Connected and disconnected contributions to pion-pion scattering

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Outline

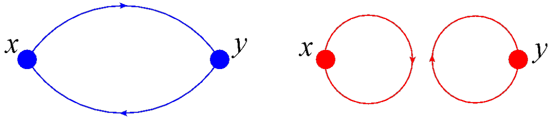
- Connected and disconnected Wick contractions in lattice QCD
- Qualitative analysis: $1/N_c$ and chiral expansions
- Quantitative calculations in partially quenched CHPT

Based on:

1. FKG, L. Liu, U.-G. Meißner, P. Wang,
   Tetraquarks, hadronic molecules, meson-meson scattering and disconnected contributions in lattice QCD,
   Phys. Rev. D 88 (2013) 074506.
2. N.R. Acharya, FKG, U.-G. Meißner, C.-Y. Seng,
   Connected and disconnected contractions in pion-pion scattering,
   Nucl. Phys. B 922 (2017) 480.

- In lattice QCD calculations, correlation functions of interpolating fields composed of quark and antiquark fields are computed
- For instance, the two-point correlation function of the scalar interpolator: $\bar{u}u$

$$\begin{aligned} &\langle \bar{u}(y)u(y)\,\bar{u}(x)u(x)\rangle \\ &= -\langle S_u(y,x)S_u(y,x)\rangle + \langle S_u(y,y)\rangle\,\langle S_u(x,x)\rangle \end{aligned}$$



- Two different types of Wick contractions: connected and disconnected

Disconnected contractions

- Disconnected diagrams are expensive to compute, and noisy (a direct computation involves $12L^3 (> 10^5)$ linear equations)

  see, e.g., Chuan Liu, *Introduction to Lattice QCD*

  "Numerically these contributions need more computational effort and higher statistics than the connected parts and many studies avoid considering such mesons or drop the disconnected pieces."

  — C. Gattringer, C. B. Lang, *Quantum Chromodynamics on the Lattice*

- Present in many circumstances:
  - ☞ Flavour-singlet mesons
  - ☞ Scalar form factors
  - ☞ Multiquark states
  - ☞ Meson-meson scattering
  - ☞ . . .

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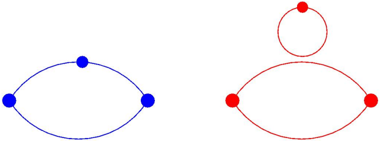

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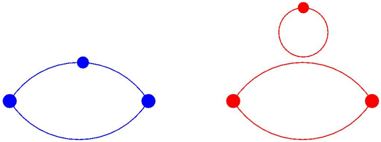
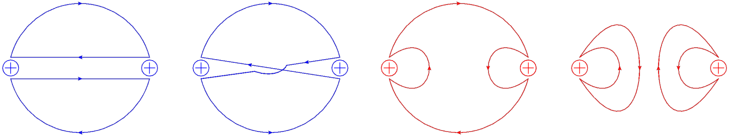




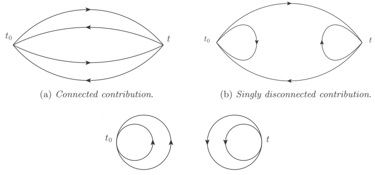
direct-connected cross-connected singly-disconnected doubly-disconnected

Lattice QCD status on scalar tetraquarks and isoscalar $\pi\pi$ scattering

- However, sometimes neglected.
  For scalar mesons, calculations using tetraquark interpolators neglecting the disconnected contribution:

  Quenched: Alford, Jaffe (2000); Mathur et al. (2007); Suganuma et al (2007); Loan, Luo, Lam (2008); Prelovsek, Mohler (2009); . . .

  Dynamical: Prelovsek et al. (2010), Alexandrou et al. (2013); . . .



(a) *Connected contribution.* (b) *Singly disconnected contribution.*
(c) *Doubly disconnected contribution.*

Example

$a_0(980)$ and κ using four-quark operators
(b) was neglected [(c) does not contribute]
no state was found
$\Rightarrow$ conclusion ?

- Methods proposed to deal with the disc. contribution, e.g.,
  stochastic LapH quark smearing method Peardon et al. (2009); Morningstar et al. (2011)
- Disconnected contribution included:
  Bali, Collins, Ehmann (2011); Lang et al. (2012); Prelovsek et al. (2013); Dudek et al. (2013); . . .
  for $I = 0$ $\pi\pi$ scattering: Fu (2013); Briceño et al. (2016); Liu et al. (2016)

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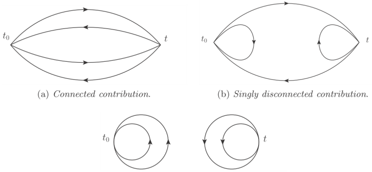

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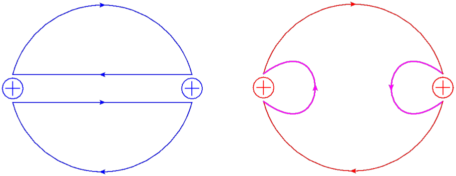
Example of tetraquarks (1)

- To see the impact of disconnected contribution, consider $\bar{q}c\bar{s}q$ $(q = u, d)$ [relevant to the charm-strange isoscalar meson $D^*_{s0}(2317)$]

$$
\begin{aligned}
&\langle \bar{q}c\bar{s}q(y)\,\bar{q}s\bar{c}q(x)\rangle \\
=\ &\langle S_q(x,y)S_c(y,x)\rangle\,\langle S_s(x,y)S_q(y,x)\rangle - \langle S_q(y,y)S_c(y,x)S_q(x,x)S_s(x,y)\rangle \\
\equiv\ &C_{\text{conn.}} - C_{\text{disc.}}
\end{aligned}
$$



- What would happen if we neglect the disconnected diagram?

Example of tetraquarks (2)

- For the isospin eigenstates, using isospin symmetry,
  - ☞ $I=0$: $j_0 = \frac{1}{\sqrt{2}}\left(\bar{u}c\bar{s}u + \bar{d}c\bar{s}d\right)$

$$\begin{aligned}\left\langle j_0^\dagger(y) j_0(x)\right\rangle &= \frac{1}{2}\left[\langle \bar{u}c\bar{s}u(y)\,\bar{u}s\bar{c}u(x)\rangle + \left\langle \bar{u}c\bar{s}u(y)\,\bar{d}s\bar{c}d(x)\right\rangle + (u\leftrightarrow d)\right] \\ &= \langle \bar{u}c\bar{s}u(y)\,\bar{u}s\bar{c}u(x)\rangle + \left\langle \bar{u}c\bar{s}u(y)\,\bar{d}s\bar{c}d(x)\right\rangle \\ &= C_{\text{conn.}} - C_{\text{disc.}} \qquad - C_{\text{disc.}}\end{aligned}$$

  - ☞ $I=1$: $j_1 = \frac{1}{\sqrt{2}}\left(\bar{u}c\bar{s}u - \bar{d}c\bar{s}d\right)$

$$\begin{aligned}\left\langle j_1^\dagger(y) j_1(x)\right\rangle &= \langle \bar{u}c\bar{s}u(y)\,\bar{u}s\bar{c}u(x)\rangle - \left\langle \bar{u}c\bar{s}u(y)\,\bar{d}s\bar{c}d(x)\right\rangle \\ &= C_{\text{conn.}} - C_{\text{disc.}} \qquad + C_{\text{disc.}} = C_{\text{conn.}}\end{aligned}$$

- Neglecting the disconnected diagram, one can only get information in the isovector channel !

  This was unfortunately done in some lattice calculation of $D_{s0}^*(2317)\ldots$

- Can we quantify contributions of the disconnected diagrams using analytical methods?

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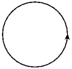
Qualitative analysis

FKG, L. Liu, U.-G. Meißner, P. Wang, PRD88(2013)074506

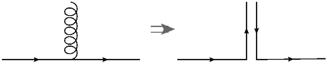
Qualitative analysis: Large N_c (1)

- Basic ingredients of the large N_c power counting: 't Hooft (1974); Witten (1979)

  - ☞ Each quark loop contributes a combinatoric factor N_c



  - ☞ Each quark-gluon vertex scales as $1/\sqrt{N_c}$



- Four-quark interpolating field: $O^{ij}_{ABCD}(x) = [\bar{q}_A(x)\Gamma^i q_B(x)][\bar{q}_C(x)\Gamma^j q_D(x)]$.

  S.Weinberg, PRL110(2013)261601; M.Knecht, S.Peris, PRD88(2013)036016; ...

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  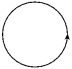

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  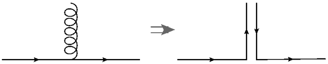

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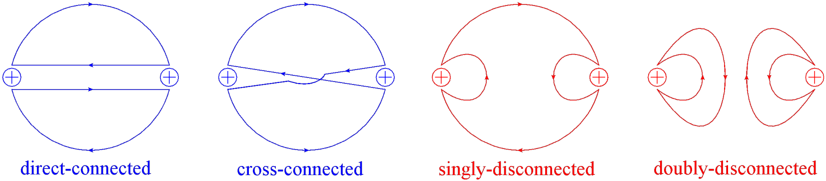
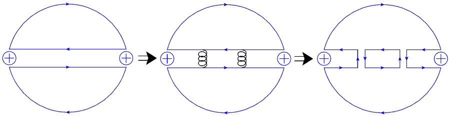


Qualitative analysis: Large N_c (2)

- Direct-connected: exchange of at least two gluons $\mathcal{O}\left(N_c^2 \frac{1}{(\sqrt{N_c})^4}\right) = \mathcal{O}\left(N_c^0\right)$



- Cross-connected: $\mathcal{O}\left(N_c\right)$
- Singly-disc. (rectangular): $\mathcal{O}\left(N_c\right)$
- Doubly-disconnected (vacuum): $\mathcal{O}\left(N_c^2 \frac{1}{(\sqrt{N_c})^4}\right) = \mathcal{O}\left(N_c^0\right)$

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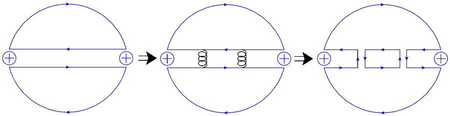

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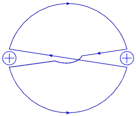

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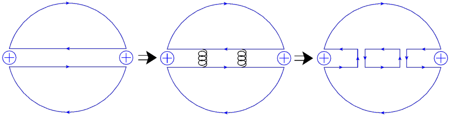

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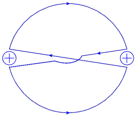

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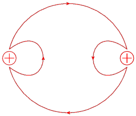

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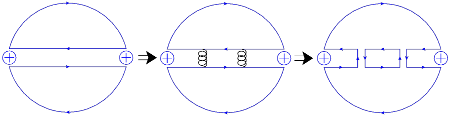

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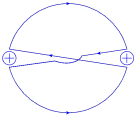

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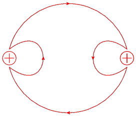

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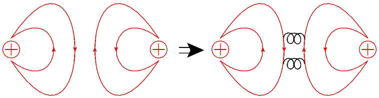
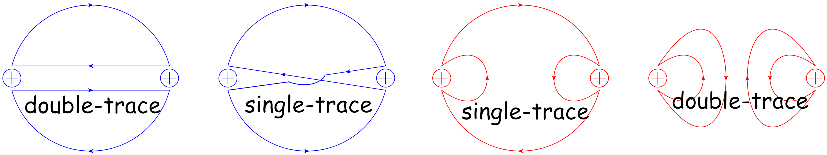


- For $\pi\pi$ scattering $\Rightarrow$ using chiral power counting
- Number of quark loops $=$ number of flavor traces

see, e.g., A.Manohar, arXiv:hep-ph/9802419



direct-connected | cross-connected | singly-disconnected | doubly-disconnected

- The leading order CHPT Lagrangian

$$\mathcal{L}_{\text{LO}} = \frac{F_0^2}{4}\left(\mathrm{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] + 2B_0\,\mathrm{Tr}\left[\mathcal{M}U^\dagger + \mathcal{M}^\dagger U\right]\right)$$

only contains single-flavor-trace terms

- double trace terms start from next-to-leading order in CHPT, e.g.

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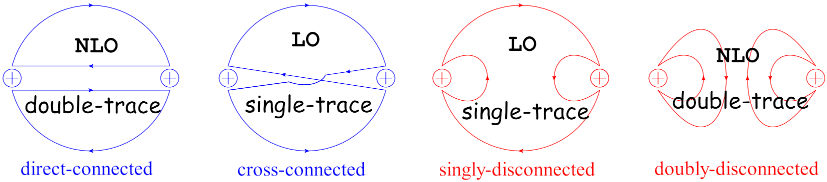

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Quantitative calculation of $\pi\pi$ scattering

N.R. Acharya, FKG, U.-G. Meißner, C.-Y. Seng, NPB922(2017)480

- Each Wick contraction can be calculated separately in partially quenched CHPT

  Bernard, Golterman (1992, 1994, 2013); Sharpe, Shoresh (2000, 2001); . . .

  For reviews, see Sharpe, arXiv:hep-lat/0607016; Golterman, arXiv:0912.4042 [hep-lat]

  For calculations of contractions, see Tiburzi (2009); Della Morte, Jüttner (2010); Jüttner (2012); . . .

- PQQCD for two light-flavors:

$$\mathcal{L} = \bar{Q}(i\not{D} - \mathcal{M})Q - \frac{1}{4}G^{a\mu\nu}G^{a}_{\mu\nu}$$

Sea + valence + ghost quarks: $Q = \left(u\ d\ j\ k \,|\, \tilde{j}\ \tilde{k}\right)^T$

in the fundamental representation of SU(4|2): special unitary graded group.

PQQCD is equivalent to QCD when $m_{\text{sea}} = m_{\text{valence}} = m_{\text{ghost}}$

- An element in SU(4|2):

$$M = \begin{pmatrix} A_{4\times 4} & B_{4\times 2} \\ C_{2\times 4} & D_{2\times 2} \end{pmatrix}, \qquad A, D\text{: commutating; } B, C\text{: anti-commutating;}$$

Supertrace: $\mathrm{Str}(M) = \mathrm{Tr}(A) - \mathrm{Tr}(D)$

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- Spontaneous chiral symmetry breaking in PQQCD (sea + valence + ghost):

$$\mathrm{SU}(4|2)_L \times \mathrm{SU}(4|2)_R \to \mathrm{SU}(4|2)_V$$

$\Rightarrow 6^2 - 1 = 35$ pseudo-Goldstone bosons:

$$U = \exp\left\{ \frac{2i}{F_0} \sum_{a=1}^{35} \phi^a T^a \right\}$$

- Alternatively, including a singlet field Φ_0 with a mass m_0 (integrated out later on by $m_0 \to \infty$), then $U = \exp\left(i\sqrt{2}\Phi/F_0\right)$, Sharpe, Shoresh (2001)

$$\Phi = \begin{pmatrix} \phi & \chi^\dagger \\ \chi & \tilde{\phi} \end{pmatrix}, \quad \phi = \begin{pmatrix} \eta_u & \pi^+ & \phi_{u\bar{j}} & \phi_{u\bar{k}} \\ \pi^- & \eta_d & \phi_{d\bar{j}} & \phi_{d\bar{k}} \\ \phi_{j\bar{u}} & \phi_{j\bar{d}} & \eta_j & \phi_{j\bar{k}} \\ \phi_{k\bar{u}} & \phi_{k\bar{d}} & \phi_{k\bar{j}} & \eta_k \end{pmatrix}, \quad \tilde{\phi} = \begin{pmatrix} \eta_{\tilde{j}} & \phi_{\tilde{j}\bar{\tilde{k}}} \\ \phi_{\tilde{k}\bar{\tilde{j}}} & \eta_{\tilde{k}} \end{pmatrix},$$

$$\chi = \begin{pmatrix} \phi_{\tilde{j}\bar{u}} & \phi_{\tilde{j}\bar{d}} & \phi_{\tilde{j}\bar{j}} & \phi_{\tilde{j}\bar{k}} \\ \phi_{\tilde{k}\bar{u}} & \phi_{\tilde{k}\bar{d}} & \phi_{\tilde{k}\bar{j}} & \phi_{\tilde{k}\bar{k}} \end{pmatrix},$$

- Spontaneous chiral symmetry breaking in PQQCD (sea + valence + ghost):

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PQCHPT (2)

- LO Lagrangian: $\mathcal{L}^{(2)} = \frac{F_0^2}{4}\mathrm{Str}\left[(\partial_\mu U^\dagger)(\partial^\mu U)\right] + \frac{F_0^2 B_0}{2}\mathrm{Str}\left[\mathcal{M}U^\dagger + U\mathcal{M}^\dagger\right]$
- NLO Lagrangian contains unphysical low-energy constants (LECs):

$$\begin{aligned}
\mathcal{L}^{(4)} = {} & L_0^{\rm PQ}\mathrm{Str}\left[(\partial_\mu U^\dagger)(\partial_\nu U)(\partial^\mu U^\dagger)(\partial^\nu U)\right] \\
& +(L_1^{\rm PQ} - \frac{1}{2}L_0^{\rm PQ})\mathrm{Str}\left[(\partial_\mu U^\dagger)(\partial^\mu U)\right]\mathrm{Str}\left[(\partial_\nu U^\dagger)(\partial^\nu U)\right] \\
& +(L_2^{\rm PQ} - L_0^{\rm PQ})\mathrm{Str}\left[(\partial_\mu U^\dagger)(\partial_\nu U)\right]\mathrm{Str}\left[(\partial^\mu U^\dagger)(\partial^\nu U)\right] \\
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\end{aligned}$$

- Independent LECs: $\underbrace{l_1, l_2, l_3, l_4, l_7}_{\text{physical}}, \underbrace{L_0^{\rm PQ}, L_3^{\rm PQ}, L_5^{\rm PQ}, L_8^{\rm PQ}}_{\text{unphysical}}$

$$l_1 \equiv 2(2L_1^{\rm PQ} + L_3^{\rm PQ}), \quad l_2 \equiv 4L_2^{\rm PQ}, \quad l_3 \equiv -4(2L_4^{\rm PQ} + L_5^{\rm PQ} - 4L_6^{\rm PQ} - 2L_8^{\rm PQ}),$$
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$$
$$
l_4 \equiv 4(2L_4^{\rm PQ} + L_5^{\rm PQ}),\quad l_7 \equiv -8(2L_7^{\rm PQ} + L_8^{\rm PQ})
$$

- Isospin symmetry + crossing $\Rightarrow$ only one independent $\pi\pi$ scattering amplitude:

$$
\begin{aligned}
T^{I=0}(s,t,u) &= 3T(s,t,u) + T(t,s,u) + T(u,t,s)\,, \\
T^{I=1}(s,t,u) &= T(t,s,u) - T(u,t,s)\,, \\
T^{I=2}(s,t,u) &= T(t,s,u) + T(u,t,s)\,.
\end{aligned}
$$

- Wick contractions for $T(s,t,u) \equiv T_{\pi^+\pi^-\to\pi^0\pi^0}(s,t,u)$
[$\pi^+ = \bar{d}u$, $\pi^- = \bar{u}d$, $\pi^0 = (\eta_u - \eta_d)/\sqrt{2}$, $\eta_u \equiv \bar{u}u$, $\eta_d \equiv \bar{d}d$]:

$$
T(s,t,u) = T_{\pi^+\pi^-\to\eta_u\eta_u}(s,t,u) - T_{\pi^+\pi^-\to\eta_u\eta_d}(s,t,u)
$$

$$
=
$$

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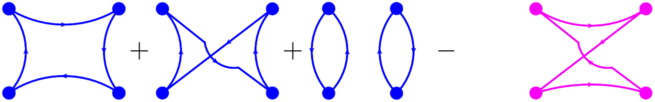
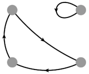
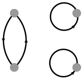
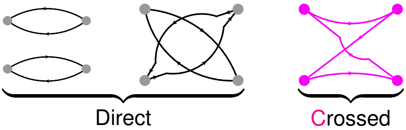
$=$ 

Contractions like  and  do not contribute

Wick contractions for $\pi\pi$ scattering (2)

- Various possible Wick contractions for $\pi\pi$ scattering:

"Connected":



Singly-disconnected:

- Isospin symmetric $\pi\pi$ scattering amplitudes:

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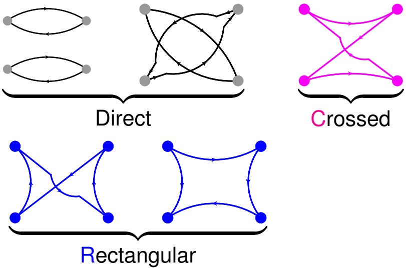

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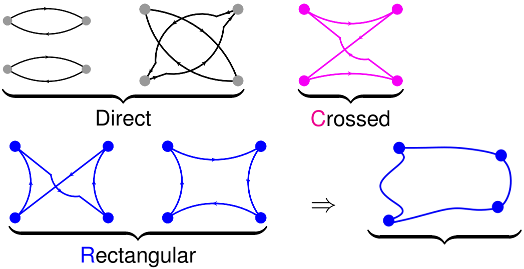

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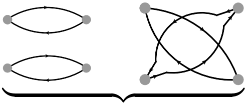
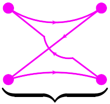
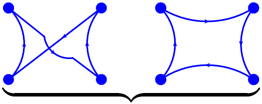


Direct



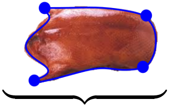
Crossed

Singly-disconnected:



Rectangular

$\Rightarrow$



Roasted Peking duck

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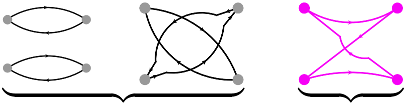
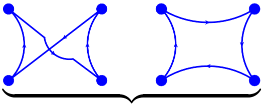


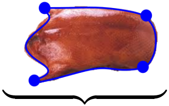
Direct | Crossed

Singly-disconnected:



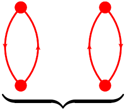
Rectangular

$\Rightarrow$



Roasted Peking duck

Doubly-disconnected:



Vacuum

difficult in lattice!

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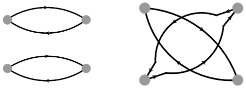
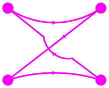
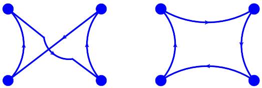


Direct



Crossed

Singly-disconnected:



Rectangular

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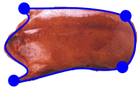

Roasted Peking duck

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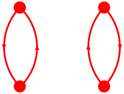

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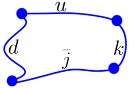
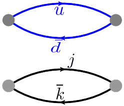
- Additional (auxiliary) flavors in PQQCD ($u, d, j, k, \tilde{j}, \tilde{k}$)
  $\Rightarrow$ Wick contractions can be calculated separately



$$(s,t,u) = T_{(u\bar{d})(d\bar{j})\to(u\bar{k})(k\bar{j})}(s,t,u)$$



$$(s,t,u) = T_{(u\bar{d})(j\bar{k})\to(u\bar{d})(j\bar{k})}(s,t,u)$$

calculable as amplitudes of “physical” processes in PQCHPT

- The other contractions can be obtained by crossing symmetry, e.g.:

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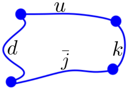

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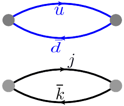

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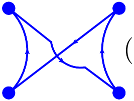
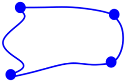
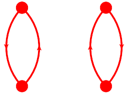


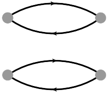
$$(s,t,u) =$$



$$(s,u,t),$$



$$(s,t,u) =$$



$$(t,s,u), \ldots$$

- Analytic expression for each Wick contraction in PQCHPT up to NLO, dependent on both physical $(\bar{l}_1, \bar{l}_2, \bar{l}_3, \bar{l}_4, \bar{l}_7)$ Colangelo, Gasser (2001); Bijnens, Ecker (2014)

  and unphysical LECs $(L_0^{\mathrm{PQ},r}, L_3^{\mathrm{PQ},r}, L_5^{\mathrm{PQ},r}, L_8^{\mathrm{PQ},r})$ Boyle et al., PRD93(2016)054502

$\bar{l}_1$	$-0.4(6)$
$\bar{l}_2$	4.3(1)
$\bar{l}_3$	3.0(8)
$\bar{l}_4$	4.4(2)
$10^3 L_0^{\mathrm{PQ},r}$	1.0(1.1)
$10^3 (L_3^{\mathrm{PQ},r} + 2L_0^{\mathrm{PQ},r})$	$-1.56(87)$
$10^3 L_5^{\mathrm{PQ},r}$	0.501(43)
$10^3 L_8^{\mathrm{PQ},r}$	0.581(22)

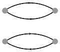
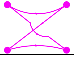
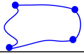
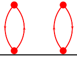
☞ $L_{5,8}^{\mathrm{PQ},r}$: fixed from the NLO meson masses and decay constants

☞ $L_{0,3}^{\mathrm{PQ},r}$: only fixed from a NNLO fitting $\Rightarrow$ large uncertainties

- Amplitudes for physical QCD processes only depend on physical LECs, standard CHPT results reproduced

Calculation of contractions in PQCHPT (3)

- $\pi\pi$ cattering lengths $a_X^{IJ} = \lim_{q^2\to 0}(q^2)^{-J}T_X^{IJ}(4M_\pi^2+4q^2)$

	$10^2 a_X^{00}$	$10^2 a_X^{20}$	$10^2 M_\pi^2 a_X^{11}$	$10^4 M_\pi^4 a_X^{02}$	$10^4 M_\pi^4 a_X^{22}$
D 	$0.35(24)$	$0.35(24)$	$0.02(26)$	$3.5(2.0)$	$3.5(2.0)$
C 	$2.41(12)$	$-4.81(23)$	0	$0.95(96)$	$-1.9(1.9)$
R 	$14.8(7)$	0	$3.59(26)$	$6.7(7.8)$	0
V 	$2.48(38)$	0	0	$0.8(7.3)$	0
Total	$20.0(2)$	$-4.46(7)$	$3.61(4)$	$11.9(8)$	$1.54(71)$

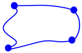
- The R-type contribution dominates as long as it contributes:
  - ☞ expected $\Leftarrow$ leading order in both chiral and $1/N_c$ expansions
  - ☞ neglecting the vacuum-type contribution would reduce the isoscalar S-wave $\pi\pi$ scattering length by about 12%.

- Can be used to make a more precise determination of the unphysical LECs $L_{0,3}^{\mathrm{PQ},r}$
- Can be used to check the accuracy of the lattice calculations, e.g.,

$$\begin{aligned} a_V^{00} - \frac{3}{2} a_D^{00} &= \frac{M_\pi^4}{\pi F_\pi^4} \left(\frac{3\bar{l}_4}{64\pi^2} - 3L_5^{\mathrm{PQ,r}} + \frac{3}{128\pi^2} \log \frac{M_\pi^2}{\mu^2} + \frac{9}{512\pi^2} \right) \\ &= (1.96 \pm 0.16) \times 10^{-2} \end{aligned}$$

Summary and outlook

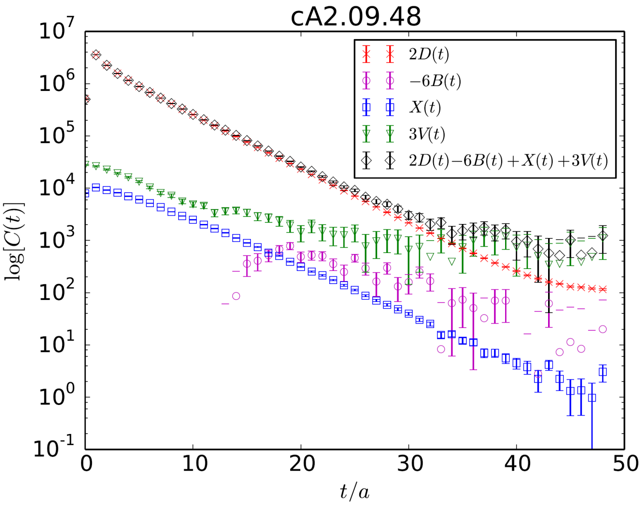
- In lattice calculations of tetraquarks and meson-meson scattering:

  The singly-disconnected (R-type) contribution  is always of leading order (dominant for isoscalar $\pi\pi$)

- Extension of the PQCHPT calculation to other scattering processes and other observables such as the parity violating $NN\pi$ coupling;
  - ☞ quantitatively assess the importance of the disconnected contribution
  - ☞ possible way to reduce the lattice QCD efforts in calculating the disconnected contribution

Thank you for your attention!

Backup slides



European Twisted Mass Collaboration

$M_\pi = 240, 330$ MeV

chiral extrapolation $\Rightarrow$

$a^{00} = 0.198(9)(6)$

Liu et al. (ETMC), PRD96(2017)054016

Notations:

Rectangular $\to$ Box;

Crossed $\to$ X