Electroweak factorization

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Motivation

- At future colliders with ultra high energy E ~ 100 TeV, electroweak (EW) showering will be enhanced by $\ln(v/E)$, v being electroweak (EW) vev
- $\ln(v/E)$ is like IR log $\ln(\Lambda_{QCD}/E)$ in QCD
- IR logs are absorbed into universal PDFs in QCD factorization
- EW IR logs modify PDF definitions?
- EW factorization? A framework to describe EW showering and hard processes

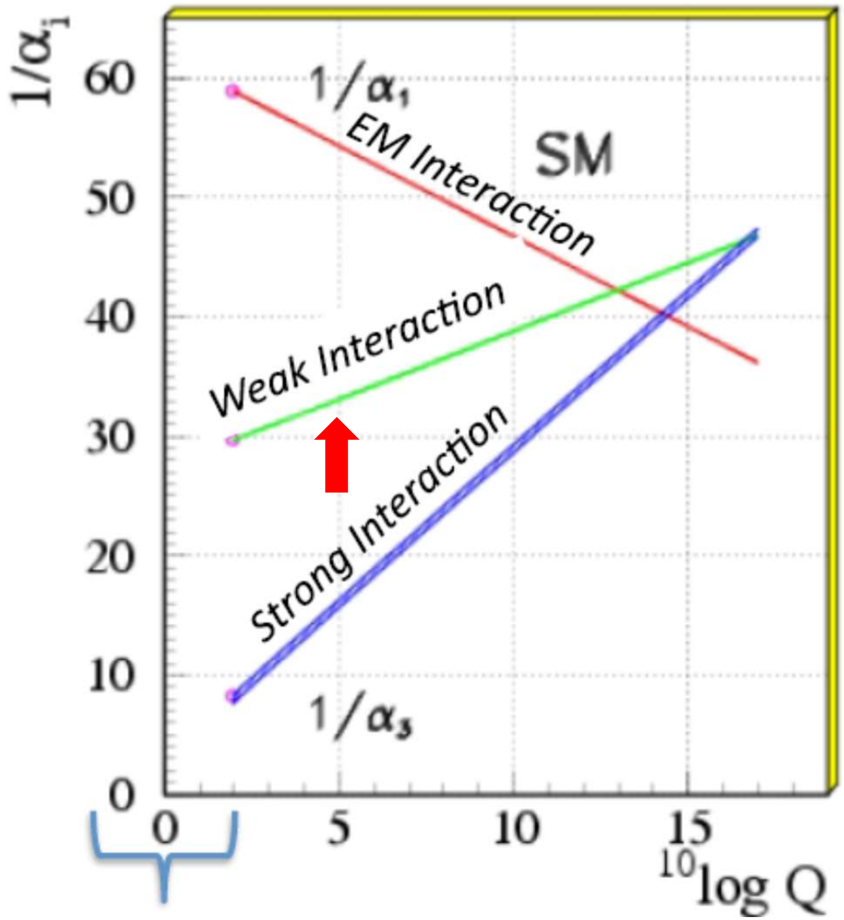
Running coupling constants

$$\alpha_W \ln^2(Q/v) \sim 1$$

at 100 TeV

EW logs become IR

at about 100 TeV

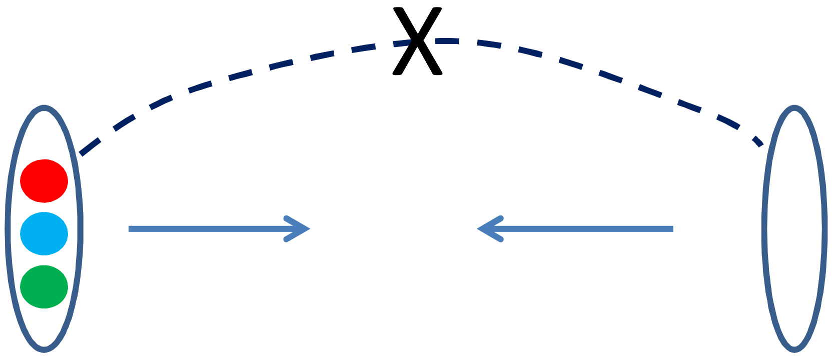


QCD vs EW

QCD	EW
SU(3)	SU(2)XU(1)
Color	isospin & hypercharge
Color-singlet bound state	none
Chiral symmetry	EW symmetry
Dynamical breaking	spontaneous breaking
QCD scale Λ_{QCD}	vev v
none (zero temp)	breaking scale $\mu_S >> v$
none	scalar, Yukawa coupling
physics scale $\geq \Lambda_{QCD}$	physics scale $\geq$ or $\leq$ v

QCD factorization

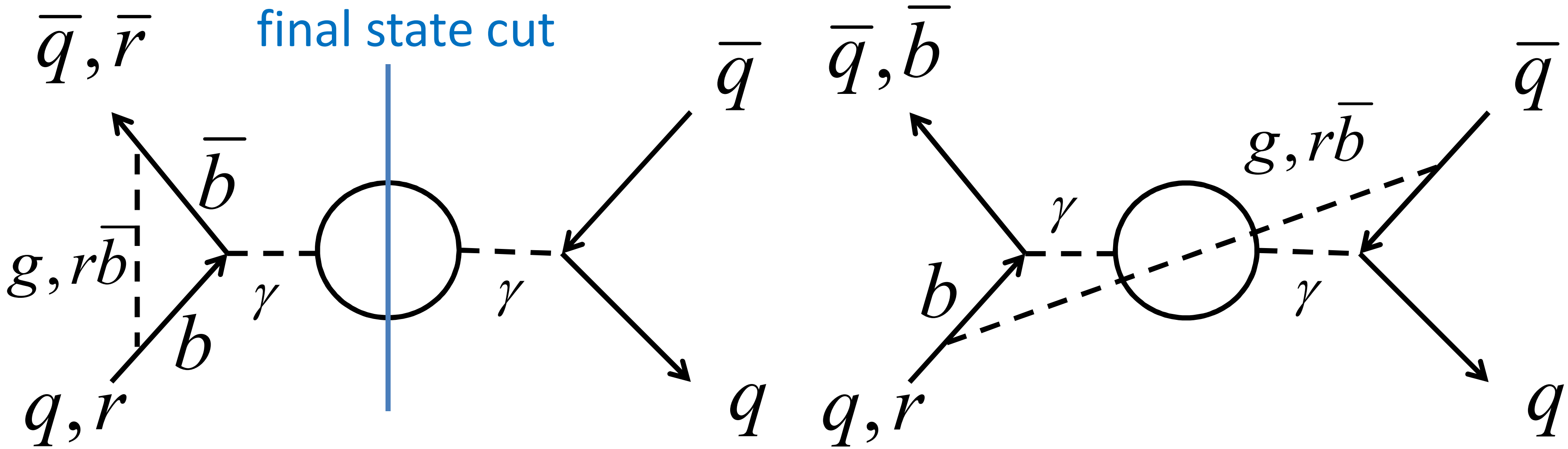
- High-energy proton beam, Lorentz contraction, color singlet implies zero color dipole in moving direction



- Soft gluons do not couple the two beams, QCD factorization
- Can define parton distribution functions (PDFs) to collect collinear gluons separately

Soft cancellation

- Soft gluons cancel between virtual and real diagrams



- Sum over colors is crucial for soft cancellation

Questions to answer

- EW factorization exists? We can manipulate isospin, but not colors
- If yes, what PDFs? Like $\phi_{uL}, \phi_{dL}, \phi_{W^+}, \phi_{W^-}, \cdots$

  proton PDFs as in QCD? 1611.00788, Chen, Han, Tweedie; 1703.08562 Bauer, Ferland, Webber
- Role of collinear scalars in constructing PDFs? PDF as nonlocal matrix element is gauge invariant
- EW symmetry breaking modifies IR structure? Massive gauge bosons become massless above μ_S

More questions

- Emergence of extra massless scalars in unbroken phase introduce new IR log?
- Yukawa couplings modify power counting of IR log?
- Connection between longitudinally polarized gauge bosons and scalars? Goldstone Equivalence Theorem

  1611.00788, Chen, Han, Tweedie
- How to match different sets of PDFs in broken and unbroken phases?

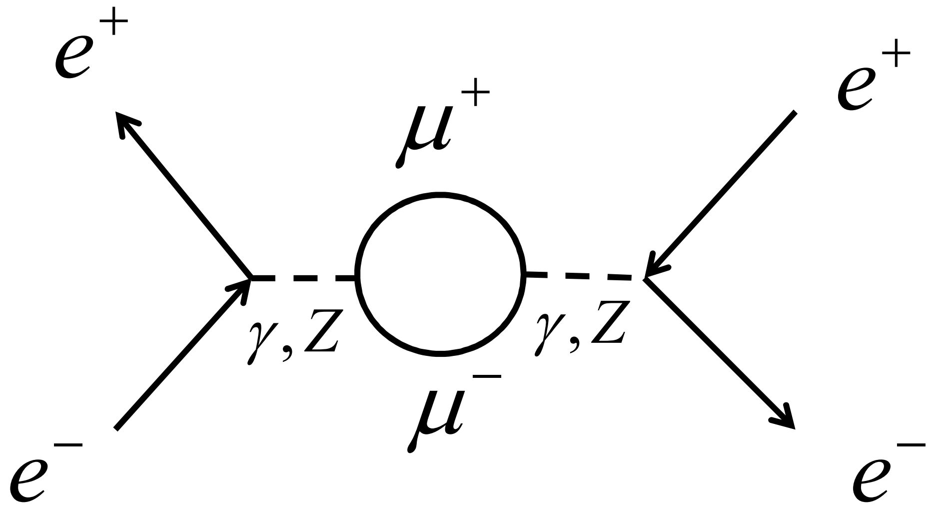
Goal of this talk

- Answer the above questions, taking

$$e^- e^+ \to \mu^- \mu^+ + X$$

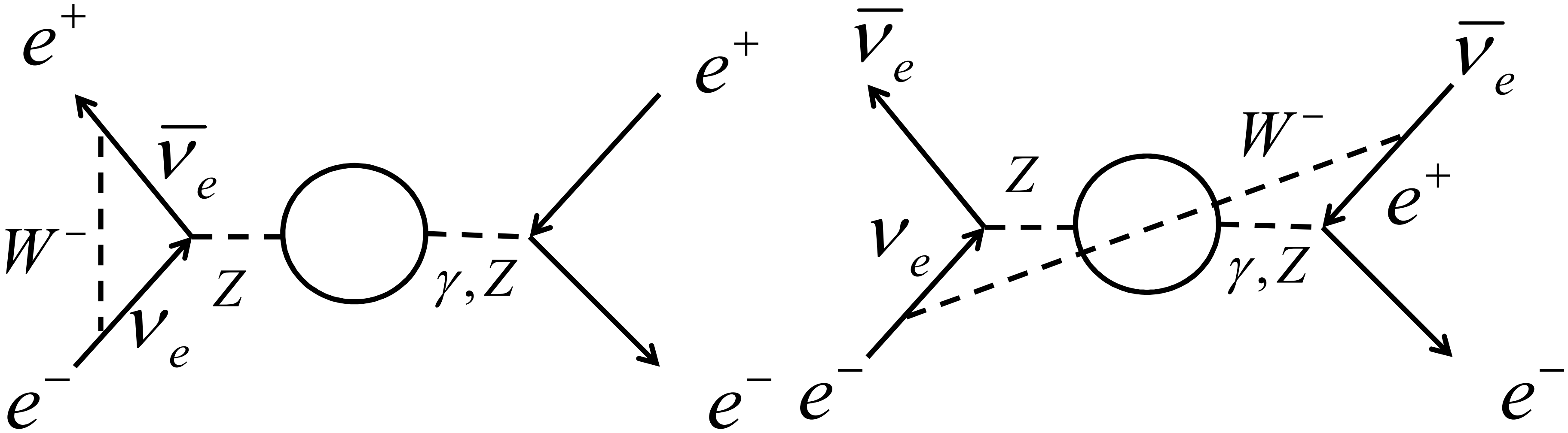
  as example
- Turn off EW showering associated with muon
- Imagine only one family for SU(2), and muon pair is just final state to be identified
- A theoretical setting to construct PDFs for electron

Leading-order



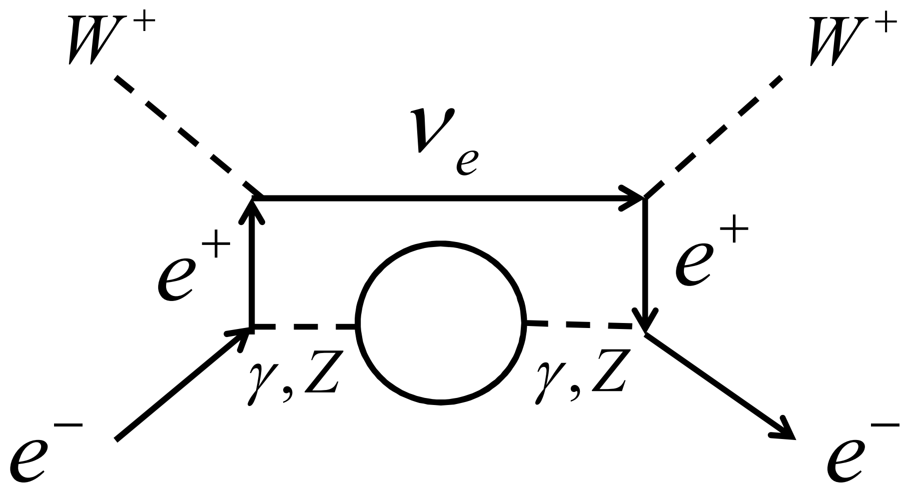
- Try to construct electron distribution in electron in broken phase
- Analyze IR log in various boson emissions from electron (do not address QED, which is trivial)

Next-to-leading order



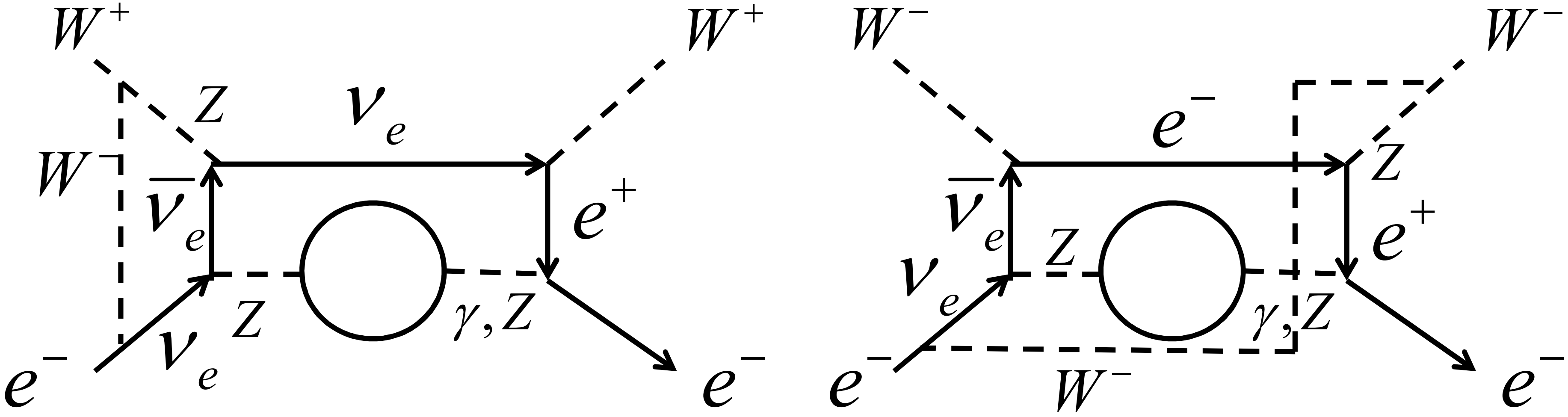
- Soft bosons, being process-dependent, must cancel between virtual and real W emissions to guarantee universality
- Must sum over partons e^+ and $\bar{\nu}_e$ (isospin)
- Can define only lepton distribution in electron

Partonic charged gauge bosons



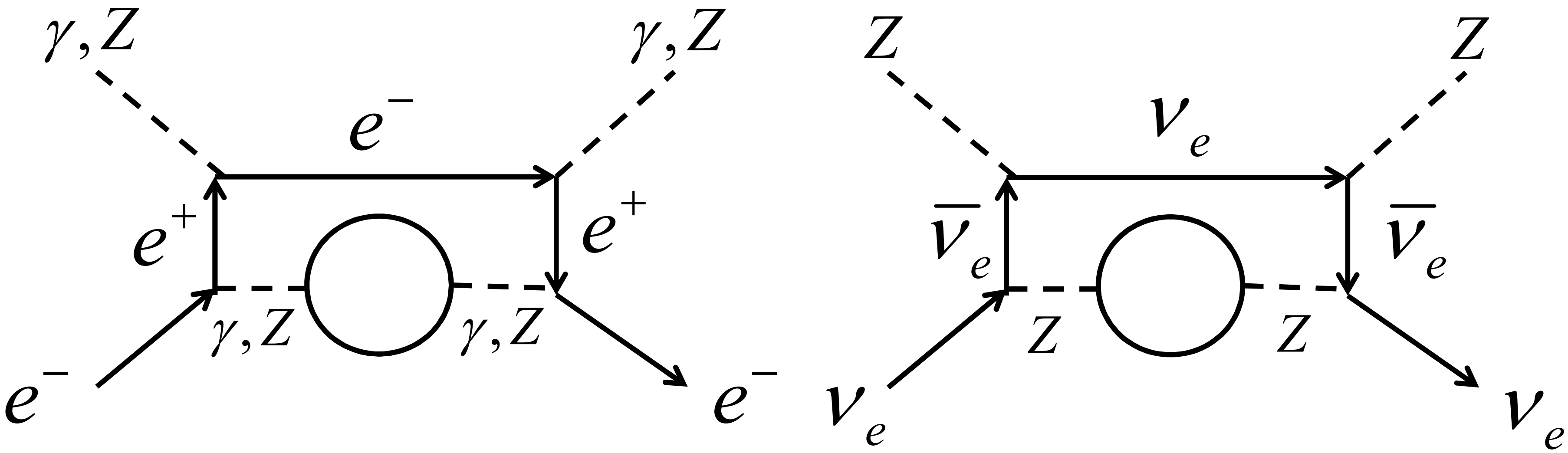
- Consider partonic gauge boson from positron
- Analyze IR log from boson emissions from electron in similar way

Soft cancellation

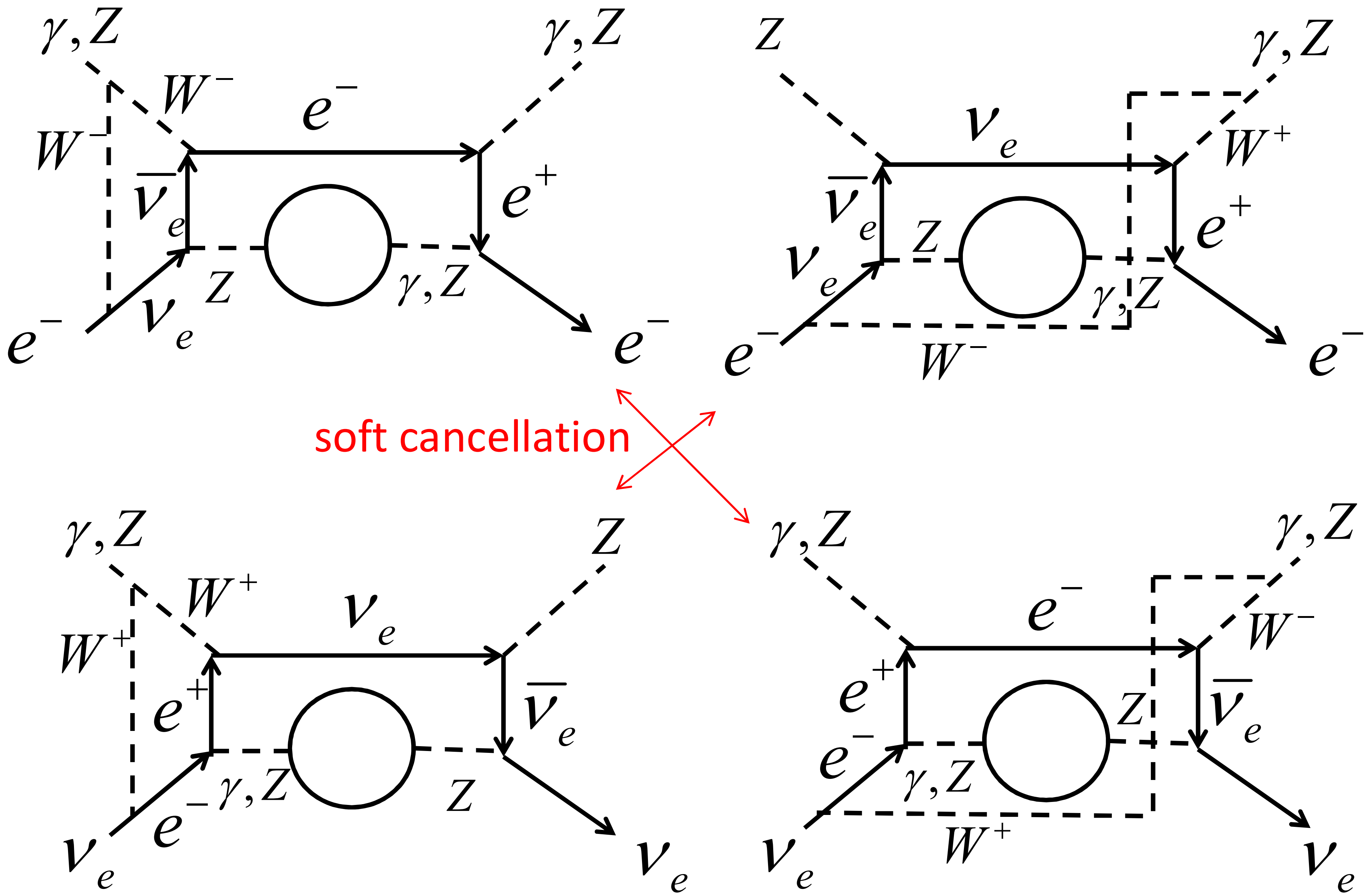


- Cancellation of soft logs require summation
- Can define only charged boson distribution
- They are just anti-particles to each other
- Neutral boson not involved, can have different distribution

Partonic neutral gauge bosons



- Must consider Z and photon simultaneously
- Need to define mixed distribution $\phi_{\gamma Z/e}$
- Must sum over partonic leptons
- Must sum over charged gauge bosons


γ, Z
W⁻
e⁻
W⁻
ν̄ₑ
e⁺
Z
γ, Z
νₑ
e⁻
e⁻
Z
γ, Z
νₑ
W⁺
ν̄ₑ
Z
e⁺
νₑ
γ, Z
e⁻
W⁻
e⁻
soft cancellation
γ, Z
Z
W⁺
νₑ
W⁺
e⁺
ν̄ₑ
γ, Z
Z
e⁻
νₑ
νₑ
γ, Z
γ, Z
e⁻
W⁻
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Z
ν̄ₑ
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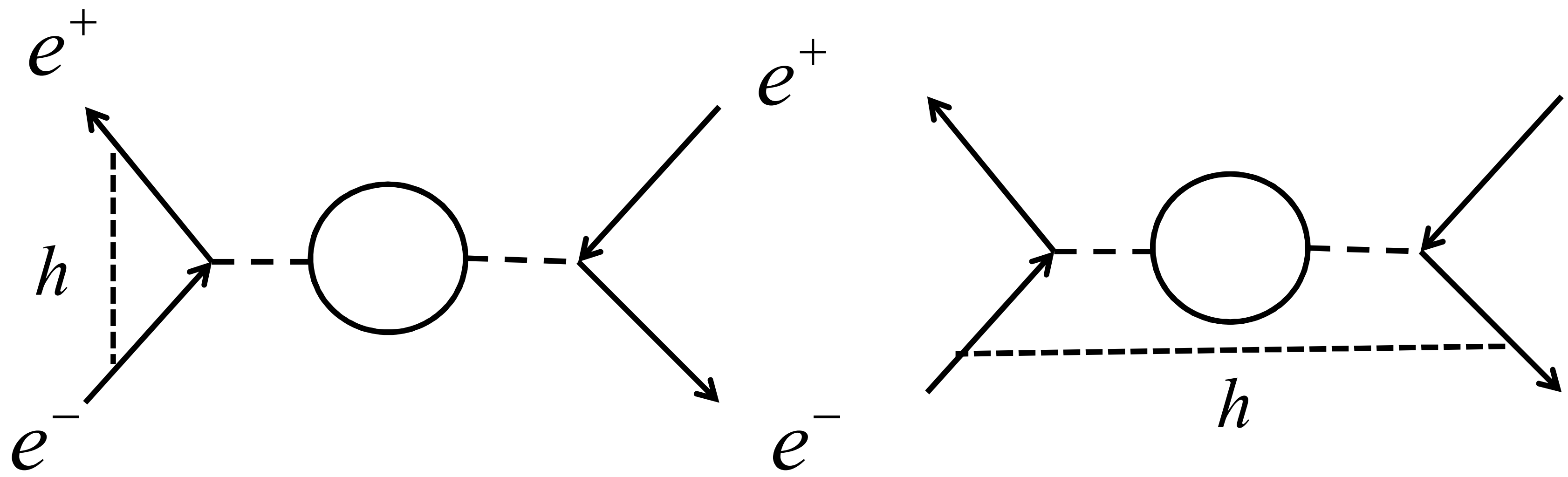
γ, Z
W⁺
νₑ
νₑ

Scalar emissions

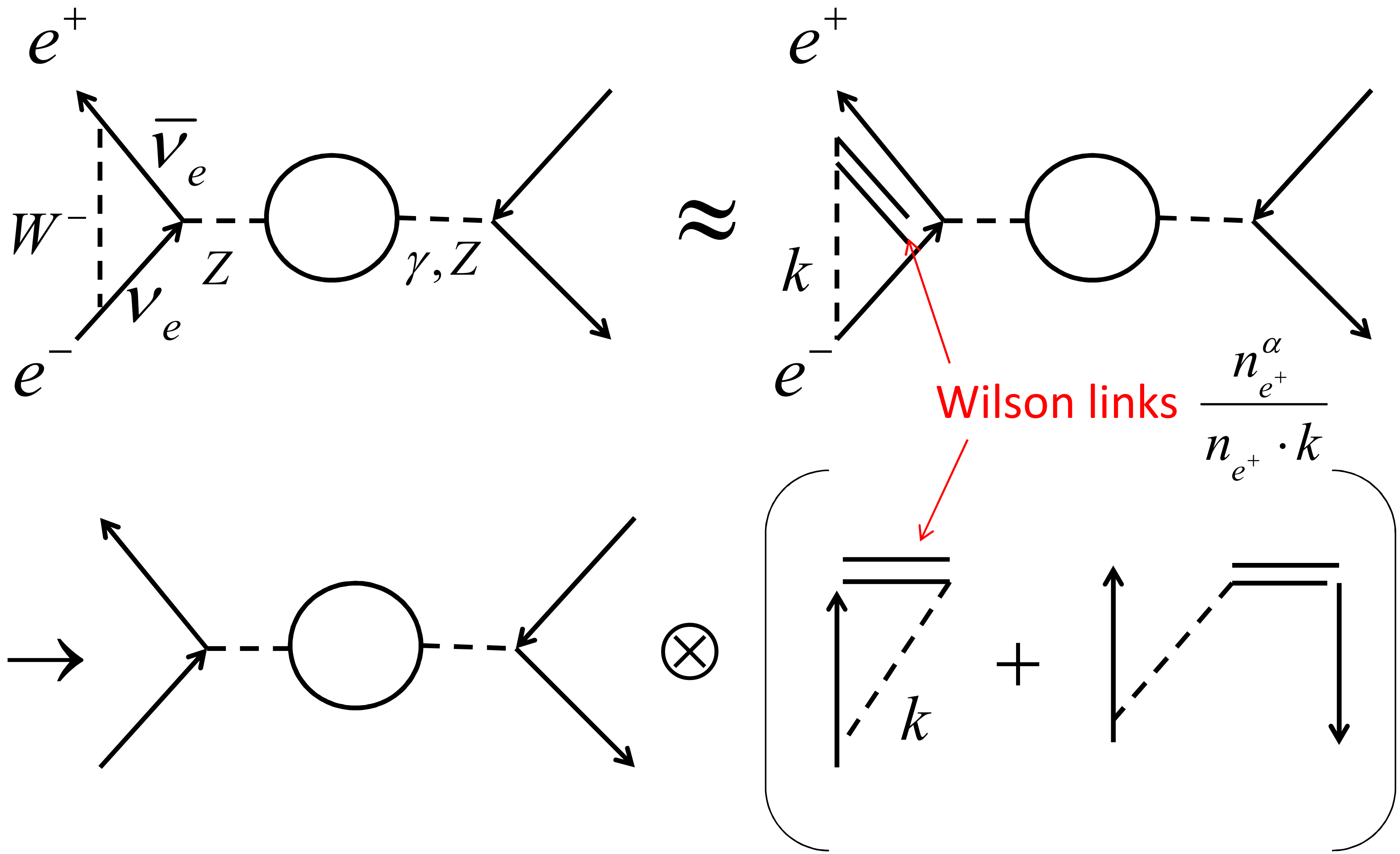


- Irreducible collinear scalar emission is IR finite due to equation $k \parallel p \Rightarrow (\not{p} - \not{k}) u_e(p) \to 0$
- Reducible collinear scalar emission gives IR log, which is power suppressed by m_e / m_W, i.e., by Yukawa coupling

Wilson links

- Eikonalization (factorization) of irreducible collinear gauge bosons leads to Wilson links
- Factorization of reducible emissions is trivial
- Wilson links render PDFs, as nonlocal matrix elements, gauge invariant
- Irreducible collinear scalar emission, being IR finite, does not contribute to Wilson links
- Reasonable, because scalar field has nothing to do with gauge invariance

Eikonalization



PDFs in broken phase

- Lepton PDF
$$\phi_{\ell/e}(x) = \frac{1}{2}\sum_s \int \frac{dy^-}{2\pi} \exp(-ixp^+y^-)$$
$$\langle e(p,s)| \sum_{\ell=e,\nu_e} \ell(y^-)W^\dagger(y^-)\frac{1}{2}\gamma^+ W(0)\ell(0)|e(p,s)\rangle$$

- Wilson links (simpler in terms of Wi and B)

$$W(y) = P\exp\left[ig\int_0^\infty dzn\cdot W_i(y+zn)\sigma_i\right]\exp\left[ig'\int_0^\infty dzn\cdot B(y+zn)I\right]$$

- Charged gauge boson PDF (neutral gauge boson PDFs, including mixed one, are similar)

$$\phi_{W/e}(x) = \frac{1}{2}\sum_s \int \frac{dy^-}{2\pi xp^+} \exp(-ixp^+y^-)$$
$$\langle e(p,s)|W^+_{i\,\nu}(y^-)W^\dagger(y^-)W(0)W_i^{\nu +}(0)|e(p,s)\rangle$$

In unbroken phase

- Massive gauge boson propagator

$$\frac{-i}{k^2 - m_W^2}\left[g^{\mu\nu} - \left(1 - \frac{1}{\lambda}\right)\frac{k^\mu k^\nu}{k^2 - m_W^2/\lambda}\right]$$

- Massless gauge boson propagator

$$\frac{-i}{k^2 - m_W^2}\left[g^{\mu\nu} - \left(1 - \frac{1}{\lambda}\right)\frac{k^\mu k^\nu}{k^2}\right]$$

- Physical mass in broken phase serves as IR regulator in unbroken phase
- Become the same in Feynman gauge $\lambda = 1$

Metric tensors of gauge bosons

- Metric tensor of real gauge boson in broken phase

$$g^{\mu\nu} - \frac{k^\mu k^\nu}{m_W^2}$$

- Metric tensor in unbroken phase

$$g^{\mu\nu} - \frac{k^\mu \bar{k}^\nu + k^\nu \bar{k}^\mu}{k \cdot \bar{k}} \qquad \bar{k}^\mu = (k^0, -\vec{k})$$

- Soft contribution cancels. In collinear region k in plus direction contracts with minus component, which is power suppressed
- Second terms are negligible, focusing on IR log

Same IR in both phases

- Loop integrands for virtual and real gauge boson emissions are identical in collinear region in both broken and unbroken phases
- EW symmetry breaking at μ_S does not modify IR structures of PDFs at $v << \mu_S$
- For high-energy electron we can construct in broken phase $\phi_{\ell/e}, \phi_{W/e}, \phi_{Z/e}, \phi_{\gamma/e}, \phi_{\gamma Z/e}$
- In unbroken phase $\phi_{\ell/e}, \phi_{W/e}, \phi_{W_3/e}, \phi_{B/e}, \phi_{BW/e}$
- PDFs in both phases can be matched perturbatively at μ_S

Goldstone Equivalence Theorem

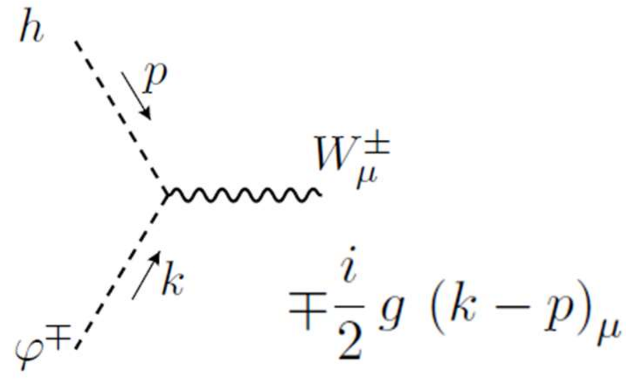
- Longitudinally polarized gauge boson emission is IR finite at leading power

$$(k^{\mu}/m_W)\gamma_{\mu}u_e(p) \propto m_e/m_W$$

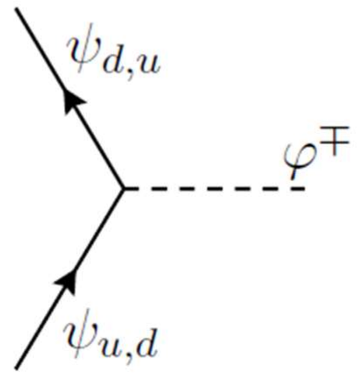
- New massless scalars emerge in unbroken phase,
- Irreducible emissions $k \parallel p \Rightarrow (\not{p} - \not{k})u_e(p) \to 0$
- Reducible suppressed by Yukawa coupling
- Consistent with IR finiteness of longitudinally polarized gauge boson emissions, both being suppressed by Yukawa coupling --- Goldstone Equivalence Theorem

Scalar PDFs

- Collinear gauge boson emission from scalar becomes leading power in unbroken phase



- Should define leading power scalar PDFs?
- For $\mathrm{W} \to \varphi h$ splitting, k-p contracts to physical polarization of partonic W, power suppressed
- Collinear emissions of scalars from fermion remain subleading



Scalar PDFs remain higher power

- Consider evolution of scalar PDFs from broken phase to unbroken phase
- It involves either higher power source (scalar PDFs) or higher power splitting kernels (collinear scalar emissions from fermion and gauge boson)
- Scalar PDFs remain higher power in unbroken phase

PDF up to one loop

- Factorization of differential cross section

$$\frac{d\sigma^{\mu^+\mu^-}}{dp_T dy} = \sum_{i,j=\ell,b} \int dx_i dx_j \phi_{i/e^+}(x_i,\mu)\phi_{j/e^-}(x_j,\mu) \times$$

$$b = W, \gamma, Z, \gamma Z \qquad H_{i,j\to\mu^+\mu^-+X}(x_i,x_j,\mu) + \mathcal{O}(v/E) ,$$

- From $\ell \to \ell + b$,

hypercharge

$$\phi_{\ell/\ell}(x,\mu) = \delta(1-x) + \left(\frac{3}{4}g^2 + g'^2 Y^2\right)\frac{1}{8\pi^2}\ln\left(\frac{\mu^2}{v^2}\right) \times$$

$$\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right] ,$$

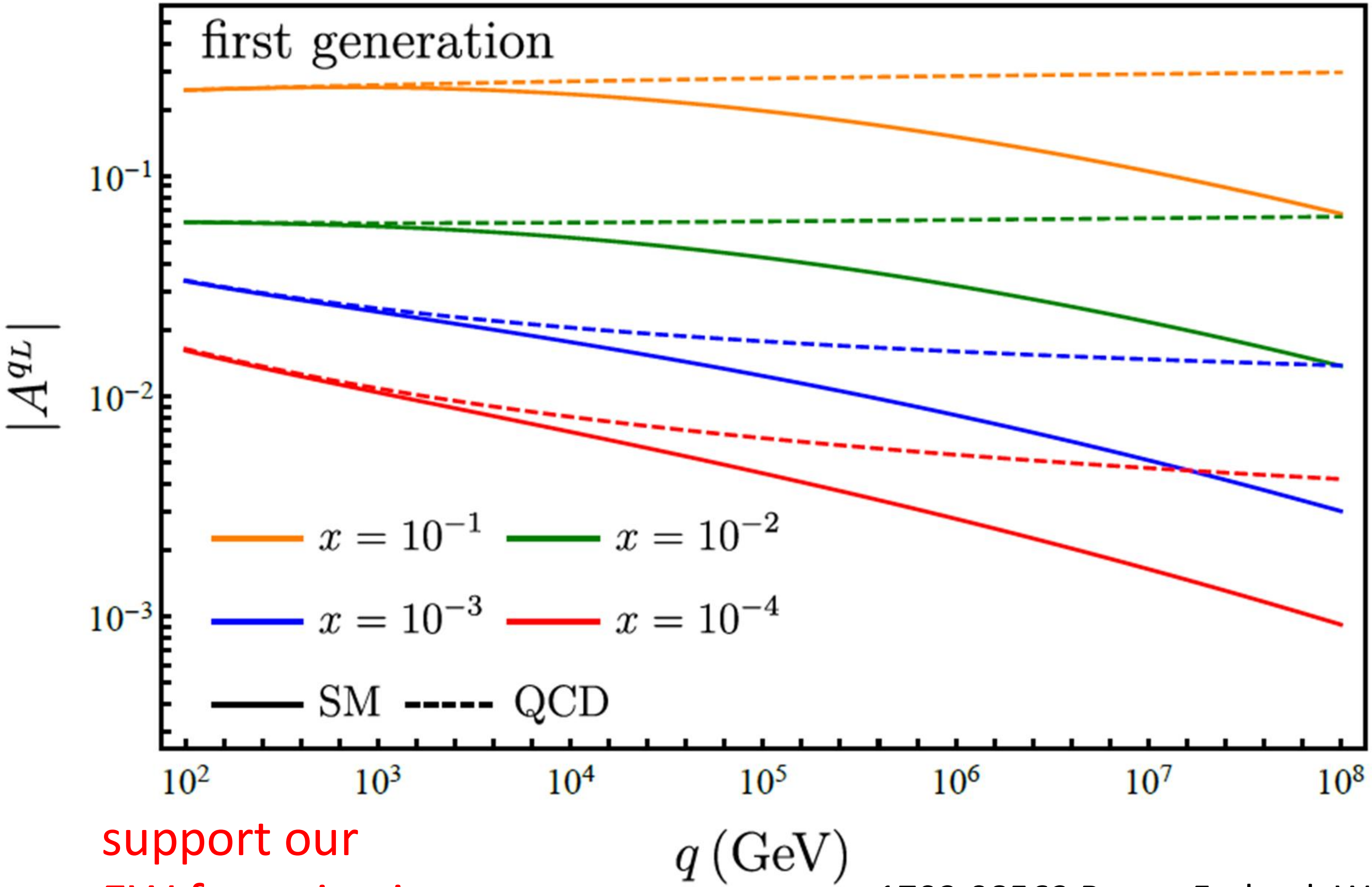
collinear log

Answers to the questions

- EW factorization exist? Yes, in nontrivial way
- What PDFs? Sum over isospin, $\phi_{uL}, \phi_{dL}, \phi_{W^+}, \phi_{W^-}, \cdots$ do not make sense. Bauer et al claimed double log in splitting kernels, implying existence of soft bosons---consequence of no sum over isospin
- Role of scalars in constructing PDFs? Wilson links do not collect collinear scalars, and scalar PDFs are subleading power in lepton scattering
- EW symmetry breaking modifies IR structure? Not at leading power

$$A^{q_L} = \frac{f_{u_L} - f_{d_L}}{f_{u_L} + f_{d_L}}$$

I=1 suppressed



support our
EW factorization

1703.08562 Bauer, Ferland, Webber

More answers

- Emergence of extra scalars in unbroken phase introduce new IR log? Not at leading power
- Yukawa couplings modify power counting of IR log? Yes, reducible collinear scalars are subleading in lepton scattering, but leading in proton scattering (with partonic top)
- Connection between longitudinally polarized gauge bosons and scalars? Both are power suppressed, consistent with Goldstone Equivalence Theorem
- How to match different sets of PDFs in broken and unbroken phases? Perturbative matching

Back-up slides

Summary of PDFs

- For high-energy electron we can construct in broken phase

$$\phi_{\ell/e}, \phi_{W/e}, \phi_{Z/e}, \phi_{\gamma/e}, \phi_{\gamma Z/e}$$

- In unbroken phase

$$\phi_{\ell/e}, \phi_{W/e}, \phi_{W_3/e}, \phi_{B/e}, \phi_{BW/e}$$

- They are perturbatively matched at μ_S
- Mixing is needed too

$$\begin{pmatrix} \phi_{\gamma/e} \\ \phi_{Z/e} \\ \phi_{\gamma Z/e} \end{pmatrix} = \begin{pmatrix} c_W^2 & s_W^2 & c_W s_W \\ s_W^2 & c_W^2 & -c_W s_W \\ -2c_W s_W & 2c_W s_W & c_W^2 - s_W^2 \end{pmatrix} \begin{pmatrix} \phi_{B/e} \\ \phi_{W_3/e} \\ \phi_{BW/e} \end{pmatrix}$$