Calculation for the mass difference between the long- and short-lived K mesons for physical quark masses with lattice QCD

Bigeng Wang
RBC-UKQCD Collaborations

Department of Physics
Columbia University in the City of New York

December 18th, 2018 @PKU

Special thanks to Norman Christ, Ziyuan Bai, Chris Sachrajda, Chulwoo Jung, Xu Feng.

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Mattia Bruno
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

UC Boulder

Oliver Witzel

Columbia University

Ziyuan Bai
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu
Bigeng Wang
Tianle Wang
Evan Wickenden
Yidi Zhao

University of Connecticut

Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
Cheng Tu

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Tadeusz Janowski
Richard Kenway
Julia Kettle
Fionn O'haigan
Brian Pendleton
Antonin Portelli
Tobias Tsang
Azusa Yamaguchi

KEK

Julien Frison

University of Liverpool

Nicolas Garron

MIT

David Murphy

Peking University

Xu Feng

University of Southampton

Jonathan Flynn
Vera Guelpers
James Harrison
Andreas Juettner
James Richings
Chris Sachrajda

Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

York University (Toronto)

Renwick Hudspith

Outline

The Standard Model

Three types of interactions

1. Electromagnetic(QED)
2. Strong(QCD)
3. Weak: least understood; good checks for new physics:
    - Unitarity of CKM matrix
    - CP violation

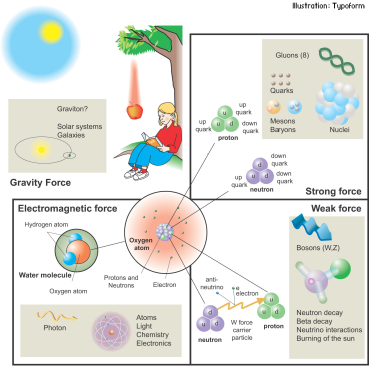


Figure: from https://www.nobelprize.org/prizes/physics/2004/popular-information/

Although weak interaction itself can be treated precisely with perturbation theory, many interesting weak interaction processes involve mesons and baryons(QCD related).

$K^0 - \overline{K^0}$ Mixing and Δm_K

$K^0(S=-1)$ and $\overline{K}^0(S=+1)$ mix through second order weak interactions:

$$i\frac{d}{dt}\begin{pmatrix}K^0(t)\\ \overline{K}^0(t)\end{pmatrix} = (M - \frac{i}{2}\Gamma)\begin{pmatrix}K^0(t)\\ \overline{K}^0(t)\end{pmatrix}, \qquad (1)$$

Long-lived (K_L) and short-lived (K_S) are the two eigenstates:

$$K_S \approx \frac{K^0 - \overline{K}^0}{\sqrt{2}}, \quad K_L \approx \frac{K^0 + \overline{K}^0}{\sqrt{2}}. \qquad (2)$$

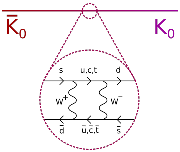


Figure: figure from wikipedia

Physics Motivation

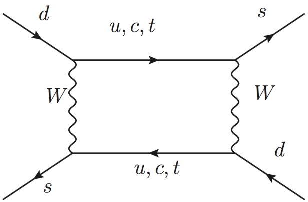
$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2ReM_{0\bar{0}}$$

To second order of the weak Hamiltonian:

$$M_{0\bar{0}} = \langle \overline{K}^0 | H_W^{\Delta S=2} | K^0 \rangle + \mathcal{P} \sum_n \frac{\langle \overline{K}^0 | H_W^{\Delta S=1} | n \rangle \langle n | H_W^{\Delta S=1} | K^0 \rangle}{m_K - E_n}$$

- This quantity is:
  1. sensitive to new physics: 2nd order weak interaction, precisely measured
  $$\Delta m_{K,exp} = 3.483(6) \times 10^{-12}\ \text{MeV}$$
  2. highly non-perturbative: contributions from distance as large as $\frac{1}{m_\pi}$
- Prediction based on the standard model?
  - Perturbation theory?
  - Lattice QCD?

Calculate Δm_K: Perturbation Theory



The contribution is separated into two parts:

- Short distance $<\frac{1}{m_c}$: integrate out W and m_c
- Long distance $>\frac{1}{m_c}$

There are several difficulties:

- Strong coupling $\alpha_S \sim O(1)$ when energy is close to Λ_{QCD}
  Convergence problem: Gives about 36 % discrepancy between the Next-Next- Leading-Order (NNLO) calculation and the Next-Leading-Order (NLO) calculation.

  A. J. Buras and J. Girrbach, Eur. Phys. J. **C73**(9), 2560 (2013), 1304.6835.
- systematic error not well controlled

Calculate Δm_K: Lattice QCD

Pros

1. Solves QCD problems non-perturbatively
2. From first principles

Challenges

1. Lattice artifacts:
    - Finite volume
    - Finite lattice spacing: short distance cutoff
2. "High" computational cost

Δm_K calculation

1. Long-distance dominating(GIM mechanism)
2. Non-perturbative

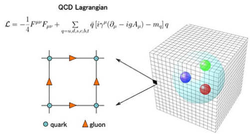


Figure: from wikipedia

Δm_K is one of RBC-UKQCD collaboration's calculations of long-distance contributions in kaon physics. It is closely related to other kaon physics calculations like ϵ_K and rare kaon decays.

Z. Bai, N.H. Christ, X. Feng, A. Lawson, A. Portelli and C.T. Sachrajda, Phys. Rev. D **98**, 074509

Kaon Mixing: Long-distance Contribution on Lattice

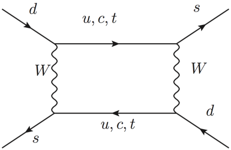
- No direct weak simulation on lattice:
  - From full weak Hamiltonian at W scale, integrate out W and Z, to get effective $\Delta S = 1$ Hamiltonian:

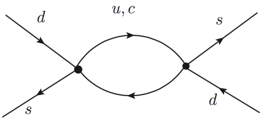
$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'}) \qquad (3)$$

  where the $Q_i^{qq'}{}_{i=1,2}$ are current-current opeartors, defined as:

$$Q_1^{qq'} = (\bar{s}_i \gamma^\mu (1-\gamma_5) d_i)(\bar{q}_j \gamma^\mu (1-\gamma_5) q'_j),$$

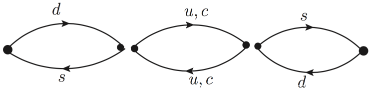
$$Q_2^{qq'} = (\bar{s}_i \gamma^\mu (1-\gamma_5) d_j)(\bar{q}_j \gamma^\mu (1-\gamma_5) q'_i),$$





Short Distance Effect: $V - A$ and GIM Mechanism

Short distance effect: Ultraviolet divergences as the two H_W approach each other:

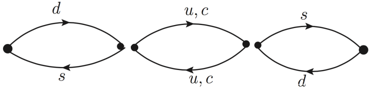


GIM mechanism removes both quadratic and logarithmic divergences

- $$\frac{\not{p} - m_u}{\not{p}^2 + m_u^2} \sim \frac{1}{p} \to \left(\frac{\not{p} - m_u}{\not{p}^2 + m_u^2} - \frac{\not{p} - m_c}{\not{p}^2 + m_c^2} \right) \sim \frac{1}{p^2} \tag{4}$$

Short Distance Effect: $V - A$ and GIM Mechanism

Short distance effect: Ultraviolet divergences as the two H_W approach each other:

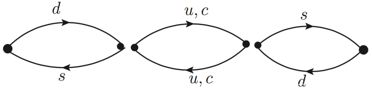


GIM mechanism removes both quadratic and logarithmic divergences

- $$\frac{\not{p} - m_u}{\not{p}^2 + m_u^2} \sim \frac{1}{p} \to \left(\frac{\not{p} - m_u}{\not{p}^2 + m_u^2} - \frac{\not{p} - m_c}{\not{p}^2 + m_c^2} \right) \sim \frac{1}{p^2} \tag{4}$$
- Due to the $(V - A)$ structure of the operator Q_i
$$\gamma^\mu (1 - \gamma^5) \frac{\not{p}(m_c^2 - m_u^2)}{(\not{p}^2 + m_u^2)(\not{p}^2 + m_c^2)} \gamma^\nu (1 - \gamma^5) \sim \frac{1}{p^3}. \tag{5}$$

Short Distance Effect: $V - A$ and GIM Mechanism

Short distance effect: Ultraviolet divergences as the two H_W approach each other:



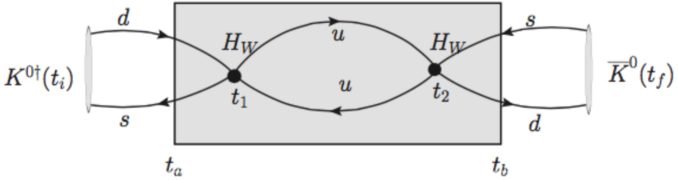
GIM mechanism removes both quadratic and logarithmic divergences

- $$\frac{\not{p} - m_u}{\not{p}^2 + m_u^2} \sim \frac{1}{p} \to \left(\frac{\not{p} - m_u}{\not{p}^2 + m_u^2} - \frac{\not{p} - m_c}{\not{p}^2 + m_c^2} \right) \sim \frac{1}{p^2} \tag{4}$$
- Due to the $(V - A)$ structure of the operator Q_i
$$\gamma^\mu (1 - \gamma^5) \frac{\not{p}(m_c^2 - m_u^2)}{(\not{p}^2 + m_u^2)(\not{p}^2 + m_c^2)} \gamma^\nu (1 - \gamma^5) \sim \frac{1}{p^3}. \tag{5}$$
- All short distance contributions from distance of order $\frac{1}{m_c}$, negligible when systematic error $\sim m_c a$ is acceptable.

1. Introduction to Δm_K
   - Kaon Mixing in the Standard Model
   - Kaon Mixing on Lattice
   - Discussion: Short-distance Effect in Δm_K

2. Calculating Δm_K on Lattice
   - From Integrated Correlator to Δm_K^{lat}
   - From Δm_K^{lat} to physical Δm_K

From Integrated Correlator to Δm_K^{lat}



- Δm_K is given by:

$$\begin{aligned}\Delta m_K &\equiv m_{K_L} - m_{K_S} \\ &= 2\mathcal{P}\sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n}\end{aligned} \tag{6}$$

- The integrated correlator is defined as:

$$\mathcal{A} = \frac{1}{2} \sum_{t_2=t_a}^{t_b} \sum_{t_1=t_a}^{t_b} \langle 0 | T\{\overline{K}^0(t_f) H_W(t_2) H_W(t_1) K^0(t_i)\} | 0 \rangle \tag{7}$$

From Integrated Correlator to Δm_K^{lat}

- If we insert a complete set of intermediate states

$$\mathcal{A} = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \{ -T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n} \} \tag{8}$$

  we identify the coefficient of the term linear in the size of integration box $T = t_b - t_a + 1$ as proportional to the expression for Δm_K

- Therefore, by fitting the coefficient of T from integrated correlators we can obtain:

$$\Delta m_K^{lat} \equiv 2 \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \tag{9}$$

Subtract States with Lower energies

$$\mathcal{A}(T) = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \{-T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n}\}$$

- Before doing a linear fitting with respect to T, the second term in the curly bracket has to be removed.
- For an intermediate state $|n\rangle$ with energy E_n larger than m_K, for large enough T, the contribution from the second term is negligible.
- For a state $|n\rangle$ with energy E_n smaller than or close to m_K, we need to subtract its contribution.
  In our case of physical quark masses, $|0\rangle$, $|\pi\pi\rangle$, $|\eta\rangle$ and $|\pi\rangle$ states need to be subtracted.

Introduction of $\bar{s}ds$ and $\bar{s}\gamma_5 d$ Operators

- With the freedom of adding the operators $\bar{s}d$ and $\bar{s}\gamma_5 d$ to the weak Hamiltonian with properly chosen coefficients c_s and c_p, we are able to remove two of the contributions.
- Here we choose c_s and c_p to satisfy:

$$\langle 0|H_W - c_p\bar{s}\gamma_5 d|K^0\rangle = 0, \quad \langle \eta|H_W - c_s\bar{s}d|K^0\rangle = 0.$$

- As a result, the original $\Delta S = 1$ effective weak Hamiltonian and therefore the current-current operators should be modified to be :

$$Q_i' = Q_i - c_{pi}\bar{s}\gamma_5 d - c_{si}\bar{s}d$$

with c_{pi} and c_{si} are calculated on lattice:

$$c_{si} = \frac{\langle \eta|Q_i|K^0\rangle}{\langle \eta|\bar{s}d|K^0\rangle}, \quad c_{pi} = \frac{\langle 0|Q_i|K^0\rangle}{\langle 0|\bar{s}\gamma_5 d|K^0\rangle}.$$

Calculation of Δm_K^{lat}

$$\mathcal{A} = N_K^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \{-T + \frac{e^{(m_K-E_n)T} - 1}{m_K - E_n}\}$$

Recall

$$H'_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1'^{qq'} + C_2 Q_2'^{qq'})$$

The fitting of the integrated correlator further breaks into fitting of the integrated correlator with Q_1 and Q_2:

$$\mathcal{A}_{ij}(T) = N_K^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0 | Q_i | n \rangle \langle n | Q_j | K^0 \rangle}{m_K - E_n} \{-T + \frac{e^{(m_K-E_n)T} - 1}{m_K - E_n}\}.$$

We therefore have:

$$\mathcal{A}(T) = \lambda_u^2 \sum_{i,j=1,2} C_i C_j \mathcal{A}_{ij}(T), \quad \lambda_u = V_{ud} V^*_{us}$$

Renormalization

We fit each $\mathcal{A}_{ij}(T)$ separately and obtain the k_{ij}, coefficient of the linear term of T. The value of Δm_K from the lattice should be:

$$\Delta m_K^{lat} = \frac{G_F^2}{2}\lambda_u^2 \sum_{i,j=1,2} (-2) \times C_i^{lat} C_j^{lat} k_{ij}. \tag{10}$$

Renormalization of lattice operator $Q_{1,2}$:

- Non-perturbative Renormalization: from lattice to RI-SMOM $Z^{lat\to RI}$

Renormalization

We fit each $\mathcal{A}_{ij}(T)$ separately and obtain the k_{ij}, coefficient of the linear term of T. The value of Δm_K from the lattice should be:

$$\Delta m_K^{lat} = \frac{G_F^2}{2}\lambda_u^2 \sum_{i,j=1,2} (-2) \times C_i^{lat} C_j^{lat} k_{ij}. \tag{10}$$

Renormalization of lattice operator $Q_{1,2}$:

- Non-perturbative Renormalization: from lattice to RI-SMOM $Z^{lat\to RI}$
- Perturbation theory: from RI-SMOM to $\overline{MS}$ $\quad (1+\Delta r)^{RI\to\overline{MS}}$

C. Lehner, C. Sturm, Phys. Rev. D 84(2011), 0140

Renormalization

We fit each $\mathcal{A}_{ij}(T)$ separately and obtain the k_{ij}, coefficient of the linear term of T. The value of Δm_K from the lattice should be:

$$\Delta m_K^{lat} = \frac{G_F^2}{2}\lambda_u^2 \sum_{i,j=1,2}(-2)\times C_i^{lat} C_j^{lat} k_{ij}. \tag{10}$$

Renormalization of lattice operator $Q_{1,2}$:

- Non-perturbative Renormalization: from lattice to RI-SMOM $Z^{lat\to RI}$
- Perturbation theory: from RI-SMOM to $\overline{MS}$ $(1+\Delta r)^{RI\to\overline{MS}}$

  C. Lehner, C. Sturm, Phys. Rev. D 84(2011), 0140

- Use Wilson coefficients in the $\overline{MS}$ scheme

  G. Buchalla, A.J. Buras and M.E. Lautenbacher, arXiv:hep-ph/9512380

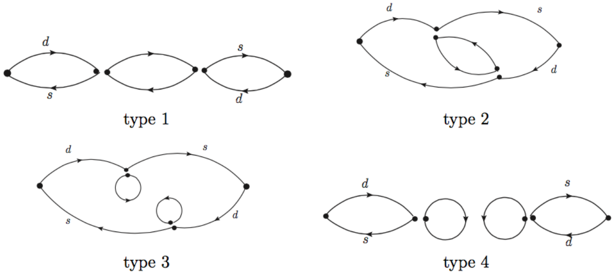
$$C_i^{lat} = C_a^{\overline{MS}}(1+\Delta r)_{ab}^{RI\to\overline{MS}} Z_{bi}^{lat\to RI}.$$

Contractions: 4-point Correlators

$$\mathcal{A}_{ij}(T) = \frac{1}{2}\sum_{t_2=t_a}^{t_b}\sum_{t_1=t_a}^{t_b}\langle 0|T\{\overline{K}^0(t_{snk})Q_j'(t_2)Q_i'(t_1)K^0(t_{src})\}|0\rangle, \quad (11)$$

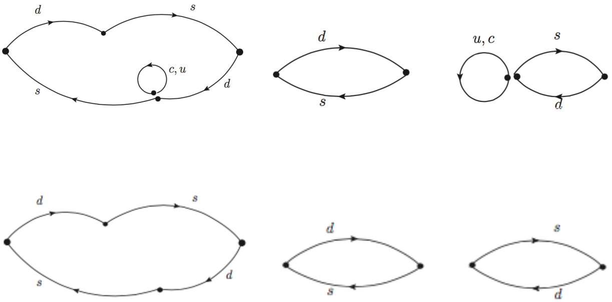
where $Q_i' = Q_i - c_{pi}\bar{s}\gamma_5 d - c_{si}\bar{s}d$.

- For contractions among Q_i, there are four types of diagrams to be evaluated.



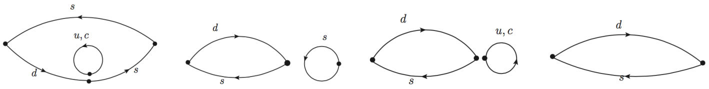
Contractions: 4-point Correlators

In addition, there are "mixed" diagrams from the contractions between the $c_s \bar{s} d$, $c_p \bar{s} \gamma^5 d$ operators and Q_i operators.



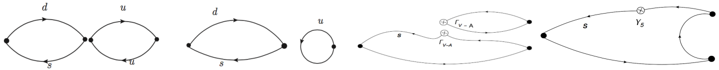
3-point Correlators

- Coefficients: $c_{si} = \frac{\langle \eta | Q'_i | K^0 \rangle}{\langle \eta | \bar{s} d | K^0 \rangle}, \quad c_{pi} = \frac{\langle 0 | Q'_i | K^0 \rangle}{\langle 0 | \bar{s} \gamma_5 d | K^0 \rangle}.$



- Explicit subtractions:

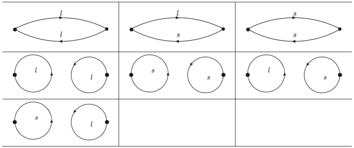
$$\frac{\langle \bar{K}^0 | Q'_i | \pi \rangle \langle \pi | Q'_j | K^0 \rangle}{m_K - E_\pi} \frac{e^{(m_K - E_\pi)T} - 1}{m_K - E_\pi} \text{ and } \frac{\langle \bar{K}^0 | Q'_i | \pi\pi \rangle \langle \pi\pi | Q'_j | K^0 \rangle}{m_K - E_{\pi\pi}} \frac{e^{(m_K - E_{\pi\pi})T} - 1}{m_K - E_{\pi\pi}}.$$

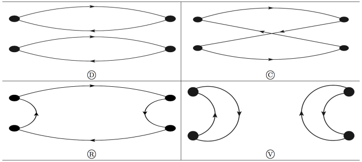


2-point Correlators

2-point correlators are used to compute the mass and normalization factor of π, K^0 η and $\pi\pi$ states.

$$C(t) \sim |\langle n|O_n^\dagger(0)|0\rangle|^2 e^{-E_n t}$$





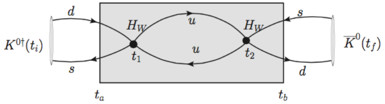
Calculate Propagators on Lattice

The contractions shown in previous section can be written as products of quark propagators.

- from the source point y to the sink point x:

$$S(x, y) = D^{-1}(x, y).$$

- There are $64^3 \times 128 \sim 10^7$ sites on lattice.



- For a certain source distribution $b(y)$, the propagator to x is given by:

$$S(x) = \sum_y S(x, y) b(y),$$

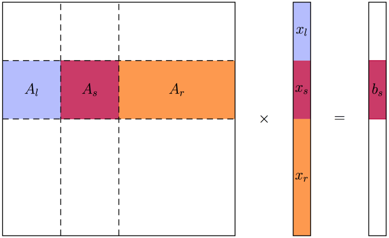
and $S(x)$ can be obtained by solving a $\mathbf{A}\vec{a} = \vec{b}$ problem using CG:

$$\sum_y D(x, y) S(y) = b(x).$$

Calculate Propagators on Lattice

$S(x)$ can be obtained by solving a $\mathbf{A}\vec{a} = \vec{b}$ problem using CG:

$$\sum_y D(x,y)S(y) = b(x).$$



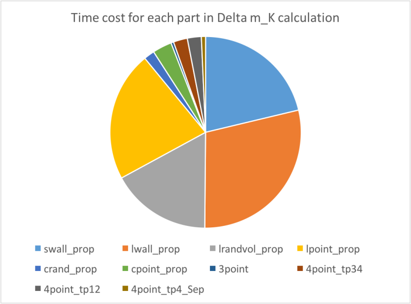


Figure on the left: from Jiqun Tu

Data and Data Analysis: Sample AMA Correction

- We use Sample All Mode Averaging (AMA) to reduce the computational cost.

  T. Blum, T. Izubuchi, and E. Shintani, Phys. Rev. D88(9), 094503 (2013)

data type	CG stop residual
sloppy	$1e-4$
exact	$1e-8$

  The difference between the "exact" and the "sloppy" result for a same quantity(e.g. a strange propagator) is used as a correction.
- Usually AMA correction is performed on each configuration, among different time slices
- Our Sample AMA correction is applied among configurations
- We do only "sloppy" measurements on most configurations and do both "sloppy" and "exact" measurements on some other configurations to serve as corrections.

Super-jackknife Method

For a certain quantity Y, a pion correlator as an example

- N_s "sloppy" measurements $\{y_i\}_{i=1,\ldots,N_s}$
  N_c corrections $\{\Delta y_i\}_{i=1,\ldots,N_c}$

y_1	y_2	$\cdots$	y_{N_s-1}	y_{N_s}	Δy_1	Δy_2	$\cdots$	Δy_{N_c-1}	Δy_{N_c}

$\underbrace{\qquad\qquad\qquad}_{N_s \text{ measurements}}$ $\underbrace{\qquad\qquad\qquad}_{N_c \text{ measurements}}$

- Jackknife the raw data to get two jackknife ensembles:
  $Y_i = \frac{1}{N_s-1}\sum_{j\neq i} y_j$, $\Delta Y_i = \frac{1}{N_c-1}\sum_{j\neq i}\Delta y_j$.

Y_1	Y_2	$\cdots$	Y_{N_s-1}	Y_{N_s}	ΔY_1	ΔY_2	$\cdots$	ΔY_{N_c-1}	ΔY_{N_c}

$\underbrace{\qquad\qquad\qquad}_{N_s \text{ jackknife elements}}$ $\underbrace{\qquad\qquad\qquad}_{N_c \text{ jackknife elements}}$

- We then combine the two jackknife ensembles to form a super-jackknife ensemble with $N_s + N_c$ elements.

$Y_1 + \overline{\Delta Y}$	$Y_2 + \overline{\Delta Y}$	$\cdots$	$Y_{N-1} + \overline{\Delta Y}$	$Y_N + \overline{\Delta Y}$	$\overline{Y} + \Delta Y_1$	$\overline{Y} + \Delta Y_2$	$\cdots$	$\overline{Y} + \Delta Y_{N_2-1}$	$\overline{Y} + \Delta Y_{N_2}$

$\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{(N_s + N_c) \text{ super-jackknife elements}}$

Status of RBC-UKQCD Calculations of Δm_k

- **"Long-distance contribution of the $K_L - K_S$ mass difference"**, **N. H. Christ, T. Izubuchi, C. T. Sachrajda, A. Soni and J. Yu**

  Phys. Rev. D 88(2013), 014508

  Development of techniques and exploratory calculation on a $16^3 \times 32$ lattice with unphysical masses($m_\pi = 421 MeV$) including only connected diagrams

Status of RBC-UKQCD Calculations of Δm_k

- **"Long-distance contribution of the $K_L - K_S$ mass difference"**,
  N. H. Christ, T. Izubuchi, C. T. Sachrajda, A. Soni and J. Yu
  Phys. Rev. D 88(2013), 014508
  Development of techniques and exploratory calculation on a $16^3 \times 32$ lattice with unphysical masses($m_\pi = 421 MeV$) including only connected diagrams
- **"$K_L - K_S$ mass difference from Lattice QCD"**
  Z. Bai, N. H. Christ, T. Izubuchi, C. T. Sachrajda, A. Soni and J. Yu
  Phys. Rev. Lett. 113(2014), 112003
  All diagrams included on a $24^3 \times 64$ lattice with unphysical masses

Status of RBC-UKQCD Calculations of Δm_k

- **"Long-distance contribution of the $K_L - K_S$ mass difference"**,
  N. H. Christ, T. Izubuchi, C. T. Sachrajda, A. Soni and J. Yu
  Phys. Rev. D 88(2013), 014508
  Development of techniques and exploratory calculation on a $16^3 \times 32$ lattice with unphysical masses($m_\pi = 421 MeV$) including only connected diagrams
- **"$K_L - K_S$ mass difference from Lattice QCD"**
  Z. Bai, N. H. Christ, T. Izubuchi, C. T. Sachrajda, A. Soni and J. Yu
  Phys. Rev. Lett. 113(2014), 112003
  All diagrams included on a $24^3 \times 64$ lattice with unphysical masses
- **"The $K_L - K_S$ Mass Difference"**
  Z. Bai, N. H. Christ and C. T. Sachrajda, EPJ Web Conf. 175 (2018) 13017
  All diagrams included on a $64^3 \times 128$ lattice with **physical mass** on 59 configurations: $\Delta m_k = (5.5 \pm 1.7) \times 10^{-12} MeV$

Status of RBC-UKQCD Calculations of Δm_k

- **"Long-distance contribution of the $K_L - K_S$ mass difference"**,
  N. H. Christ, T. Izubuchi, C. T. Sachrajda, A. Soni and J. Yu
  Phys. Rev. D 88(2013), 014508
  Development of techniques and exploratory calculation on a $16^3 \times 32$ lattice with unphysical masses($m_\pi = 421MeV$) including only connected diagrams
- **"$K_L - K_S$ mass difference from Lattice QCD"**
  Z. Bai, N. H. Christ, T. Izubuchi, C. T. Sachrajda, A. Soni and J. Yu
  Phys. Rev. Lett. 113(2014), 112003
  All diagrams included on a $24^3 \times 64$ lattice with unphysical masses
- **"The $K_L - K_S$ Mass Difference"**
  Z. Bai, N. H. Christ and C. T. Sachrajda, EPJ Web Conf. 175 (2018) 13017
  All diagrams included on a $64^3 \times 128$ lattice with **physical mass** on 59 configurations: $\Delta m_k = (5.5 \pm 1.7) \times 10^{-12} MeV$
- Here I present an update of the results extending Z. Bai's calculation from 59 to 152 configurations.

arXiv:1812.05302

Details of the Calculation

- The calculation was performed on a $64^3 \times 128 \times 12$ lattice with 2+1 flavors of Möbius DWF and the Iwasaki gauge action with physical pion mass (136 MeV) and inverse lattice spacing $a^{-1} = 2.36$ GeV.

β	am_l	am_h	$\alpha = b + c$	L_s
2.25	0.0006203	0.02539	2.00	12

  For valence charm quark, we used $am_c \simeq 0.31$.

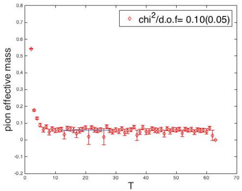
- I will compare results presented in 2017 with our updated results.

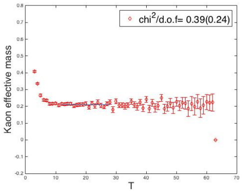
Data Set	# of Sloppy	# of Correction	# of Type12
"old 59"	52	7	11
"new 152"	116	36	36

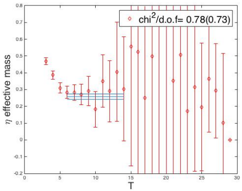
- The calulation was performed on BlueGene/Q with 8K nodes on MIRA, ANL

Results preliminary

2-point diagram







Data Set	K^0	π	η	$\pi\pi_{I=0}$
new 152	496.5(3)	135.4(3)	608(5)	268.5(1.2)
old 59	496.9(7)	135.9(3)	684(84)	268.3(1.5)

- These results are consistent within errors. As the statistics increase, the errors scale approximately as $\frac{1}{\sqrt{N}}$.

Results preliminary

3-point diagram: direct subtraction terms

Data Set	$\langle\pi\|Q_1\|K^0\rangle$	$\langle\pi\|Q_2\|K^0\rangle$	$\langle 0\|Q_1\|K^0\rangle$	$\langle 0\|Q_2\|K^0\rangle$
new 152	$-5.02(3)\times 10^{-4}$	$1.407(4)\times 10^{-3}$	$-1.284(3)\times 10^{-2}$	$2.449(4)\times 10^{-2}$
old 59	$-5.08(5)\times 10^{-4}$	$1.407(8)\times 10^{-3}$	$-1.289(4)\times 10^{-2}$	$2.454(7)\times 10^{-2}$

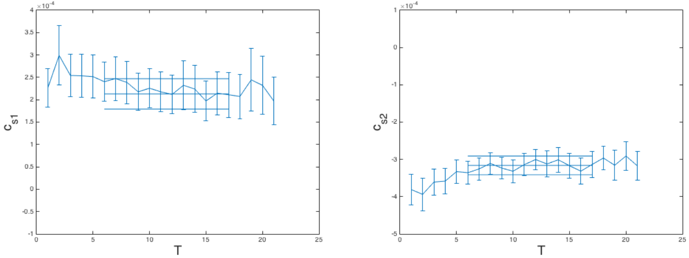
Table: The K^0 to π matrix element and the K^0 to vacuum matrix element, without subtracting the $\bar{s}d$ operator.

Data Set	$\langle\pi\pi_{I=2}\|Q_1\|K^0\rangle$	$\langle\pi\pi_{I=2}\|Q_2\|K^0\rangle$	$\langle\pi\pi_{I=0}\|Q_1\|K^0\rangle$	$\langle\pi\pi_{I=0}\|Q_2\|K^0\rangle$
new 152	$1.473(6)\times 10^{-5}$	$1.473(6)\times 10^{-5}$	$-8.7(1.5)\times 10^{-5}$	$9.5(1.5)\times 10^{-5}$
old 59	$1.471(10)\times 10^{-5}$	$1.471(10)\times 10^{-5}$	$-6.6(2.5)\times 10^{-5}$	$7.9(2.3)\times 10^{-5}$

Table: The K to $\pi\pi$ matrix element for Isospin 0 and 2. The I=2 matrix element for Q_1 and Q_2 are the same because they come from the same three point diagrams.

Results **preliminary**

3-point diagram: c_s and c_p



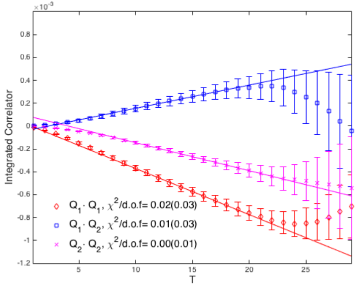
Data Set	c_{s1}	c_{s2}	c_{p1}	c_{p2}
new 152	$2.13(33) \times 10^{-4}$	$-3.16(25) \times 10^{-4}$	$-1.472(2) \times 10^{-4}$	$2.807(2) \times 10^{-4}$
old 59	$1.53(64) \times 10^{-4}$	$-2.77(42) \times 10^{-4}$	$-1.476(3) \times 10^{-4}$	$2.811(3) \times 10^{-4}$

Table: The subtraction coefficients for the scalar and pseudo-scalar operator.

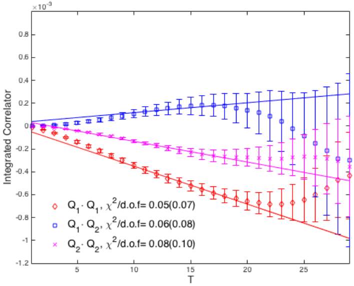
- **c_s are relatively noisy**: need more statistics

Results **preliminary**

the integrate correlator $\mathcal{A}_{ij}$ fittings: All diagrams, uncorrelated



(a) All diagrams fitting: 152 configurations



(b) All diagrams fitting: 59 configurations

Data Set	Δm_K	Δm_K(tp12)	Δm_K(tp34)	Δm_K(tp3)	Δm_K(tp4)
new 152	8.2(1.3)	8.3(0.6)	0.1(1.1)	1.58(31)	-1.28(94)
old 59	5.8(1.8)	7.0(1.3)	-1.1(1.2)	1.17(43)	-2.16(1.20)

Table: Results for Δm_K from uncorrelated fits in units of 10^{-12} MeV with fitting range 10:20.

Results preliminary

Sample AMA statistical errors

Our use of the sample AMA method reduced the computational cost of the calculation by a factor of 2.3, while the statistical error on the correction will add to the total statistical error. $\sigma \sim \sqrt{\sigma_{slp}^2 + \sigma_{corr}^2}$

Data Set	type 3&4 error from "sloppy"	type 3&4 error from correction	type 3&4 error in total
new 152	0.9	0.6	1.1
old 59	1.1	0.6	1.2

The AMA method does not contribute much to the error in our final answer.

Results **preliminary**

Systematic Errors

- Finite-volume corrections: small compared to statistical errors
  "Effects of finite volume on the $K_L - K_S$ mass difference"
  N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, arXiv:1504.01170
  Calculation gives: $\Delta m_K(FV) = -0.22(7) \times 10^{-12} MeV$

$2f(mk)$	$h = \delta + \phi$	$coth$	dh/dE	$coth \times dh/dE$	$\Delta m_K(FV)$
$-0.0035(10)$	$-0.49(6)$	$-1.85(27)$	$33.5(4)$	$-62(10)$	$-0.22(7)$

Table: The $\pi\pi_{I=0}$ contribution to Δm_K, and the terms determining the corresponding finite volume correction. The last term is the finite volume correction to the $K_L - K_S$ mass difference Δm_K, in units of $10^{-12} MeV$.

- The lattice spacing in our calculation is $a^{-1} = 2.36 GeV$, which is only twice the charm quark mass. Discretization effects are estimated to be the largest source of systematic error: $\sim (m_c a)^2$ is $\sim 25\%$.

Conclusion and Outlook

- Our **preliminary** result based on 152 configurations is

$$\Delta m_K = 7.9(1.3)(2.1) \times 10^{-12} MeV$$

to be compared to the experimental value:

$$(\Delta m_K)^{exp} = 3.483(6) \times 10^{-12} MeV$$

- We view such a comparison as premature given the possibly large and poorly estimated finite lattice spacing error.

Outlook

- Continue the calculation of Δm_K on Summit at Oak Ridge National Lab
  - Finner lattice spacing
  - Improved solver, contraction code for GPU(QUDA)

Figure: from `https://www.olcf.ornl.gov/calendar/summit-training-workshop/`

- Reduce the statistical error with measurements on larger number of configurations
- Include other elements of our kaon physics program

Thanks for your attention!

Backup slides: Propagator sources

- Wall source at time t with spin α and color a is defined as:

$$b(\vec{y}, t_y) = \begin{cases} \chi_{a\alpha}, & t_y = t \\ 0, & t_y \neq t_y, \end{cases} \tag{12}$$

where $\chi_{a\alpha}$ is a 12-component vector with 1 at spin α and color a and 0 at anywhere else.