PV nuclear coupling strengths directly from SM; models give non-convergent results while lattice QCD faces various technical difficulties
4. Recent progress:
   - In the $\Delta I=1$ channel, a soft-pion theorem is formulated that simplifies the QCD matrix element
   - PQChPT is expected to provide hints for the noisy disconnected diagrams
5. Series of follow-up work will be performed for a precise determination of the PV couplings

Back-Up Slides

Different Representations of PV Nuclear Potential

PV potential from DDH:

$$
\begin{aligned}
V_{\mathrm{DDH}}^{\mathrm{PNC}}(\vec{r}) = {} & i\frac{h_\pi^1 g_{\pi NN}}{\sqrt{2}}\left(\frac{\vec{\tau}_1 \times \vec{\tau}_2}{2}\right)_z (\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\left[\frac{\vec{p}_1 - \vec{p}_2}{2m_N}, w_\pi(r)\right] \\
& - g_\rho\left(h_\rho^0 \vec{\tau}_1\cdot\vec{\tau}_2 + h_\rho^1\left(\frac{\vec{\tau}_1 + \vec{\tau}_2}{2}\right)_z + h_\rho^2\frac{(3\tau_1^z\tau_2^z - \vec{\tau}_1\cdot\vec{\tau}_2)}{2\sqrt{6}}\right)\times\left((\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\left\{\frac{\vec{p}_1 - \vec{p}_2}{2m_N}, w_\rho(r)\right\}\right. \\
& \left. + \ i(1+\chi_V)\vec{\sigma}_1\times\vec{\sigma}_2\cdot\left[\frac{\vec{p}_1 - \vec{p}_2}{2m_N}, w_\rho(r)\right]\right) - g_\omega\left(h_\omega^0 + h_\omega^1\left(\frac{\vec{\tau}_1 + \vec{\tau}_2}{2}\right)_z\right) \\
& \times\left((\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\left\{\frac{\vec{p}_1 - \vec{p}_2}{2m_N}, w_\omega(r)\right\} + i(1+\chi_S)\vec{\sigma}_1\times\vec{\sigma}_2\cdot\left[\frac{\vec{p}_1 - \vec{p}_2}{2m_N}, w_\omega(r)\right]\right) \\
& + \left(\frac{\vec{\tau}_1 - \vec{\tau}_2}{2}\right)_z (\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\left(g_\rho h_\rho^1\left\{\frac{\vec{p}_1 - \vec{p}_2}{2m_N}, w_\rho(r)\right\} - g_\omega h_\omega^1\left\{\frac{\vec{p}_1 - \vec{p}_2}{2m_N}, w_\omega(r)\right\}\right) \\
& - g_\rho h_\rho^{1\prime} i\left(\frac{\vec{\tau}_1 \times \vec{\tau}_2}{2}\right)_z (\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\left[\frac{\vec{p}_1 - \vec{p}_2}{2m_N}, w_\rho(r)\right],
\end{aligned}
$$

Different Representations of PV Nuclear Potential

PV potential from Zhu (EFT):

$$
\begin{aligned}
V_{\mathrm{LO}}^{\mathrm{Zhu}} = & -2\frac{\tilde{\mathcal{C}}_6}{\Lambda_\chi^3} i(\vec{\tau}_1 \times \vec{\tau}_2)_z(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_S \delta(\vec{r}) + 2\frac{\mathcal{C}_3}{\Lambda_\chi^3}(\vec{\tau}_1 \cdot \vec{\tau}_2)(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_A \delta(\vec{r}) \\
& - 2\frac{\tilde{\mathcal{C}}_3}{\Lambda_\chi^3}(\vec{\tau}_1 \cdot \vec{\tau}_2) i(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_S \delta(\vec{r}) + \frac{\mathcal{C}_4}{\Lambda_\chi^3}(\tau_1^z + \tau_2^z)(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_A \delta(\vec{r}) \\
& - \frac{\tilde{\mathcal{C}}_4}{\Lambda_\chi^3}(\tau_1^z + \tau_2^z) i(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_S \delta(\vec{r}) + 2\sqrt{6}\frac{\mathcal{C}_5}{\Lambda_\chi^3}(\tau_1 \otimes \tau_2)_{20}(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_A \delta(\vec{r}) \\
& - 2\sqrt{6}\frac{\tilde{\mathcal{C}}_5}{\Lambda_\chi^3}(\tau_1 \otimes \tau_2)_{20} i(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_S \delta(\vec{r}) + 2\frac{\mathcal{C}_1}{\Lambda_\chi^3}(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_A \delta(\vec{r}) \\
& - 2\frac{\tilde{\mathcal{C}}_1}{\Lambda_\chi^3} i(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_S \delta(\vec{r}) + \frac{\mathcal{C}_2}{\Lambda_\chi^3}(\tau_1^z + \tau_2^z)(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_A \delta(\vec{r}) \\
& - \frac{\tilde{\mathcal{C}}_2}{\Lambda_\chi^3}(\tau_1^z + \tau_2^z) i(\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_S \delta(\vec{r}) + \frac{(\mathcal{C}_2 - \mathcal{C}_4)}{\Lambda_\chi^3}(\tau_1^z - \tau_2^z)(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \frac{1}{i} \overleftrightarrow{\nabla}_A \delta(\vec{r}).
\end{aligned}
$$

Different Representations of PV Nuclear Potential

PV potential from Girlanda (EFT):

$$
\begin{aligned}
V_{\mathrm{LO}}^{Girlanda} = & \left[-2\tilde{\mathcal{G}}_1\right] \frac{1}{i} \overleftrightarrow{\nabla}_S \, \delta(\vec{r}) \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2) + [2\mathcal{G}_1] \frac{1}{i} \overleftrightarrow{\nabla}_A \, \delta(\vec{r}) \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \\
& + [\mathcal{G}_2] \frac{1}{i} \overleftrightarrow{\nabla}_A \, \delta(\vec{r}) \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)(\tau_1^z + \tau_2^z) + [2\mathcal{G}_6] \frac{1}{i} \overleftrightarrow{\nabla}_A \, \delta(\vec{r}) \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)(\tau_1^z - \tau_2^z) \\
& + \left[-2\sqrt{6}\mathcal{G}_5\right] \frac{1}{i} \overleftrightarrow{\nabla}_A \, \delta(\vec{r}) \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)(\tau_1 \otimes \tau_2)_{20}.
\end{aligned}
$$

Different Representations of PV Nuclear Potential

PV potential in terms of Danilov's S-P amplitudes:

$$
\begin{aligned}
V_{\mathrm{LO}}^{\mathrm{PNC}}(\vec{r}) = {} & \Lambda_0^{{}^1S_0-{}^3P_0}\left(\frac{1}{i}\frac{\overleftrightarrow{\nabla}_A}{2m_N}\frac{\delta(\vec{r})}{m_\rho^2}\cdot(\vec{\sigma}_1-\vec{\sigma}_2)-\frac{1}{i}\frac{\overleftrightarrow{\nabla}_S}{2m_N}\frac{\delta(\vec{r})}{m_\rho^2}\cdot i(\vec{\sigma}_1\times\vec{\sigma}_2)\right) \\
& + \Lambda_0^{{}^3S_1-{}^1P_1}\left(\frac{1}{i}\frac{\overleftrightarrow{\nabla}_A}{2m_N}\frac{\delta(\vec{r})}{m_\rho^2}\cdot(\vec{\sigma}_1-\vec{\sigma}_2)+\frac{1}{i}\frac{\overleftrightarrow{\nabla}_S}{2m_N}\frac{\delta(\vec{r})}{m_\rho^2}\cdot i(\vec{\sigma}_1\times\vec{\sigma}_2)\right) \\
& + \Lambda_1^{{}^1S_0-{}^3P_0}\frac{1}{i}\frac{\overleftrightarrow{\nabla}_A}{2m_N}\frac{\delta(\vec{r})}{m_\rho^2}\cdot(\vec{\sigma}_1-\vec{\sigma}_2)(\tau_1^z+\tau_2^z)+\Lambda_1^{{}^3S_1-{}^3P_1}\frac{1}{i}\frac{\overleftrightarrow{\nabla}_A}{2m_N}\frac{\delta(\vec{r})}{m_\rho^2}\cdot(\vec{\sigma}_1+\vec{\sigma}_2)(\tau_1^z-\tau_2^z) \\
& + \Lambda_2^{{}^1S_0-{}^3P_0}\frac{1}{i}\frac{\overleftrightarrow{\nabla}_A}{2m_N}\frac{\delta(\vec{r})}{m_\rho^2}\cdot(\vec{\sigma}_1-\vec{\sigma}_2)(\vec{\tau}_1\otimes\vec{\tau}_2)_{20}
\end{aligned}
$$