Past, Present, and Future of the QGP Physics

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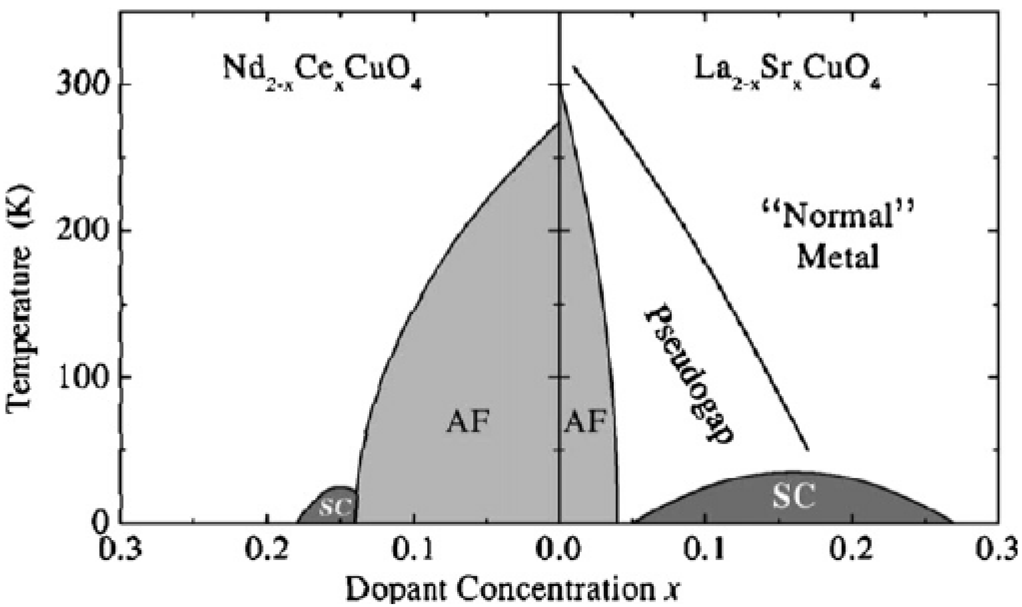
November 8, 2018

Toward Microscopic Understanding

In Condensed Matter Physics

➢ 1st Macroscopic Properties

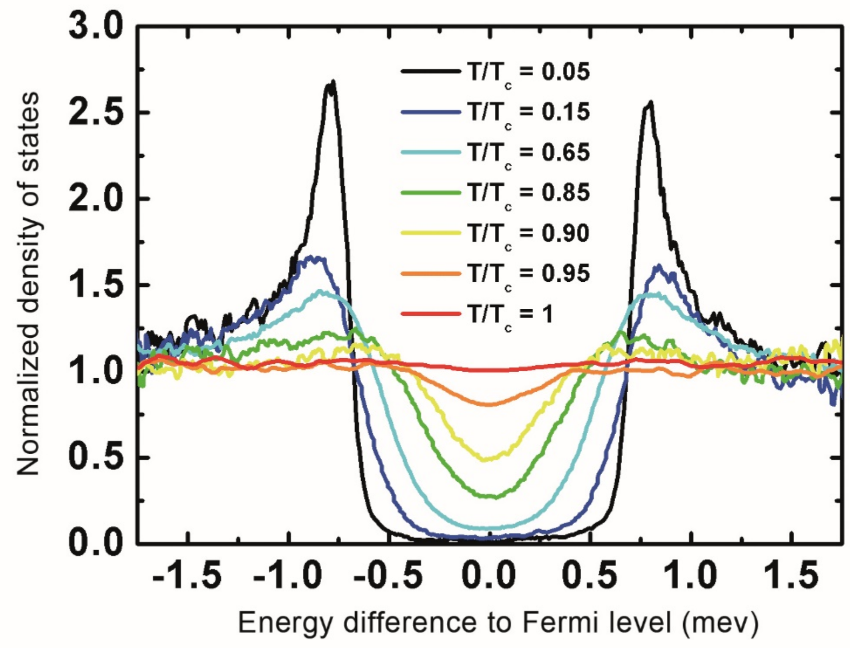
- bulk quantities: heat capacity, heat conductivity, transport coefficients, …
- phase structure



Toward Microscopic Understanding

➢ 2nd Microscopic Properties

- effective mass
- band structure, gap structure
- various correlations
- spectral function



➢ 3rd Microscopic Understanding

- (Normal) Superconductor: BCS theory
- Fractal Quantum Hall Effect: Laughlin wave function

In QGP Physics

In Condensed Matter Physics

➢ 1st Macroscopic Properties

- bulk quantities: heat capacity, heat conductivity, transport coefficients, …
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We are HERE

➢ 2nd Microscopic Properties

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We are HERE

➢ 3rd Microscopic Understanding

- (Normal) Superconductor: BCS theory
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Past and Now of QGP

- Past — *weakly interacting soup of quarks and gluons*

  expected from asymptotic freedom of QCD

  Collins and Perry (1975)

- Now — *strongly interacting system of quarks and gluons*

  from small η/s of QGP

  RHIC experiments 2004~

• Why small $\eta/s \leftrightarrow$ strongly interacting system?

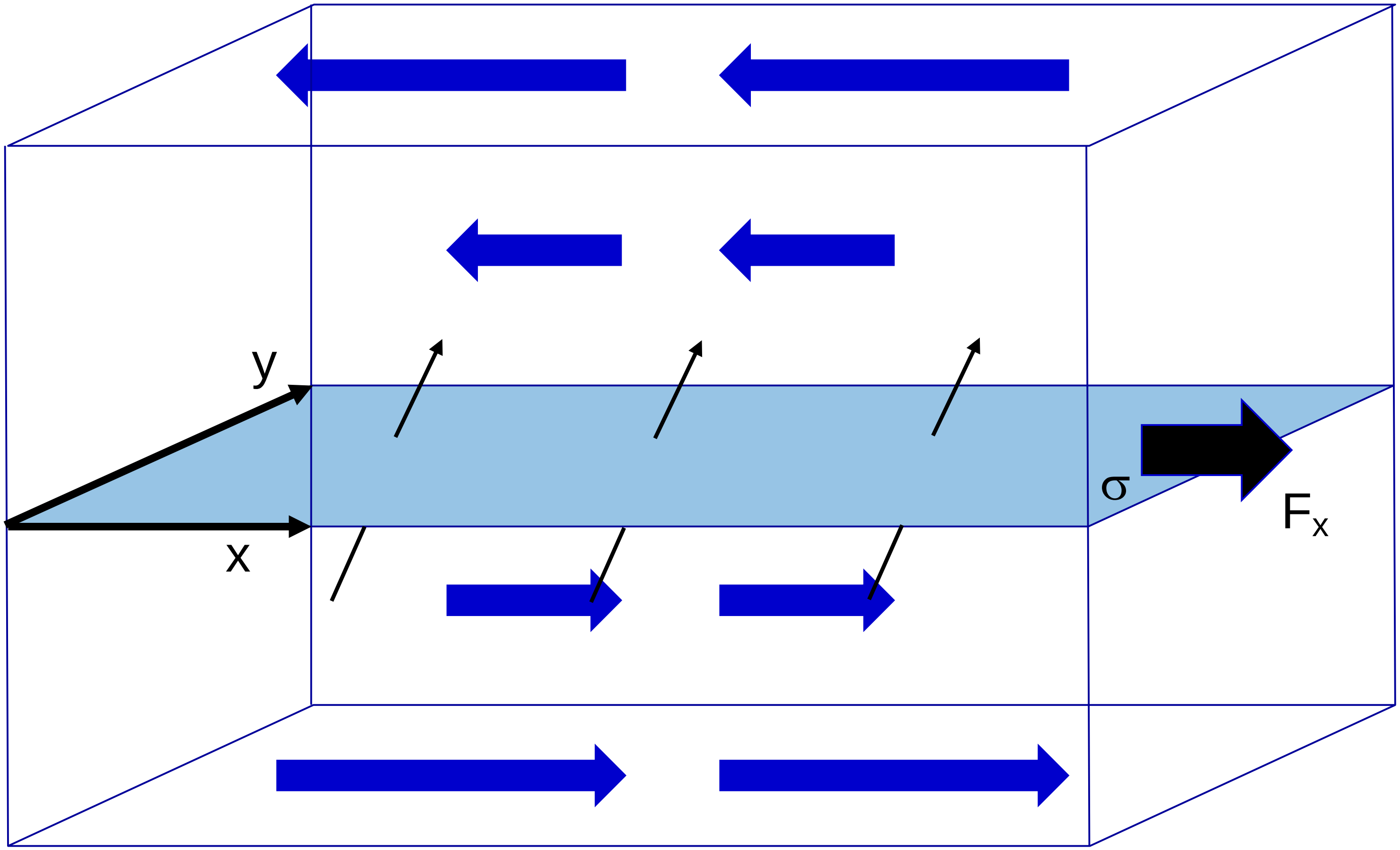
$$\eta = \frac{1}{3} n\bar{v}ml$$

This is obtained by dilute gas approximation

This approximation is not valid for strongly interacting cases

Qualitative Understanding of Shear Viscosity

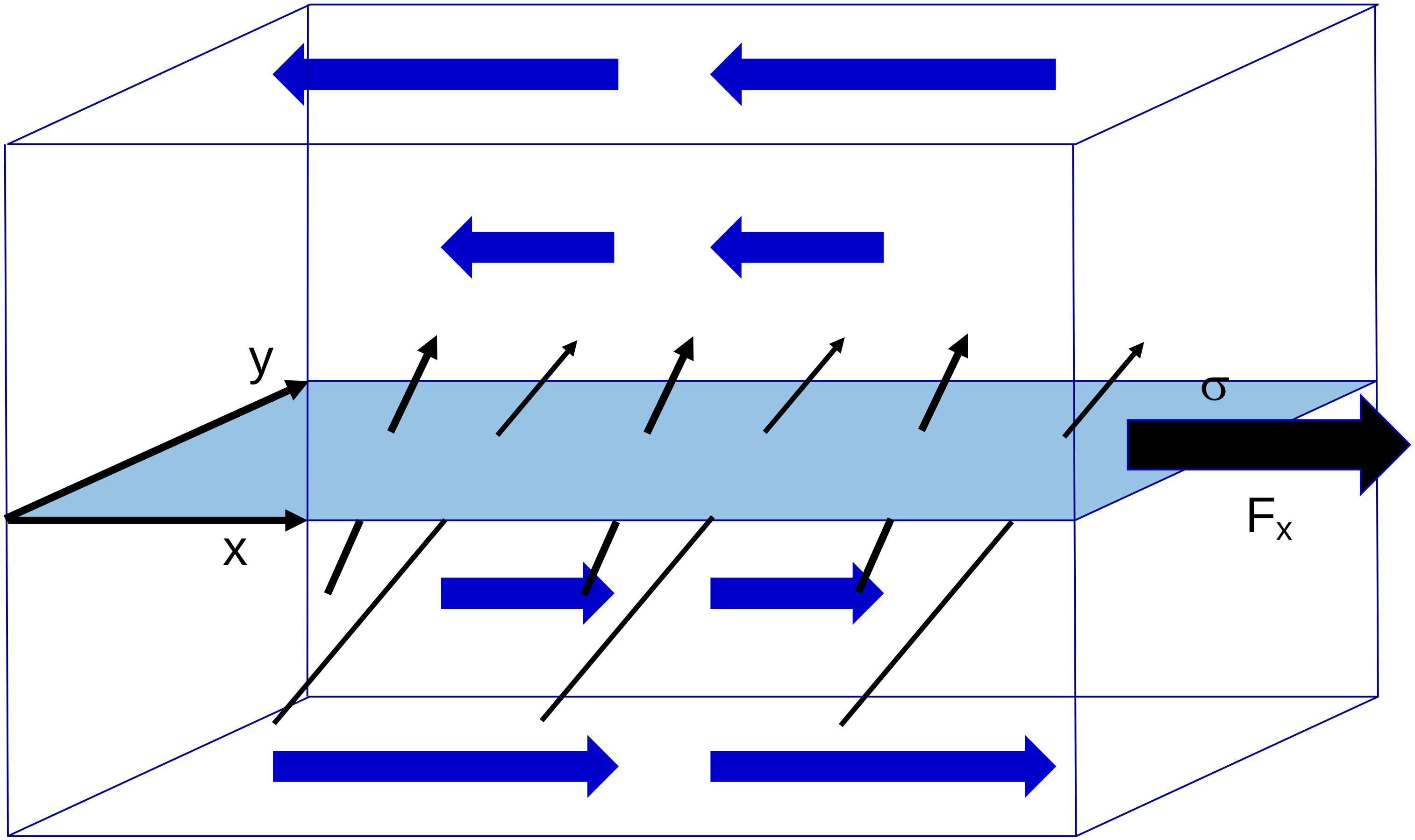
strongly interacting case



shear stress (F_x) = p_x that crosses unit surface (σ) per unit time : small

Qualitative Understanding of Shear Viscosity

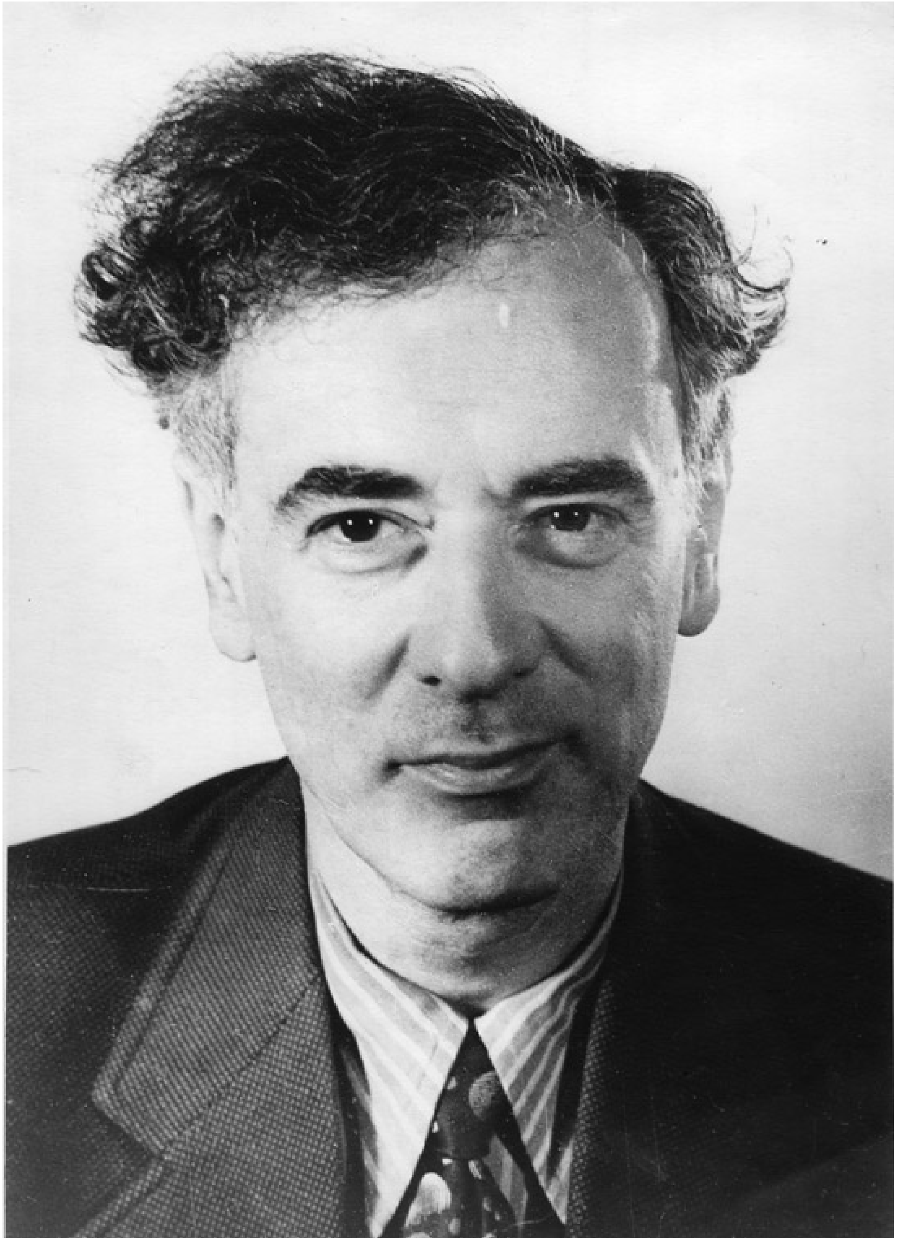
weakly interacting case



more p_x crosses unit surface (σ) per unit time : larger stress

= larger shear viscosity

Landau's Insight



74. ON MULTIPLE PRODUCTION OF PARTICLES DURING COLLISIONS OF FAST PARTICLES

1. General Relations

Collisions of ultra-fast nuclear particles can be accompanied by the appearance of a large number of new particles (many-pronged stars in cosmic radiation). Fermi[1] propounded the ingenious idea of the possibility of applying statistical methods for studying this process. However, the quantitative calculation given by him appears unconvincing to us and incorrect at several points (in particular, in regard to distribution in energy and angle).

Qualitatively the whole process of collision has the following appearance. At the moment of collision there appear a large number of particles† concentrated in a volume whose linear dimensions are determined by the range of the nuclear forces and by the energies of the colliding particles (concerning this, see below); it must be emphasised that we can speak of the number of particles at this moment only in a limited sense, since for a system with such a high density of strongly interacting particles (mesons and nucleons) the concept of the number of particles has in general no precise meaning. The "mean free path" of particles in such a system is clearly very small compared to its dimensions. In the course of time, the system expands, but the aforementioned property of the free path must be valid also for a significant part of the process of expansion. This part of the expansion process must have a hydrodynamic character, since the smallness of the mean free path permits us to consider the motion of the matter in the system in a macroscopic hydrodynamical fashion as the motion of an ideal (non-viscous and non-heat-conducting) liquid. Since the velocities in the system are comparable to the velocity of light, we are dealing, not with ordinary, but rather with relativistic hydrodynamics.

The total "number of particles" in the system is not at all constant during the course of the hydrodynamic stage of the expansion. Therefore, the number of particles in the resulting star is determined, not by the number of particles which appear at the very moment of collision (as Fermi mistakenly assumes) but rather by the number of particles in the system at the moment of transition to the second stage of the expansion—the stage of free separation of the particles. This essential point was first made by I. Ya. Pomeranchuk[2].

Л. Д. Ландау, О множественном образовании частиц при столкновениях быстрых частиц, *Известия Академии Наук СССР, Серия Физическая*, **17**, 51 (1953).

† In fact, the appearance of a large number of particles is the condition for the applicability of the method for treating the problem which is presented below, and of the associated formulas.

Landau's Insight

74. ON MULTIPLE PRODUCTION OF PARTICLES DURING COLLISIONS OF FAST PARTICLES

1. General Relations

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Strongly interacting system

"Mean Free Path" : so small

Quantum mechanically, concept of the number of particles loses its meaning

"Mean Free Path" makes sense only when it is much larger than de Broglie wave length

Hydrodynamics

Future of Hydrodynamics?

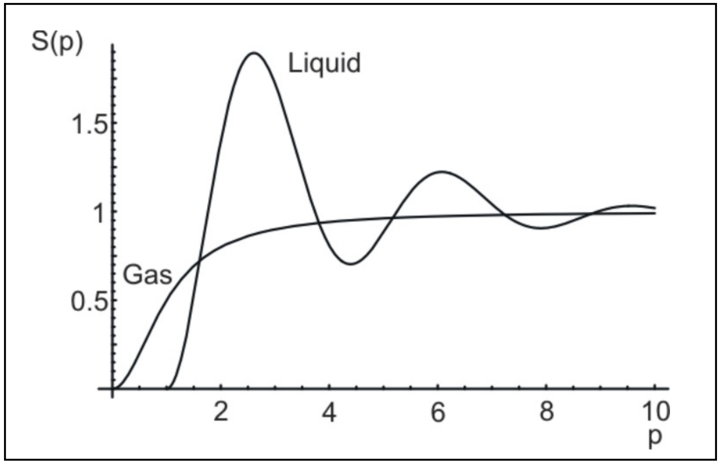
- Although “Mean Free Path” argument kills a lot of transport models, hydrodynamics is ***not the end***

  Hydrodynamics should be compared to Jellium Model
  in condensed matter physics

- What is the next step?

  - ✓ A possible answer: microscopic structure of jellium (or fluid)

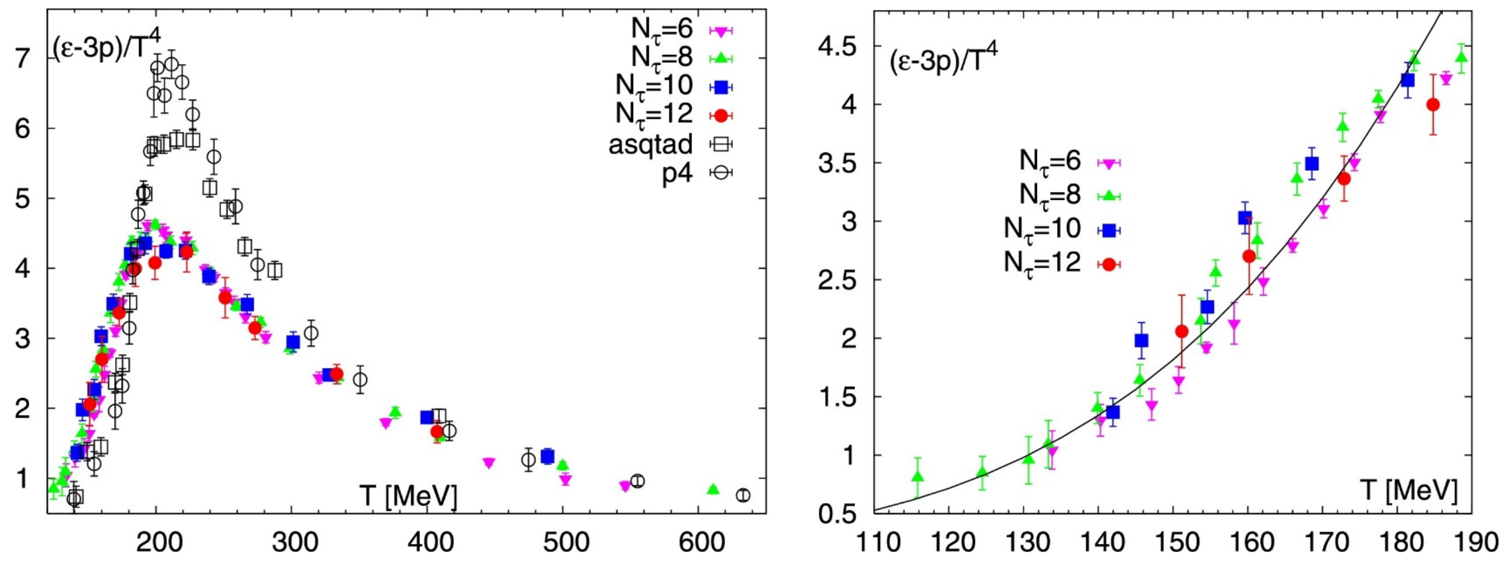
  For example, structure function:
  Fourier transform of spatial correlation



Thoma, QM2005

How can we see interaction on Lattice?

- In the following, $\mu_B=0$



$$I = \frac{e - 3p}{T^4}$$: “Interaction Measure” or “Trace Anomaly”

Naïve questions: Isn’t there interaction in hadron phase or in the vacuum?

Doesn’t there exist trace anomaly in the vacuum (strongly interacting!)?

What is shown by Lattice Calculation

On the lattice, vacuum subtraction is carried out

Since QCD vacuum is more stable than perturbative vacuum,

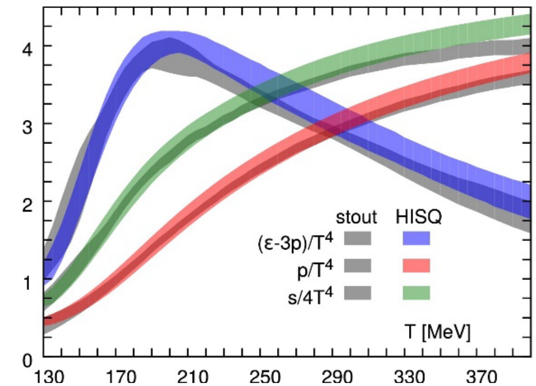
$$e_0 = \left\langle T^{00} \right\rangle_{\text{QCD vacuum}} < 0$$
$$p_0 = \left\langle T^{ii} \right\rangle_{\text{QCD vacuum}} > 0$$

From Lorentz invariance of the vacuum,

$$\left\langle T^{\mu\nu} \right\rangle_{\text{QCD vacuum}} = e_0 g^{\mu\nu}$$

What is plotted as e or p: vacuum subtracted

$$e = \left\langle T^{00} \right\rangle_T - \left\langle T^{00} \right\rangle_{T=0}$$
$$p = \left\langle T^{ii} \right\rangle_T - \left\langle T^{ii} \right\rangle_{T=0}$$



Where is interaction?

$$\langle T^{\mu\nu} \rangle_{T=0} = e_0 g^{\mu\nu}$$

i: not summed

$$Ts = e + p = \left[\langle T^{00} \rangle_T - \cancel{\langle T^{00} \rangle_{T=0}} \right] + \left[\langle T^{ii} \rangle_T - \cancel{\langle T^{ii} \rangle_{T=0}} \right] = \langle T^{00} \rangle_T + \langle T^{ii} \rangle_T$$

Entropy density s is not affected by this subtraction
(From Nernst's theorem: s=0 at T=0)

s has a direct physical meaning: $\propto$ density of degrees of freedom

Suppose a sudden phase transition from free massless pion gas to free quark-gluon plasma takes places at T_c

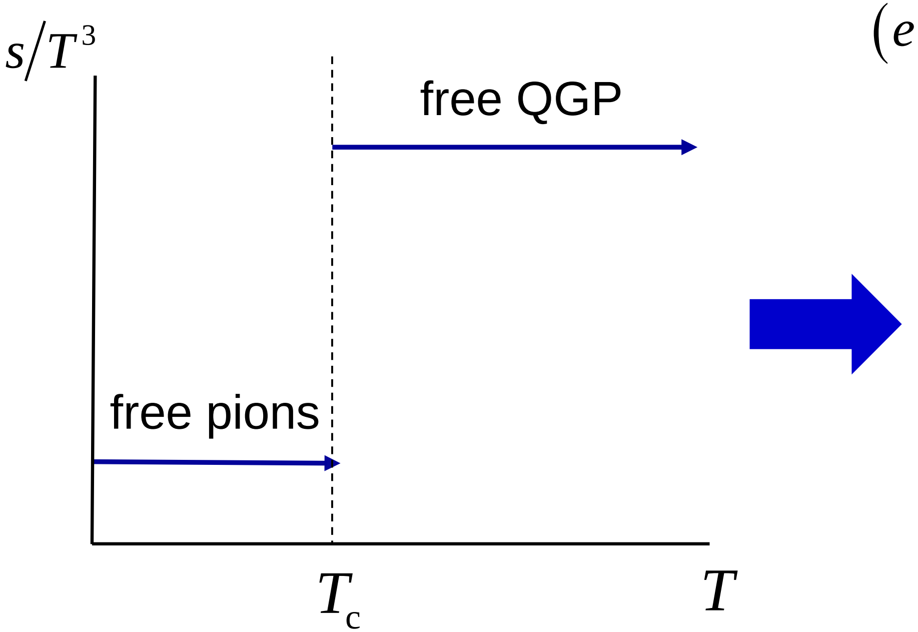
Then how does interaction measure behave?

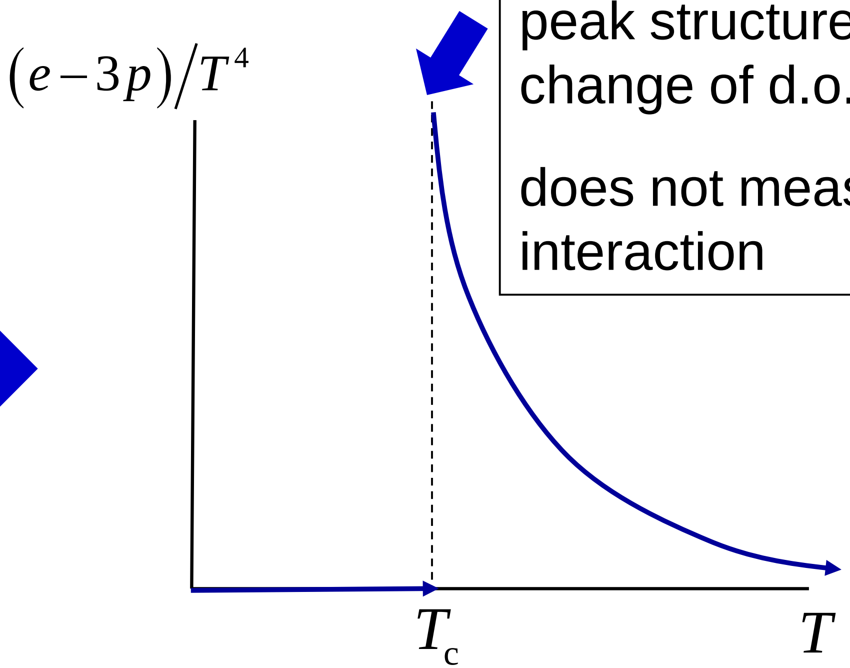
Thermodynamics

$$p(T) = \int_0^T s(t)dt + p_0$$ with p_0=0 (vacuum subtraction)

$$e(T) = Ts(T) - p(T)$$

all needed is entropy density (entropy monism)





peak structure:
change of d.o.f.

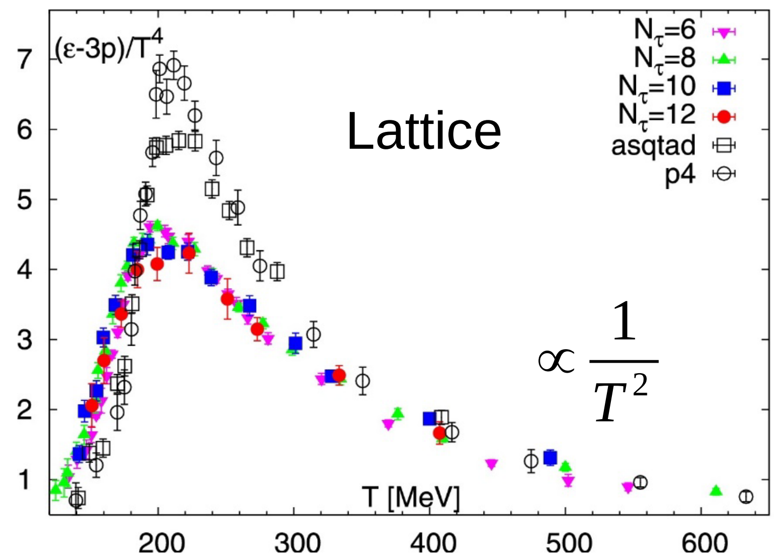
does not measure
interaction

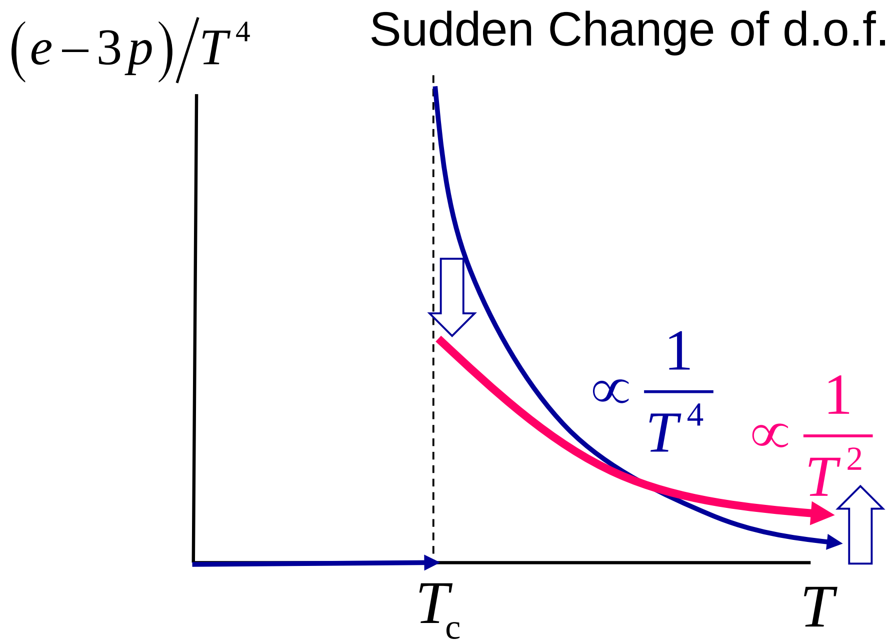
Hatsuda and M.A. (1997)

Furthermore, e-$3p$ is increasing
if $d(T)$ is increasing ($s(T)=d(T)T^3$)

Then, where is interaction?

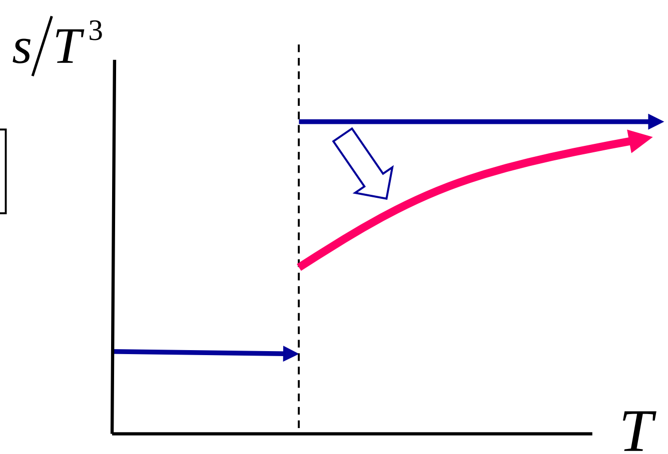
Slow Fall-off of "Interaction Measure"





Since all needed is $s(T)$, this can be explained by the behavior of $s(T)$

Slow rise of $d(T)=s(T)/T^3$



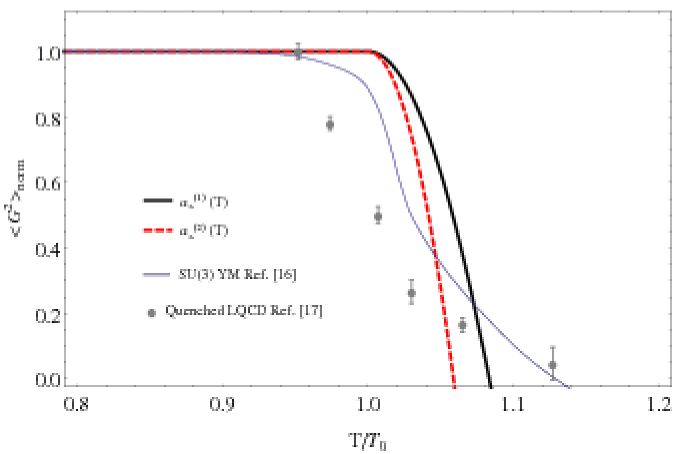
Trace Anomaly

- Trace Anomaly (up to fermion contribution)

$$T^{\mu}_{\mu} = \frac{\beta(g)}{2g} G^{a}_{\mu\nu} G^{\mu\nu,a}$$ identity

$$\frac{\alpha_s}{\pi} \left\langle G^{a}_{\mu\nu} G^{\mu\nu,a} \right\rangle_{T=0} \sim (360\mathrm{MeV})^4 \qquad \beta(g) < 0$$

$\left\langle G^{a}_{\mu\nu} G^{\mu\nu,a} \right\rangle_T$ decreases around the phase transition

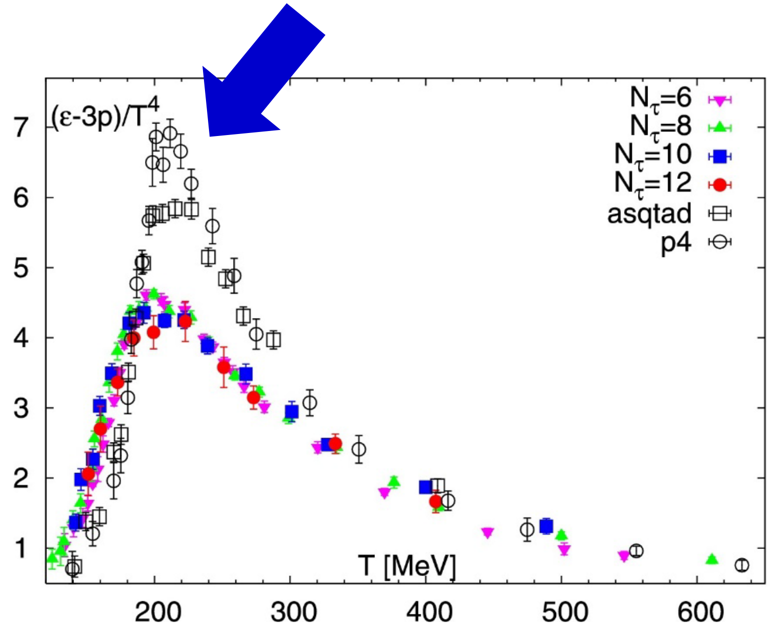


$$\left\langle T^{\mu}_{\mu} \right\rangle_T = \frac{\beta(g)}{2g} \left\langle G^{a}_{\mu\nu} G^{\mu\nu,a} \right\rangle_T$$

approaches zero
(and eventually becomes positive)

Trace Anomaly?

Although this quantity is also called "Trace Anomaly", this quantity is *not* trace anomaly without vacuum subtraction

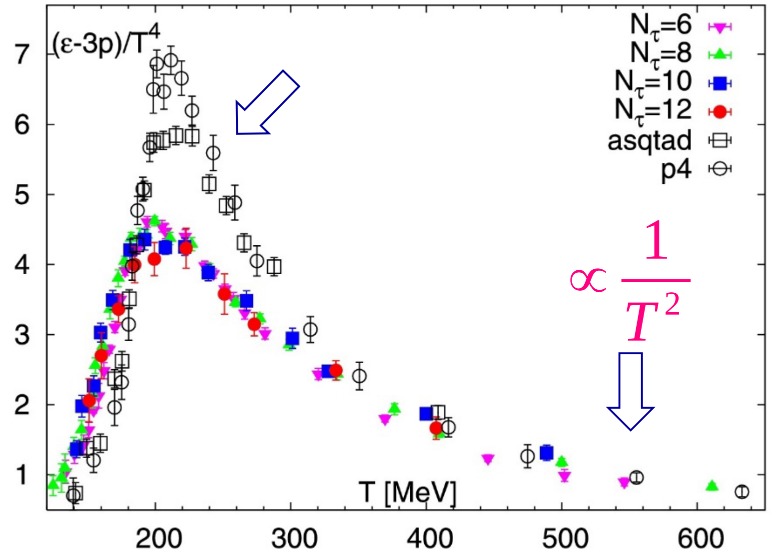


What we are seeing is

$$\frac{\left\langle T_\mu^\mu \right\rangle_T - \left\langle T_\mu^\mu \right\rangle_{T=0}}{T^4} = \frac{\beta(g)}{2gT^4}\left[\underbrace{\left\langle G_{\mu\nu}^a G^{\mu\nu,a} \right\rangle_T}_{\text{decreasing}} - \underbrace{\left\langle G_{\mu\nu}^a G^{\mu\nu,a} \right\rangle_{T=0}}_{>0} \right]$$

- This peak is due to "disappearance or decrease" of Trace Anomaly
- It is not appropriate to interpret this peak as appearance of "Trace Anomaly"

Although QGP is strongly interacting,



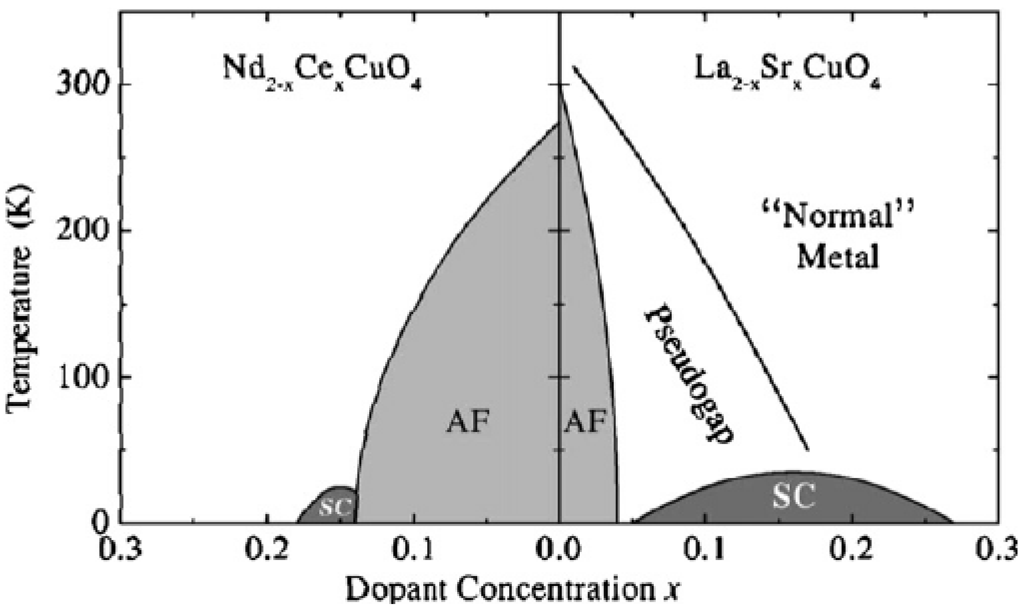
In conclusion, we cannot interpret this figure is showing that QGP around T_c is strongly interacting or anomalous (in the meaning of field theory)

Toward Microscopic Understanding

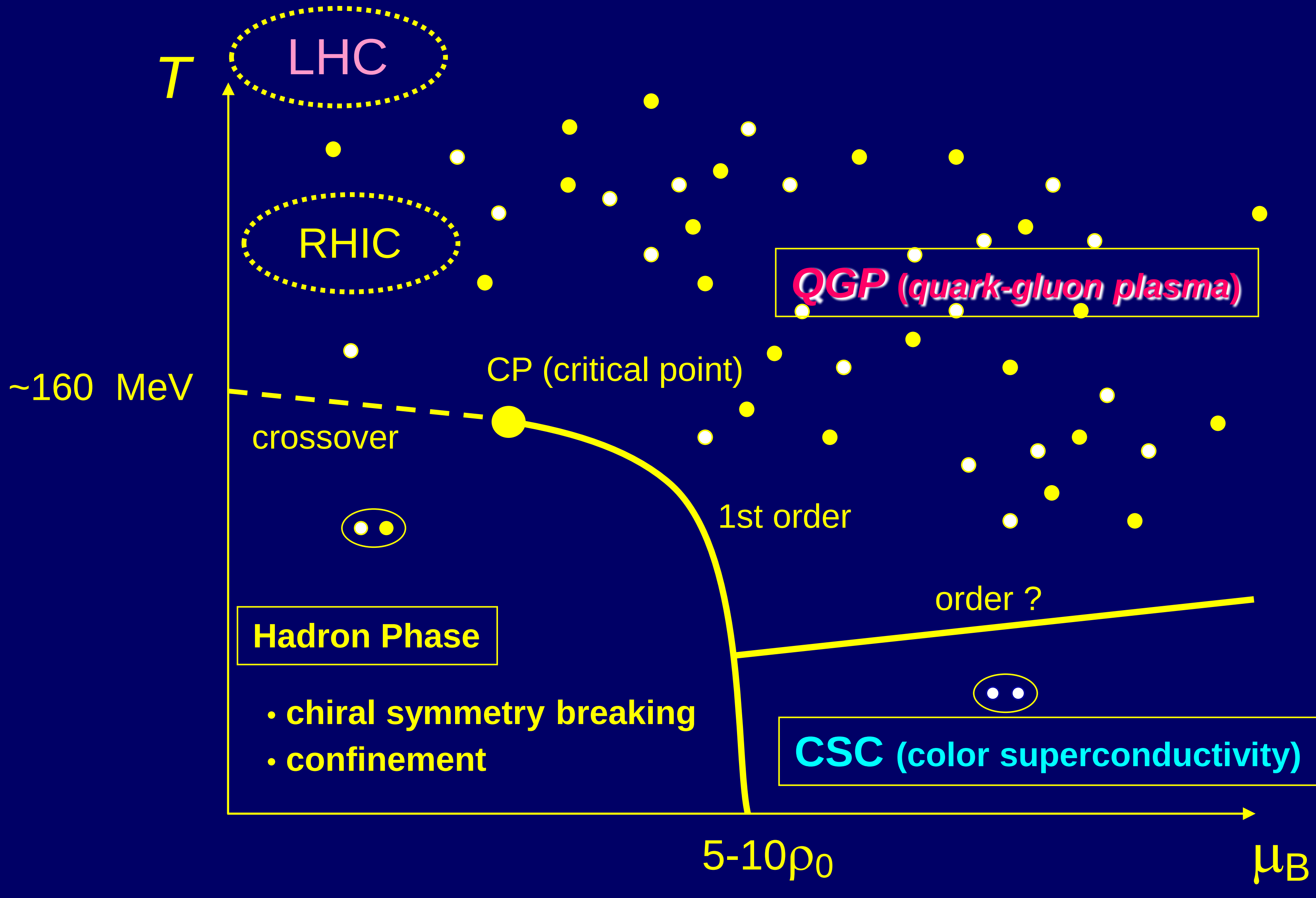
In Condensed Matter Physics

➢ 1st Macroscopic Properties

- bulk quantities: heat capacity, heat conductivity, transport coefficients, …
- phase structure



QCD Phase Diagram



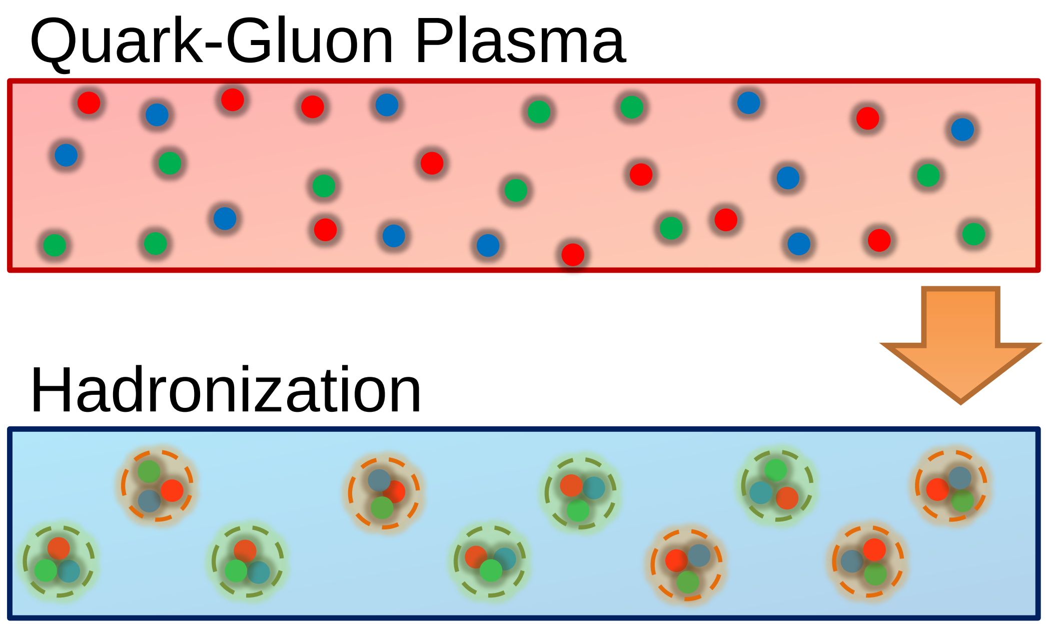
Conserved Charge Fluctuations

The original idea of conserved charge fluctuations

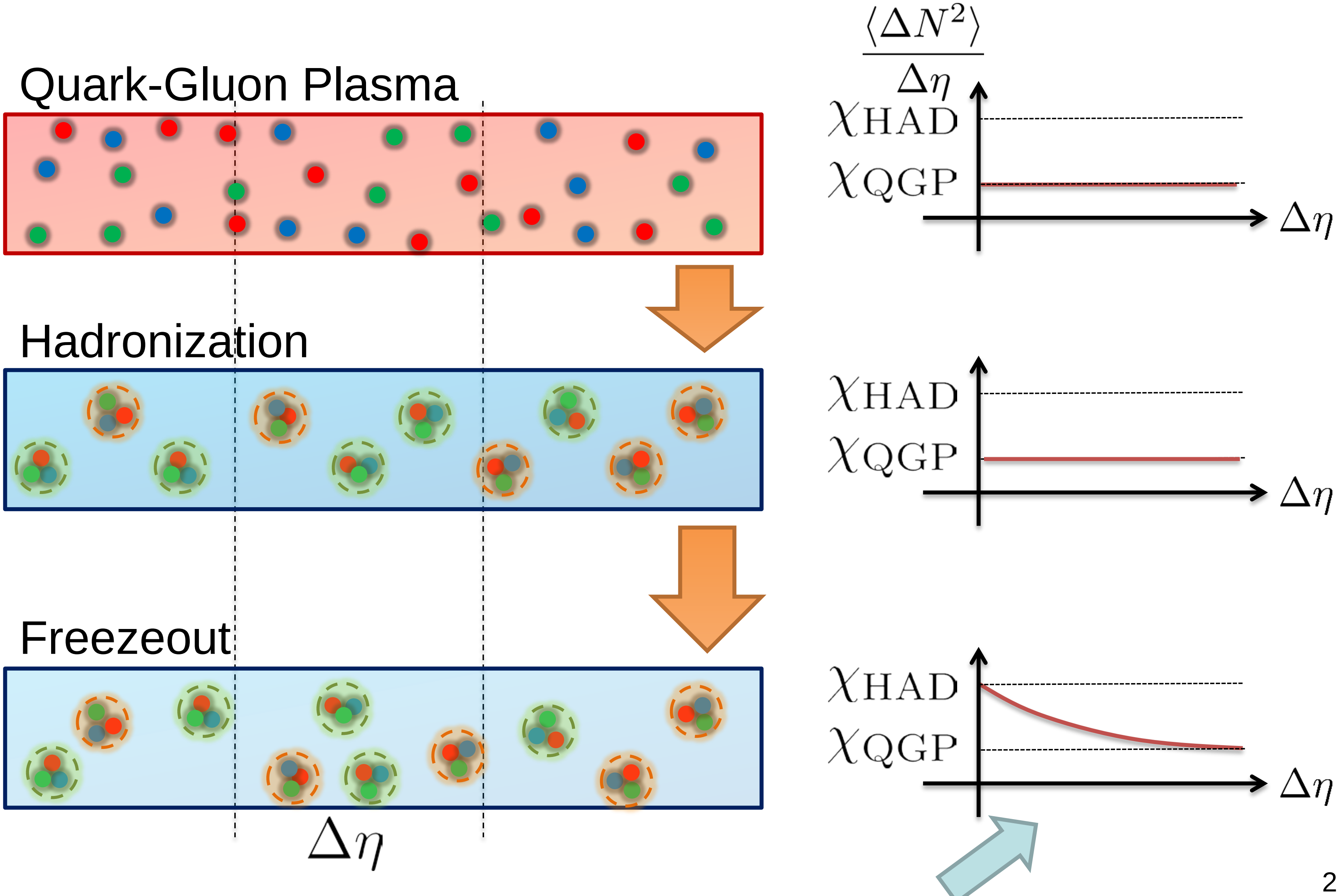
M.A, Heinz, Müller, Jeon, Koch (2000)

Conserved charge fluctuations change only through diffusion

→ Thus, in particular, they are not affected by phase transition



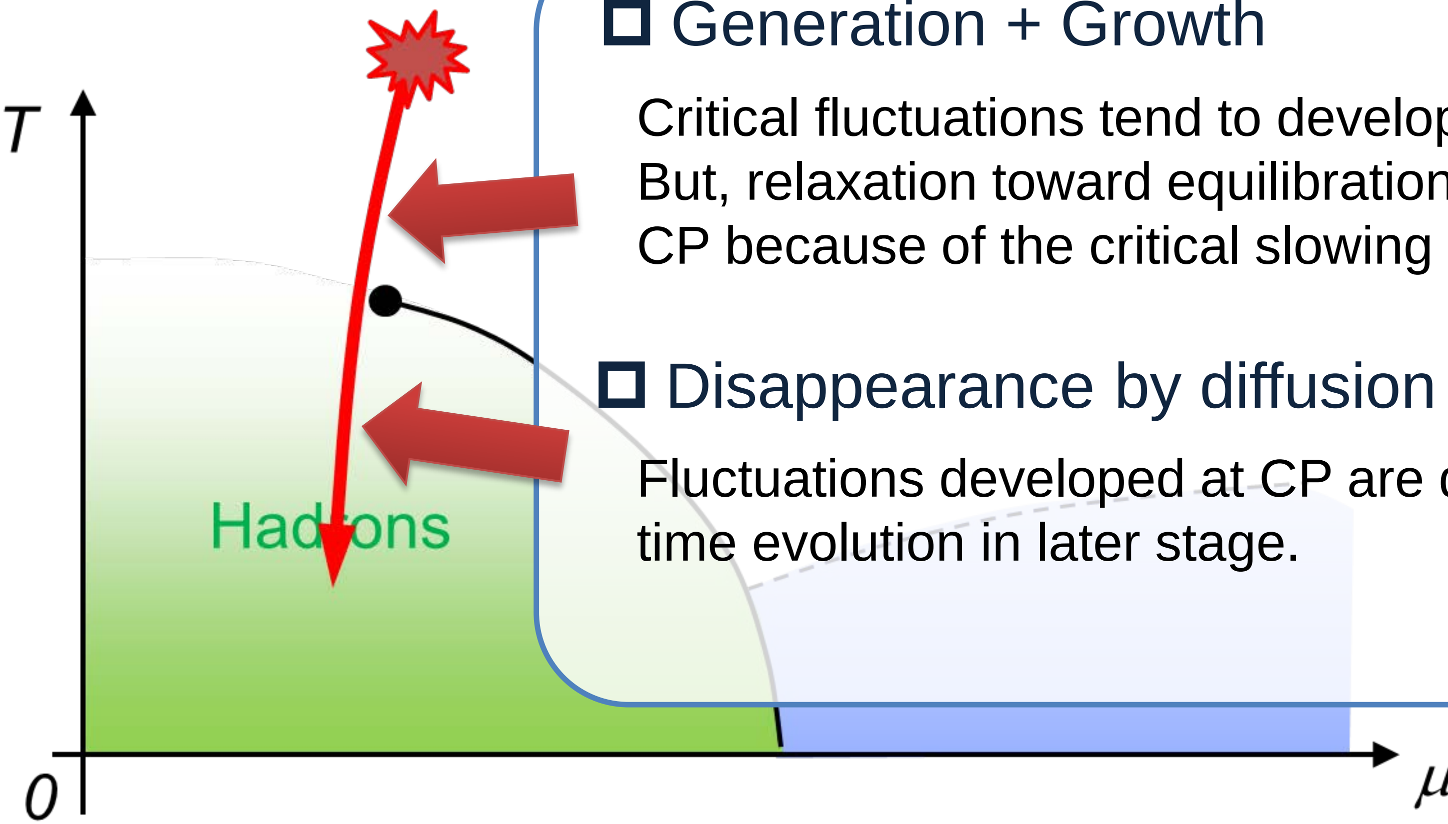
Time Evolution of C.C. fluctuation



In the $\Delta\eta$ dependence of C.C. Fluctuation, history of system is encoded

Critical Phenomena + Time Evolution

Experiments cannot observe critical fluctuation in equilibrium directly.



- **Generation + Growth**

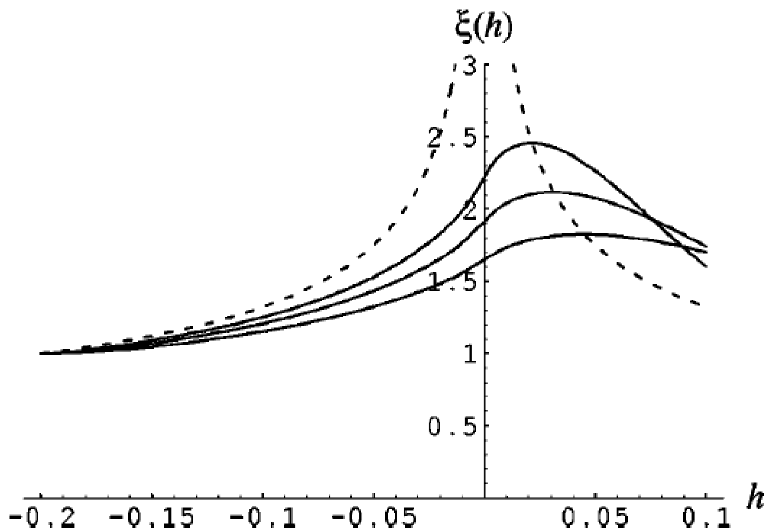
  Critical fluctuations tend to develop.
  But, relaxation toward equilibration is slow around CP because of the critical slowing down.

- **Disappearance by diffusion**

  Fluctuations developed at CP are diffused by the time evolution in later stage.

Correlation Length of Non-Conserved Quantity

Time evolution of correlation length
around CP with critical slowing down



~ distance from CP

Berdnikov, Rajagopal (2000)
Nonaka, M.A. (2004)

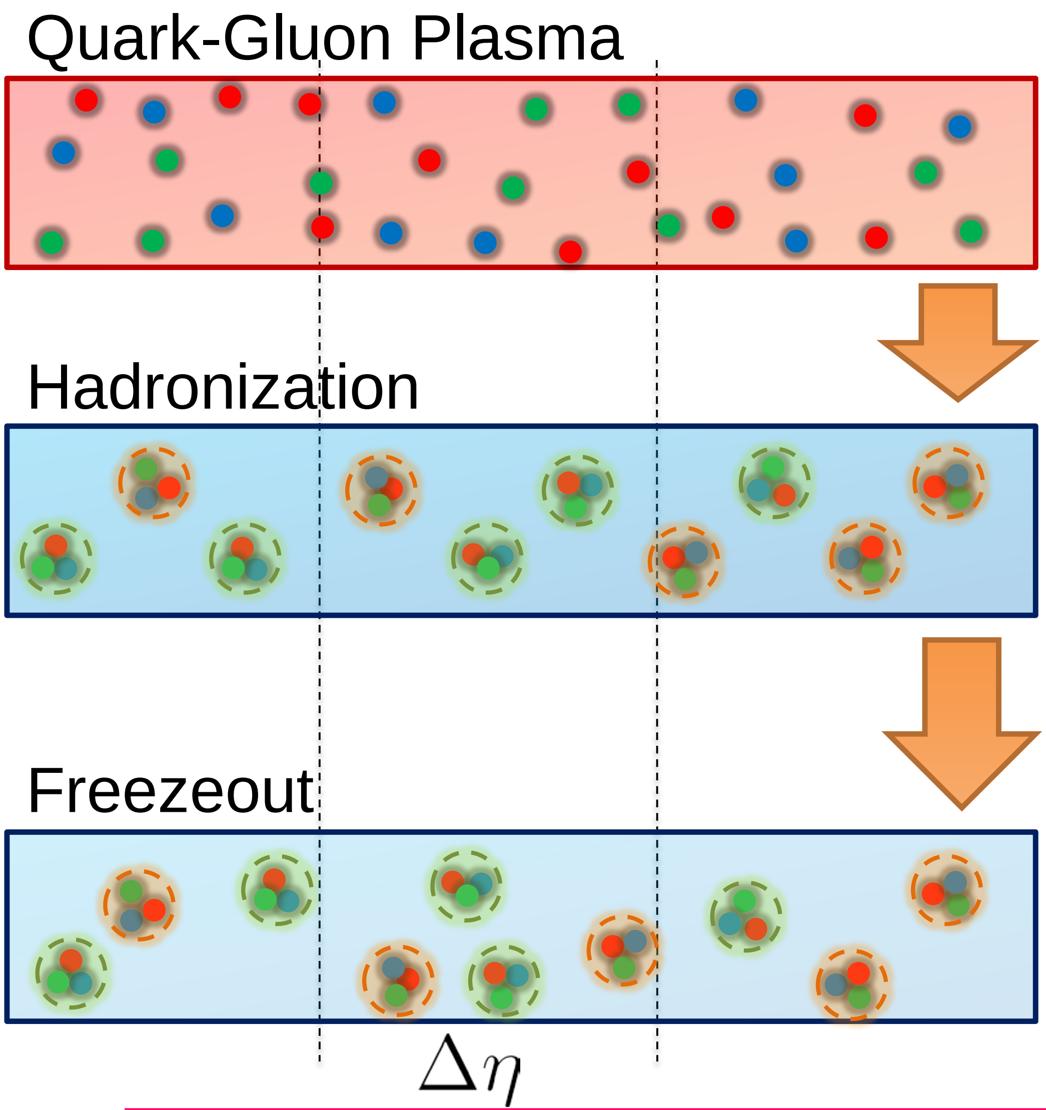
usual argument

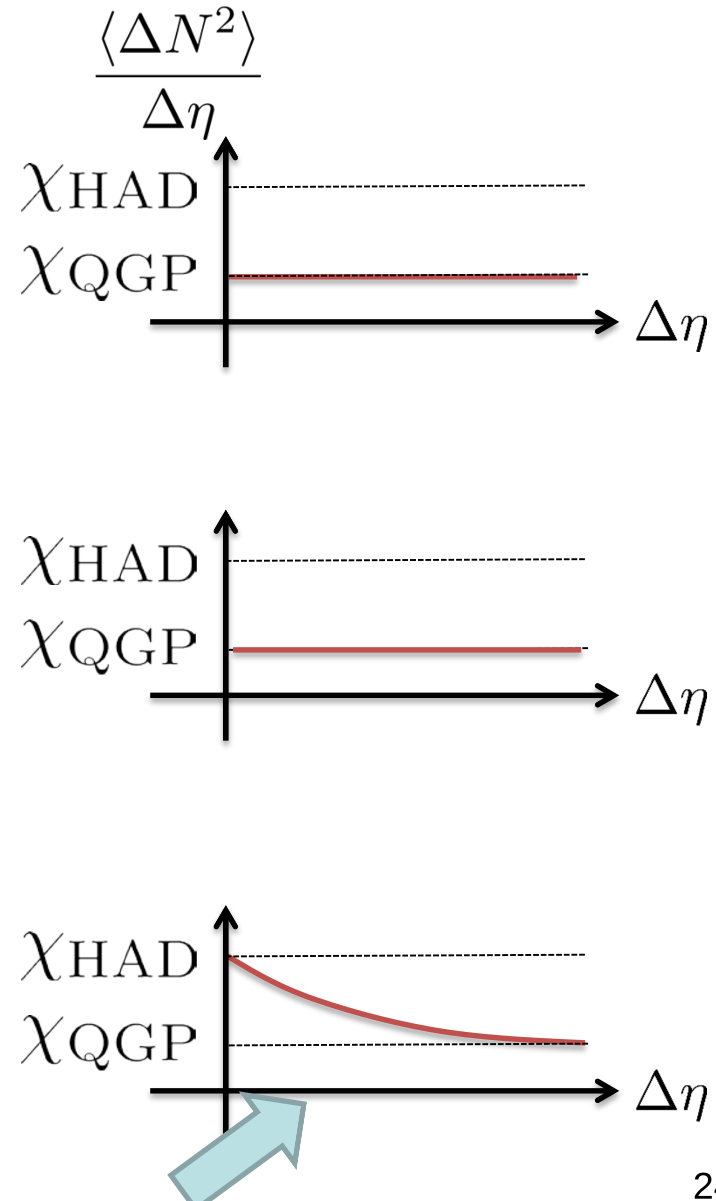
$K_2 \sim \xi^2, \quad K_3 \sim \xi^{4.5}, \quad K_4 \sim \xi^7$ ➡ higher order cumulants are advantageous

- This ξ is m_σ^{-1}, not a conserved quantity (not diffusive mode)
- Conserved charge cumulants change more slowly
- In HI collisions, ξ and conserved charge cumulants are *not synchronized*
- Furthermore, conserved charge cumulants are *scale dependent*

Time Evolution of C.C. fluctuation

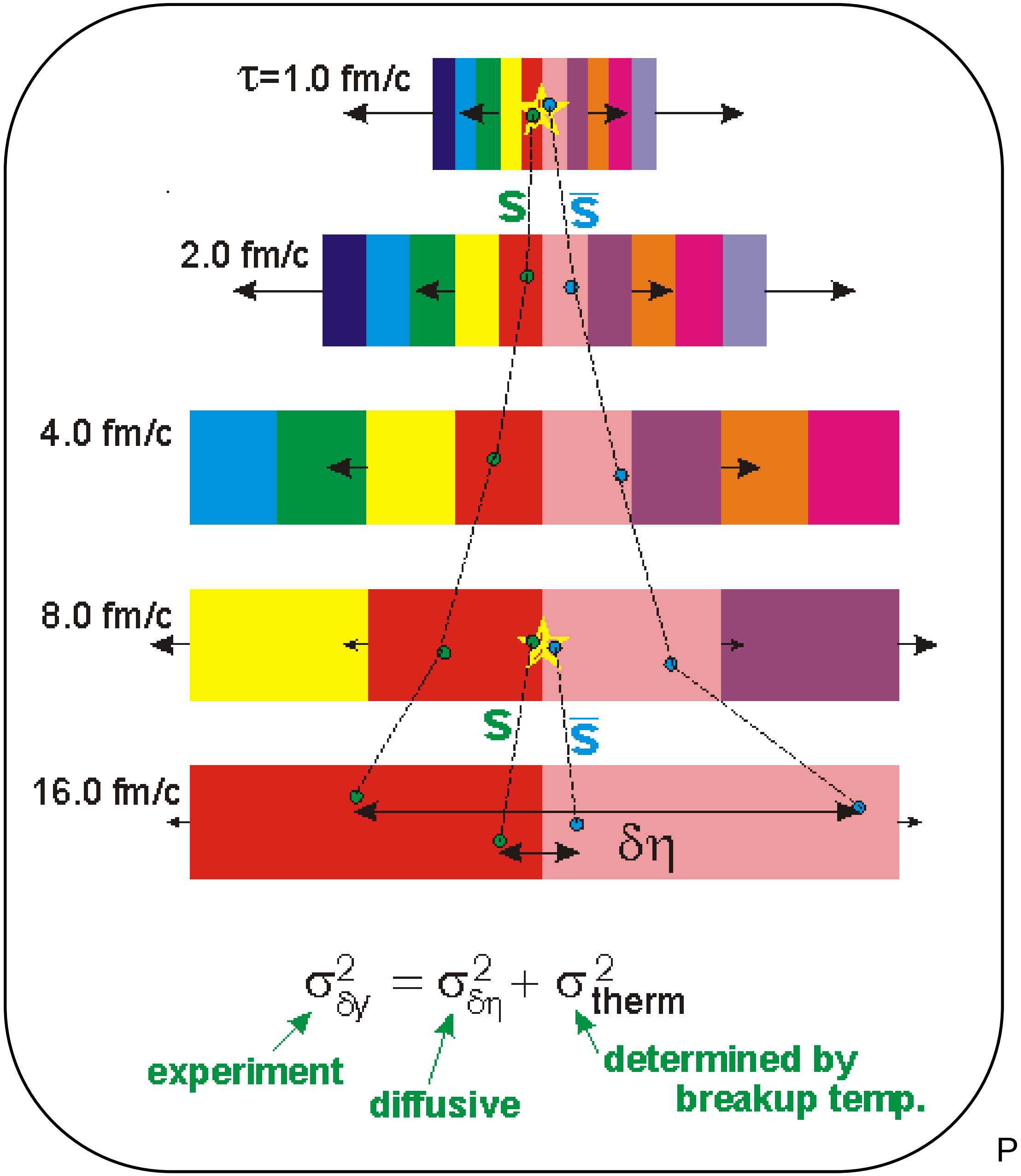
Conserved charge cumulants are *scale dependent*





In the $\Delta\eta$ dependence of C.C. Fluctuation, history of system is encoded

Similarity with Balance Function



Information at Larger Δy
= Earlier Stage

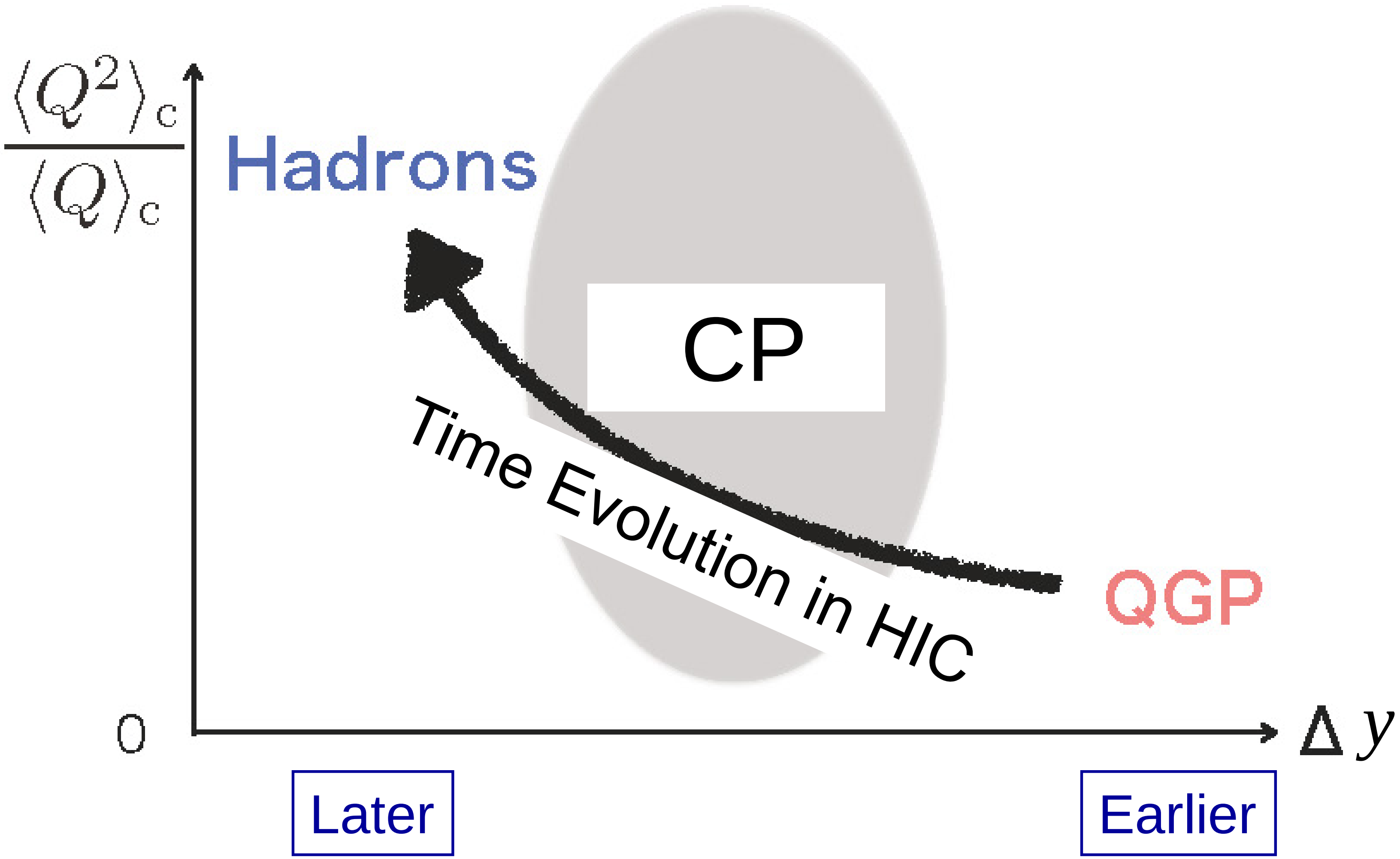
Local Charge Conservation

Diffusion

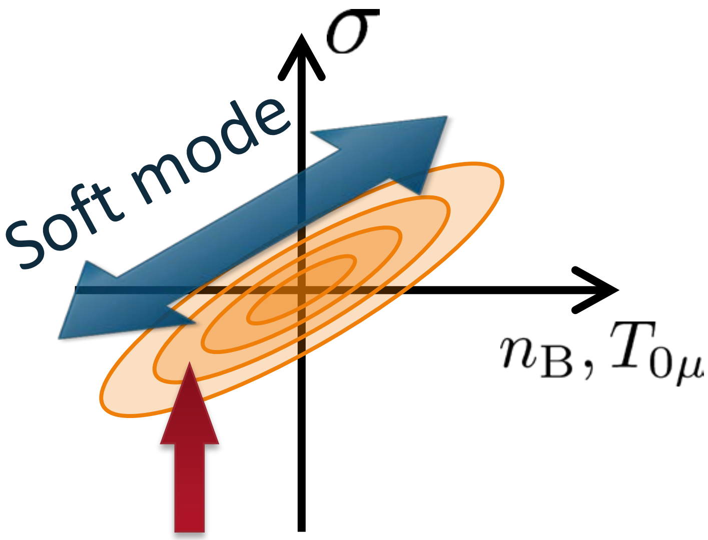
Information at Smaller Δy
= Later Stage

Pratt, QM2002

Critical Fluctuation and Δy Dependence



Critical Phenomena and Diffusive Mode



σ: fast damping

Effective Potential

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \cdots$$

Soft mode of QCD CP

$$\begin{pmatrix} \dot{\sigma} \\ \dot{n} \end{pmatrix} = \begin{pmatrix} \Gamma_{\sigma\sigma} & \Gamma_{\sigma n} \\ \Gamma_{n\sigma} & \Gamma_{nn} \end{pmatrix} \begin{pmatrix} \frac{\delta F}{\delta \sigma} \\ \frac{\delta F}{\delta n} \end{pmatrix} + \begin{pmatrix} \xi_\sigma \\ \xi_n \end{pmatrix}$$

cf. Onsager relation $\sim k^2$

Evolution of baryon number density (slow and small k)

$$\frac{\partial}{\partial \tau}\delta n(\eta,\tau) = D_\eta(\tau)\frac{\partial^2}{\partial \eta^2}\delta n(\eta,\tau) + \frac{\partial}{\partial y}\xi(\eta,\tau)$$

$$\langle \xi(y_1,\tau_1)\xi(y_2,\tau_2)\rangle = 2\chi_\eta(\tau_1)D_\eta(\tau_1)\delta(y_1-y_2)\delta(\tau_1-\tau_2)$$

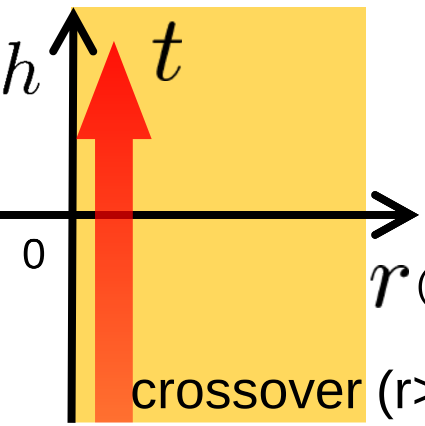
$D_Y(\tau)$, $\chi_\eta(\tau)$:parameters characterizing criticality

$D_\eta(\tau) = D_C(\tau)/\tau^2$, $\chi_\eta(\tau) = \tau\chi_C(\tau)$

Parametrizing D and χ: critical + regular

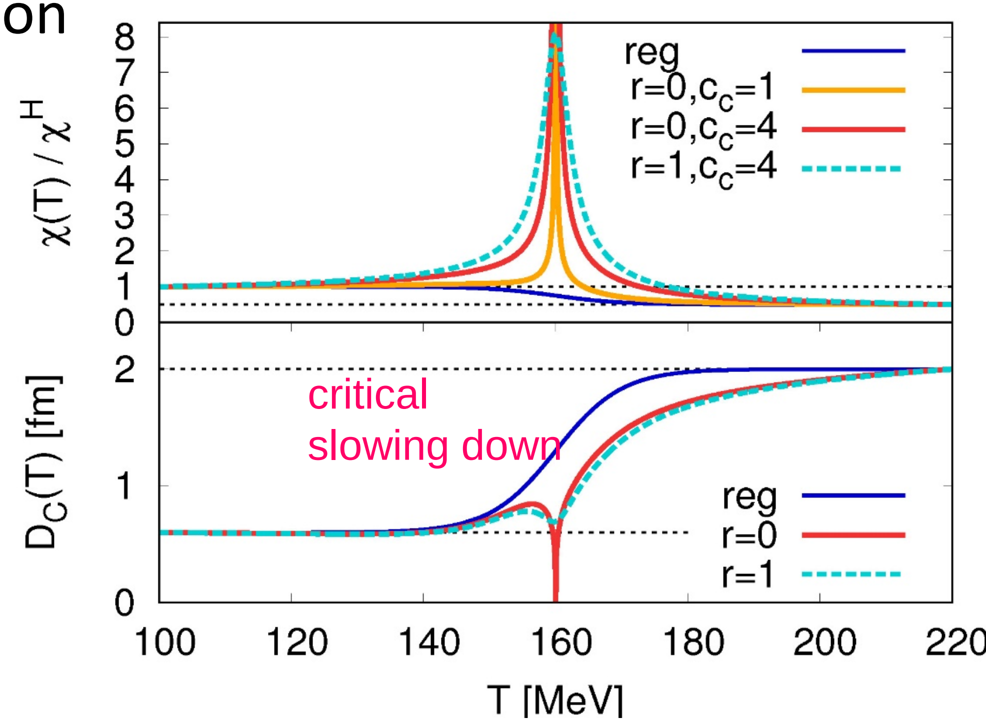
Berdnikov, Rajagopal (2000)
Nonaka, M.A. (2004)
Stephanov (2011)
Mukherjee, Venugopalan, Yin (2015)

- model-H (3d-Ising)
- $\chi \sim \xi^{1.96}, \ D \sim \xi^{-1.044}$
- mapping to (T,μ) / time evolution



1D Bjorken expansion

$$c_s^2 = 0.2$$



- $\chi_{\rm QGP} / \chi_{\rm hadron} = 0.5$
- QCD CP at T=160MeV
- kinetic f.o. at T=100MeV

Cumulant and Correlation Function

$$Q = \int_V dx\, n(x)$$

total charge

charge density

$$\langle \delta Q^2 \rangle = \int_V dx dy \langle \delta n(x) \delta n(y) \rangle$$

2nd order cumulant (fluctuation)

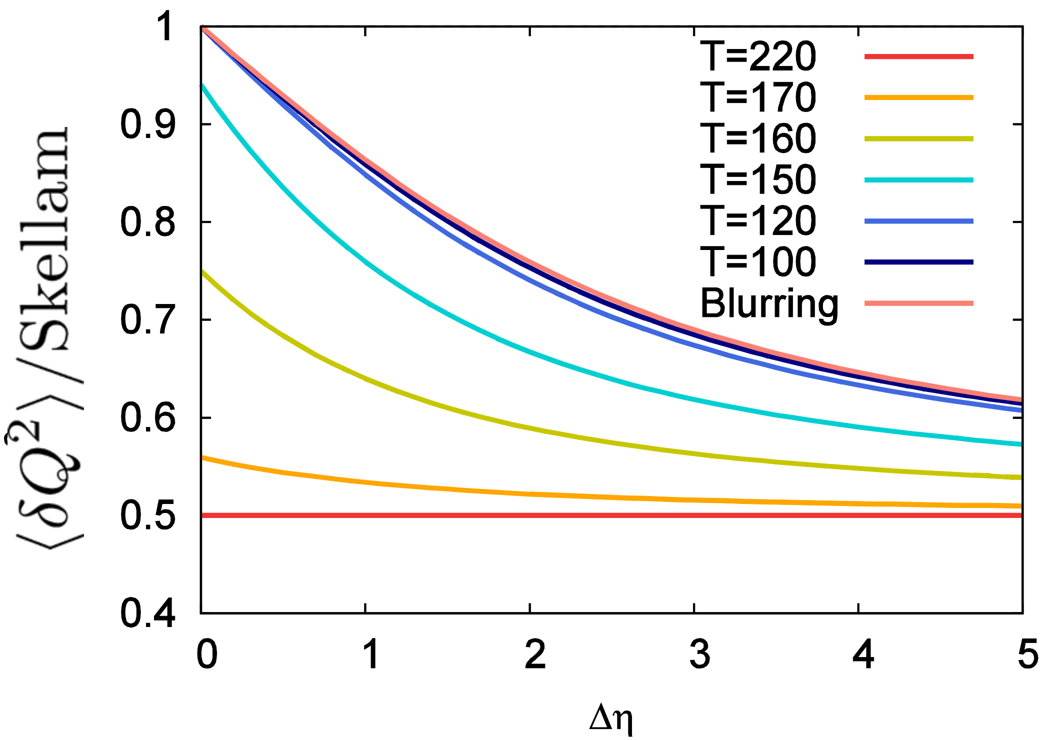
correlation function

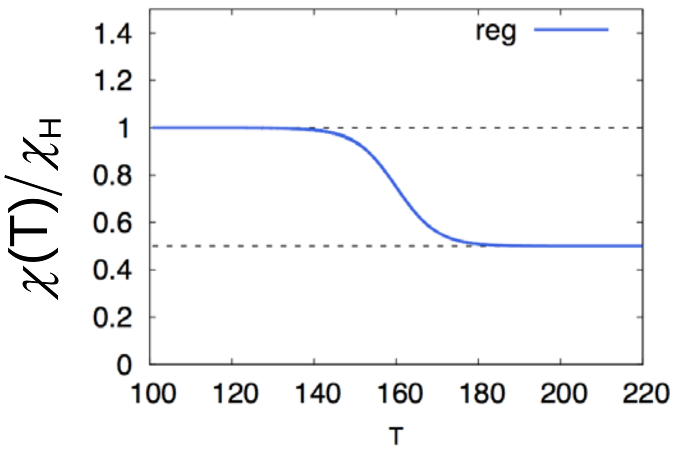
1-to-1 correspondence

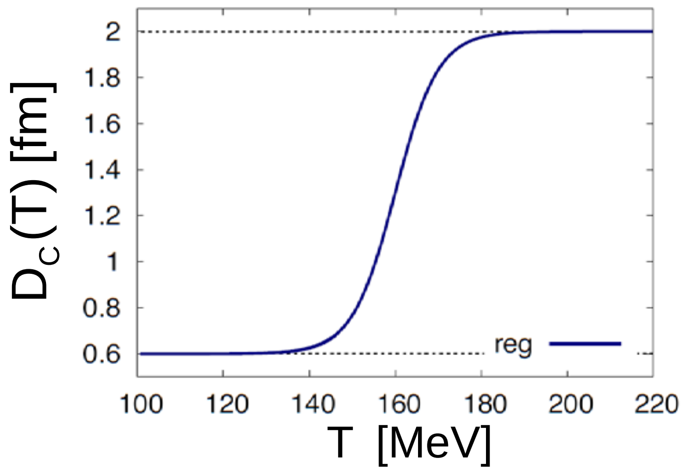
1-dim case

$$\langle \delta Q^2 \rangle_{\Delta y} = \int_{\Delta y} dy (\Delta y - |y|) \langle \delta n(y) \delta n(0) \rangle$$

Time Evolution 1, Fluctuation: No CP



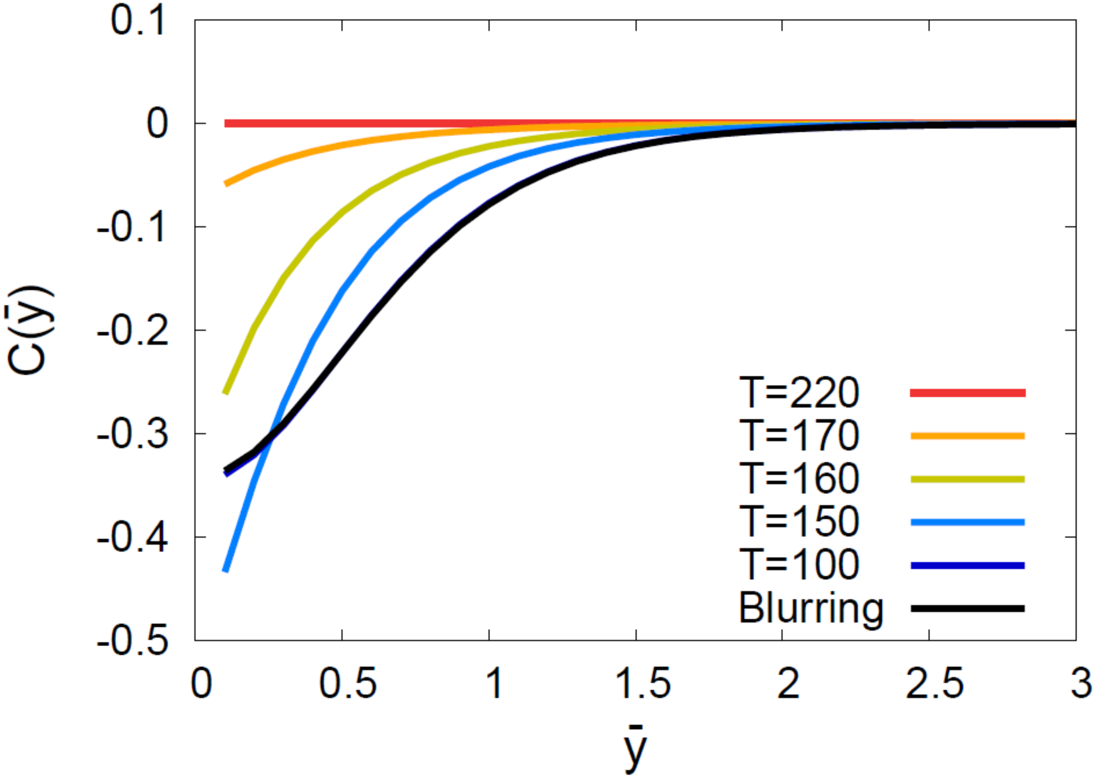


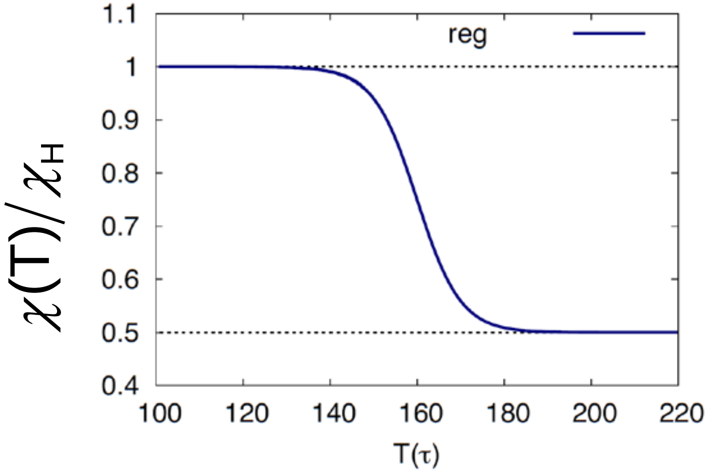


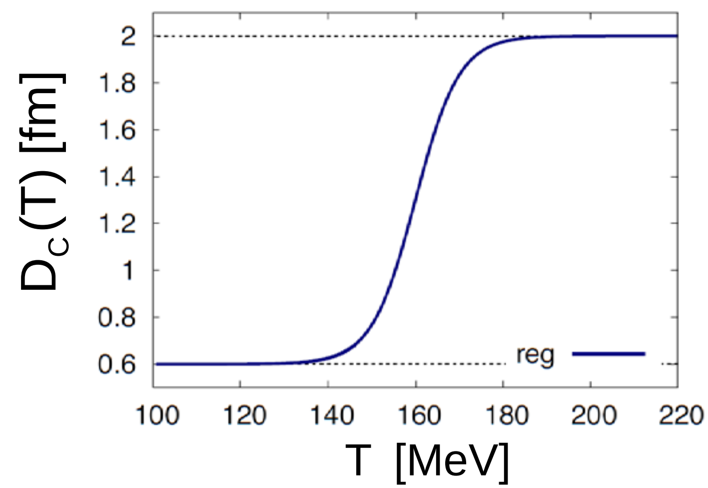
Sakaida, Fujii, Kitazawa, M.A. (2017)

Time Evolution 1, Correlation: No CP

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\rm hadron}$$



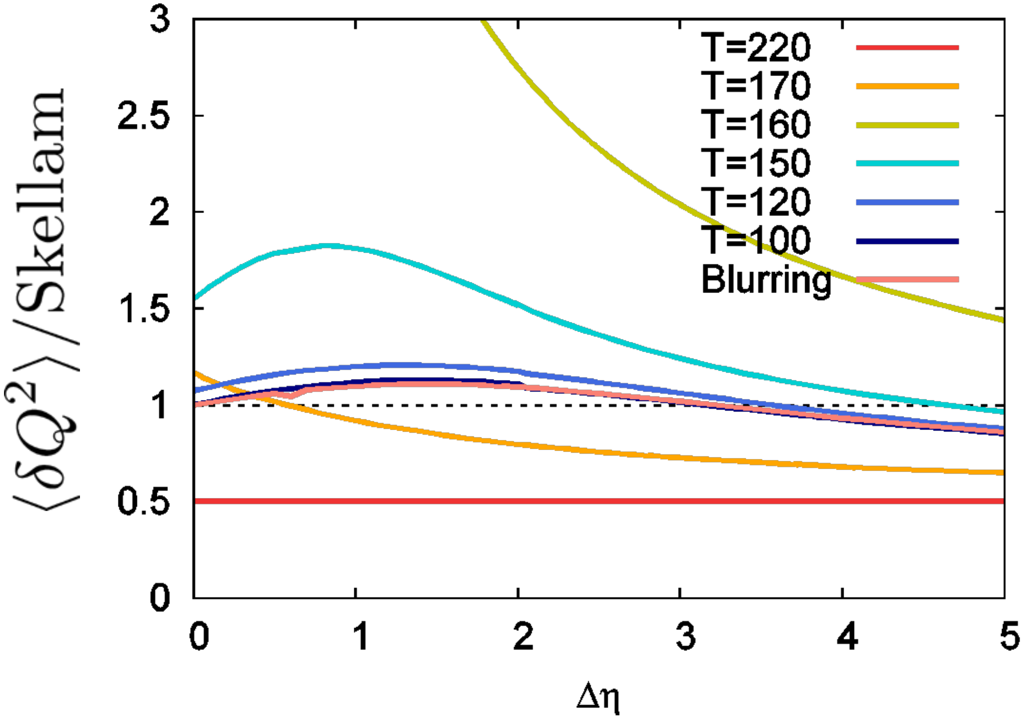


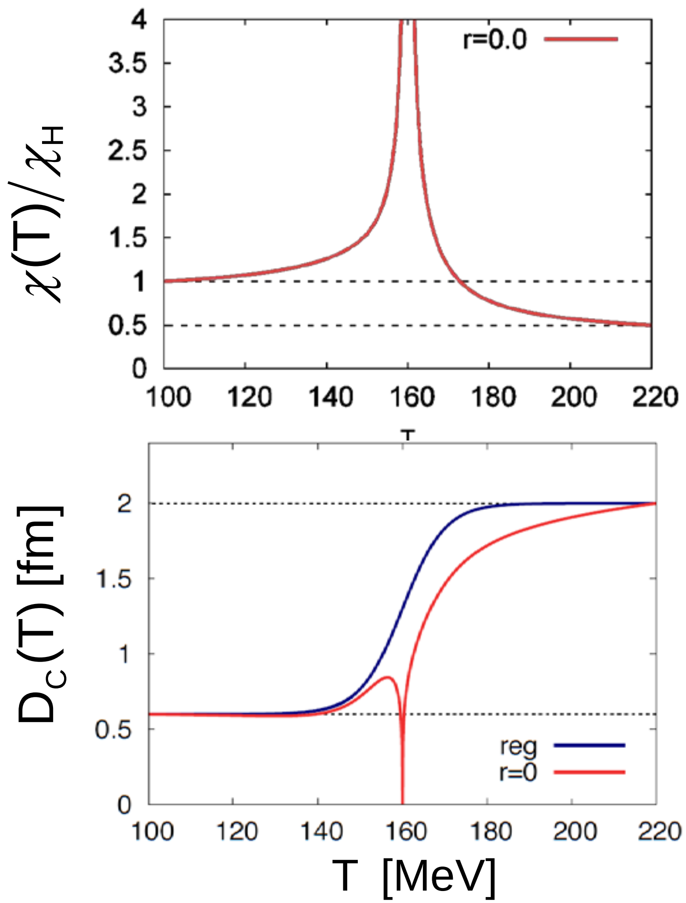


monotonically decreasing

Analytic result: $\chi(\tau)$ monotonically increasing ⇒ $C(\bar{y})$ monotonically increasing

Time Evolution 2, Fluctuation: With CP

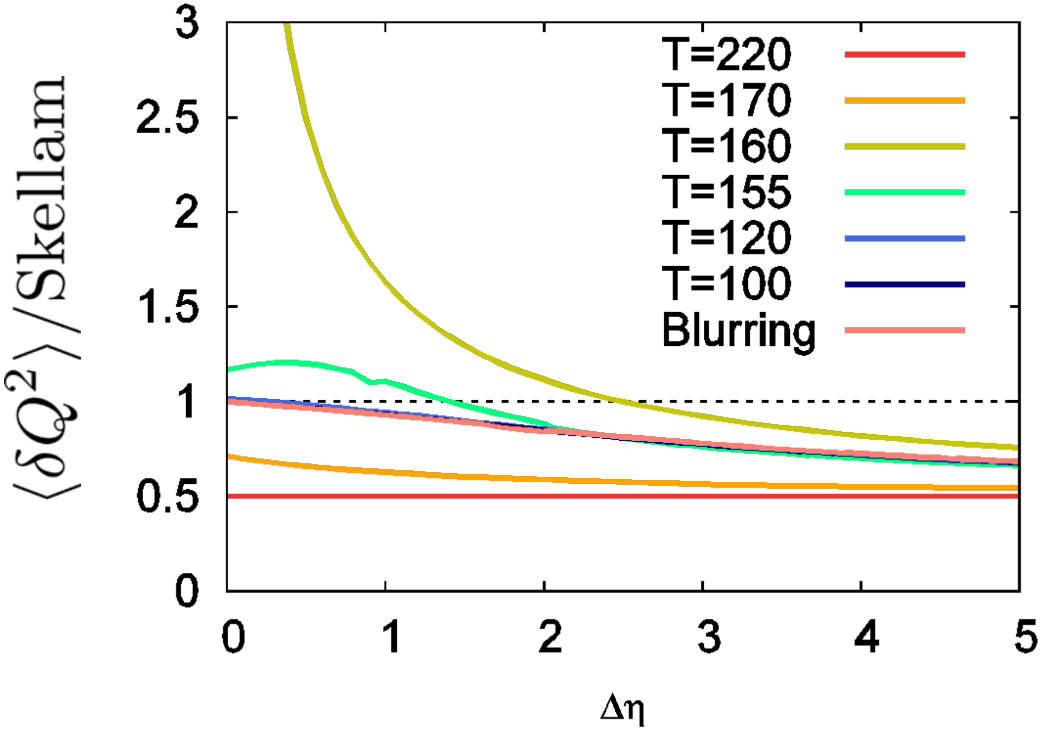


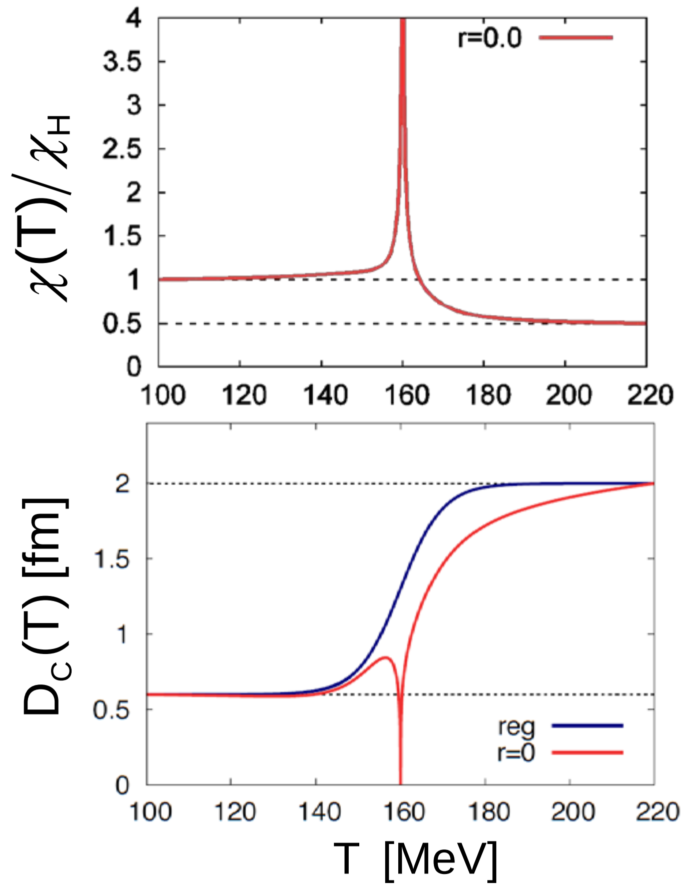


- Non-monotonic $\Delta\eta$ dependence manifests itself

  ➡ Robust experimental evidence of the existence of a peak in χ(T)

With Narrower Critical Region



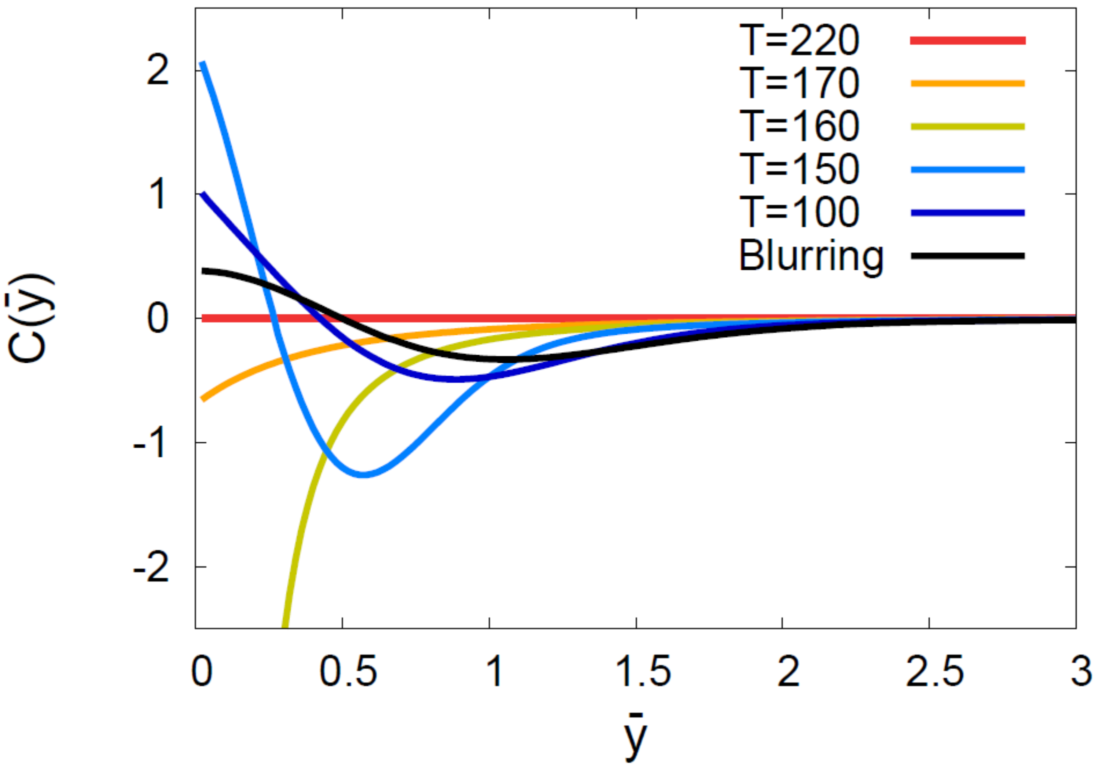


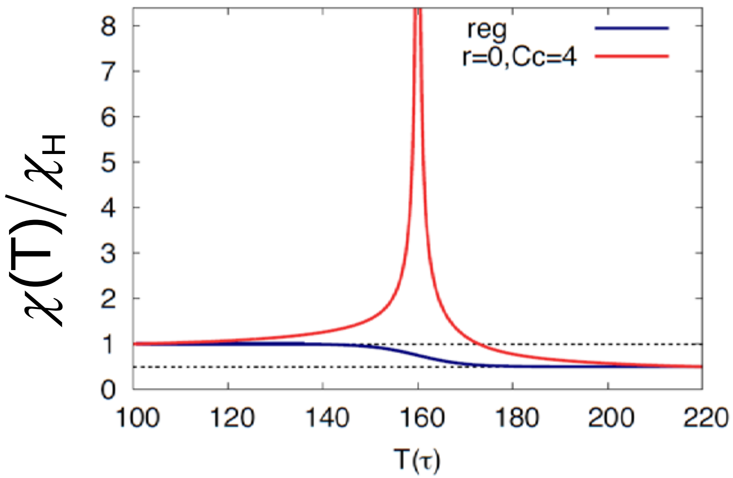
non-monotonic behavior

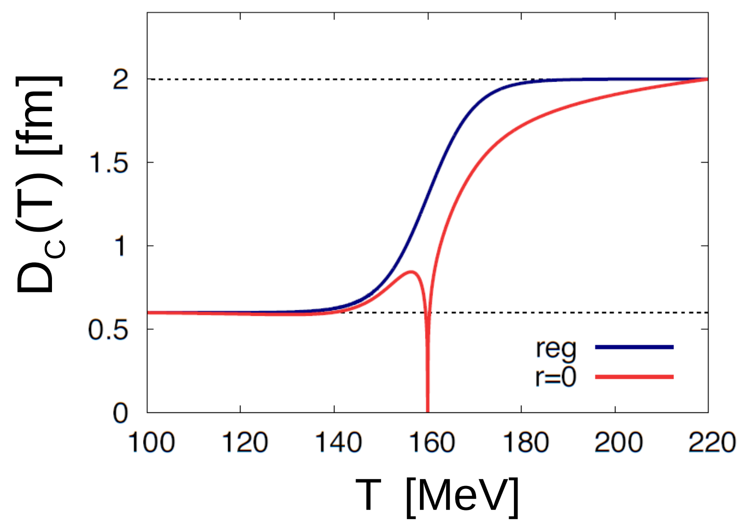
Peak in $\chi_2(T)$

Time Evolution 2, Correlation: With CP

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\mathrm{hadron}}$$







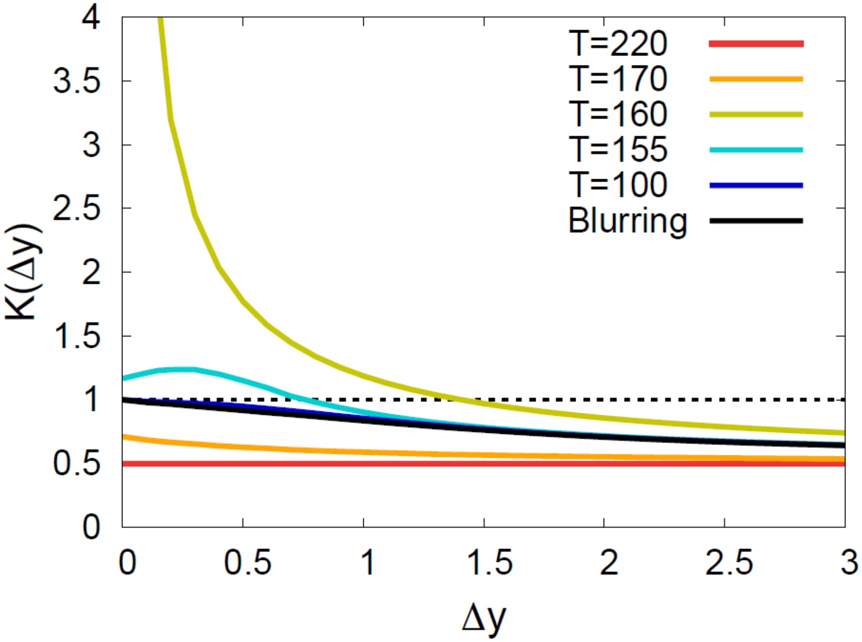
non-monotonic Δy dep.

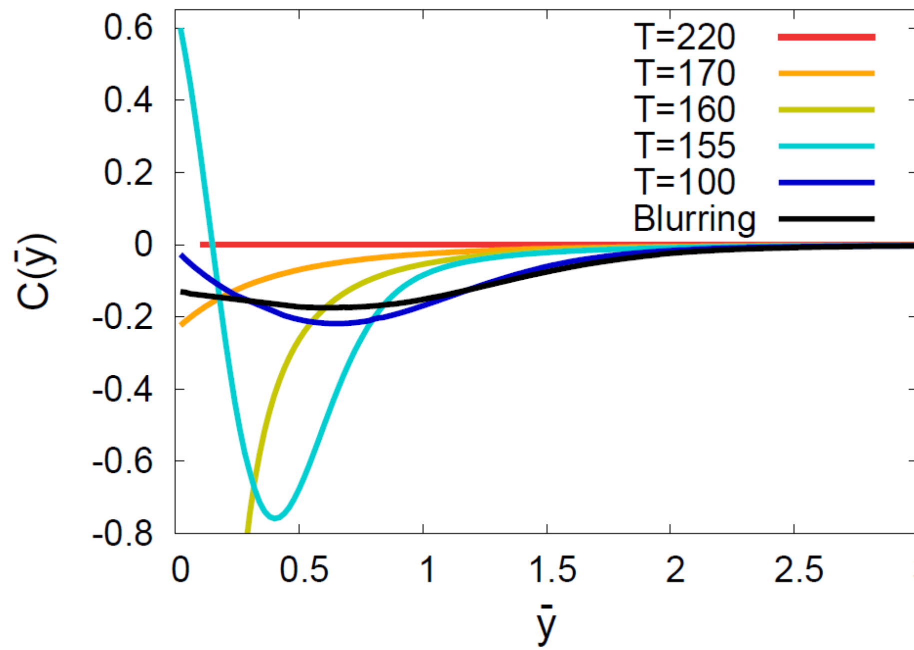
Analytic result

$C(\Delta y)$ non-monotonic → $\chi(\tau)$ non-monotonic

Comparison: Fluctuation and Correlation

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\mathrm{eq.}} \quad C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\mathrm{hadron}}$$





- Non-monotonicity in $K(\Delta y)$ disappears
- But $C(y)$ is still non-monotonic

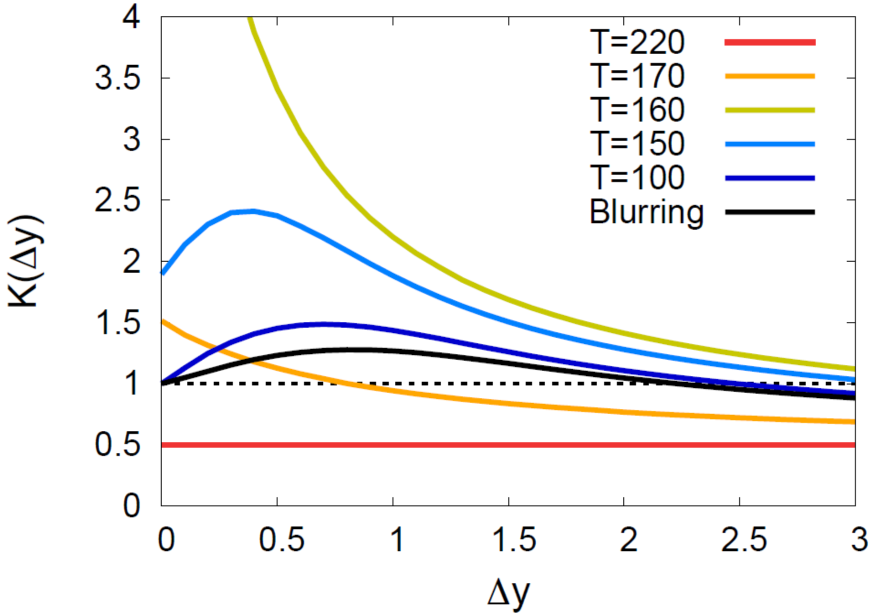
Analytic result

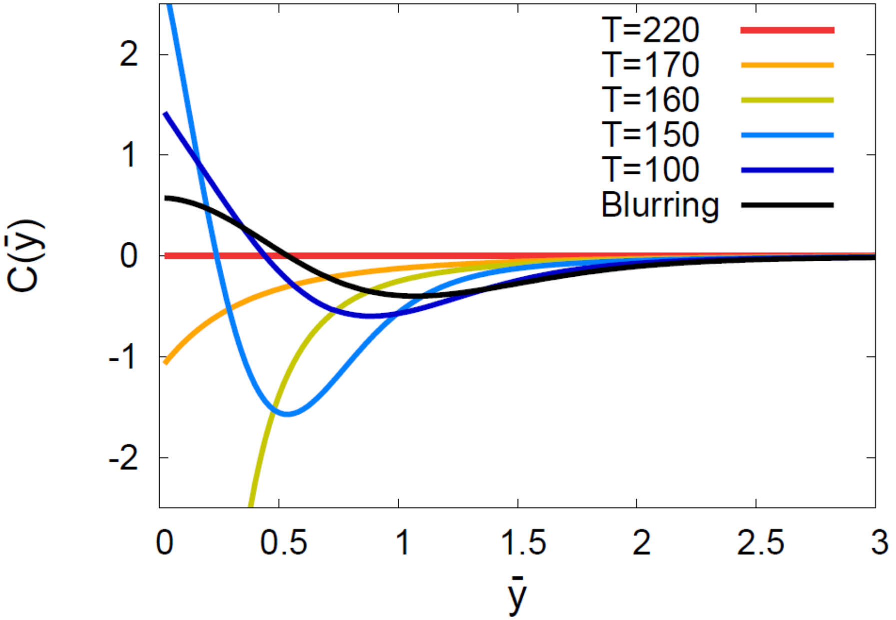
$K(\Delta y),\ C(\bar{y})$ monotonic ⇒ no information on $\chi(\tau)$

$C(\bar{y})$ is better to see non-monotonicity

Away from CP (Crossover)

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\rm eq.} \qquad C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\rm hadron}$$

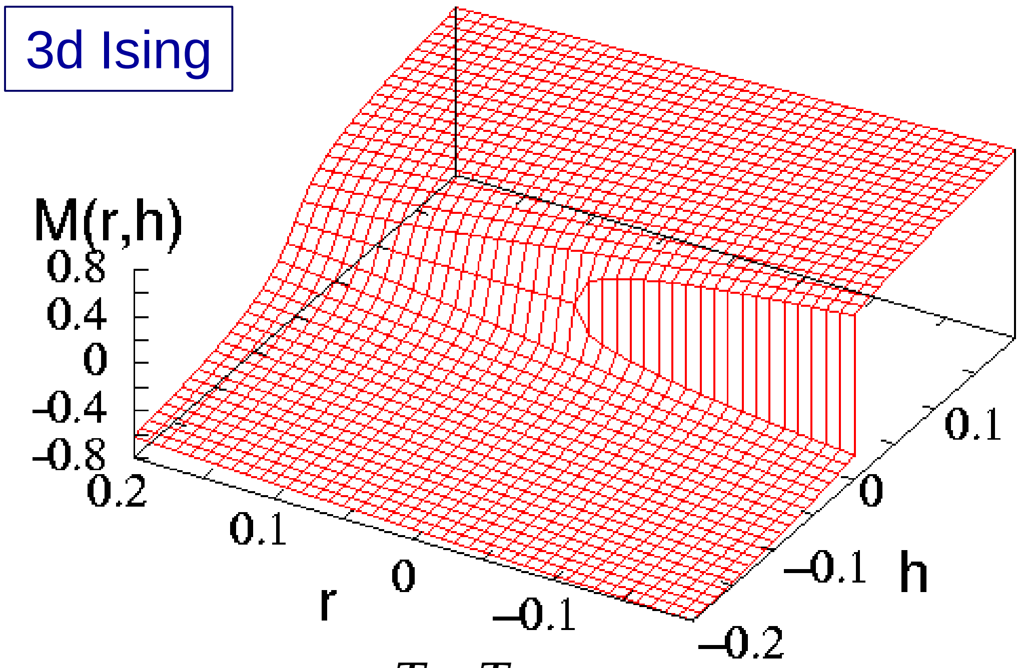




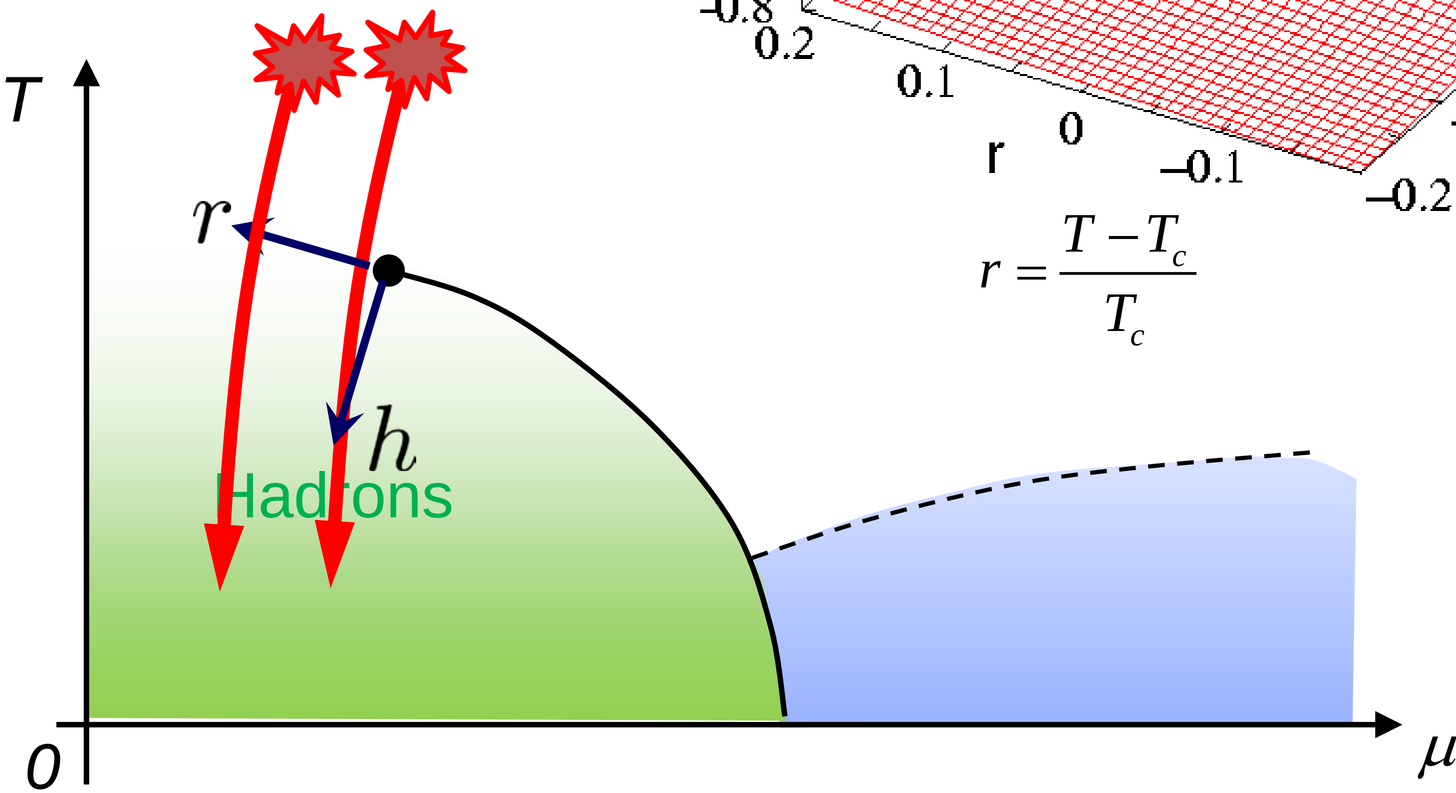
- Signal of the critical enhancement can be clearer along paths away from the CP

Away from the CP: Weaker critical slowing down

Mapping from 3-d Ising to QCD

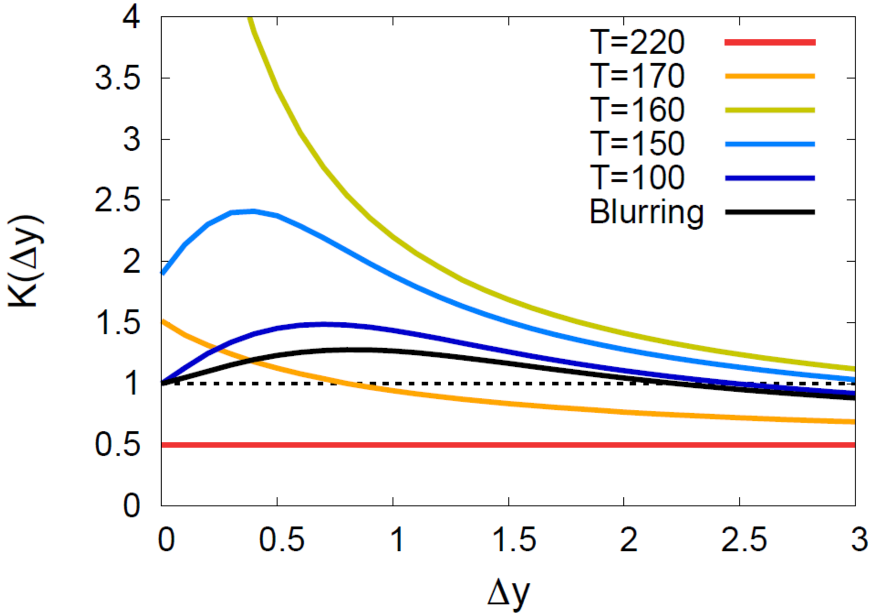


$$r = \frac{T - T_c}{T_c}$$

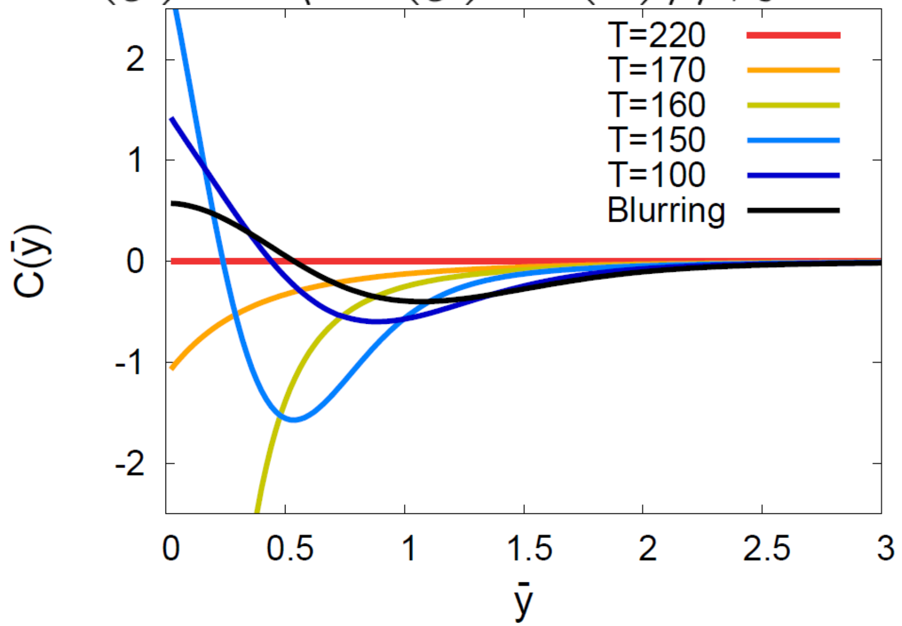


Away from CP (Crossover)

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$



$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$



- Signal of the critical enhancement can be clearer along paths away from the CP

Away from the CP: Weaker critical slowing down

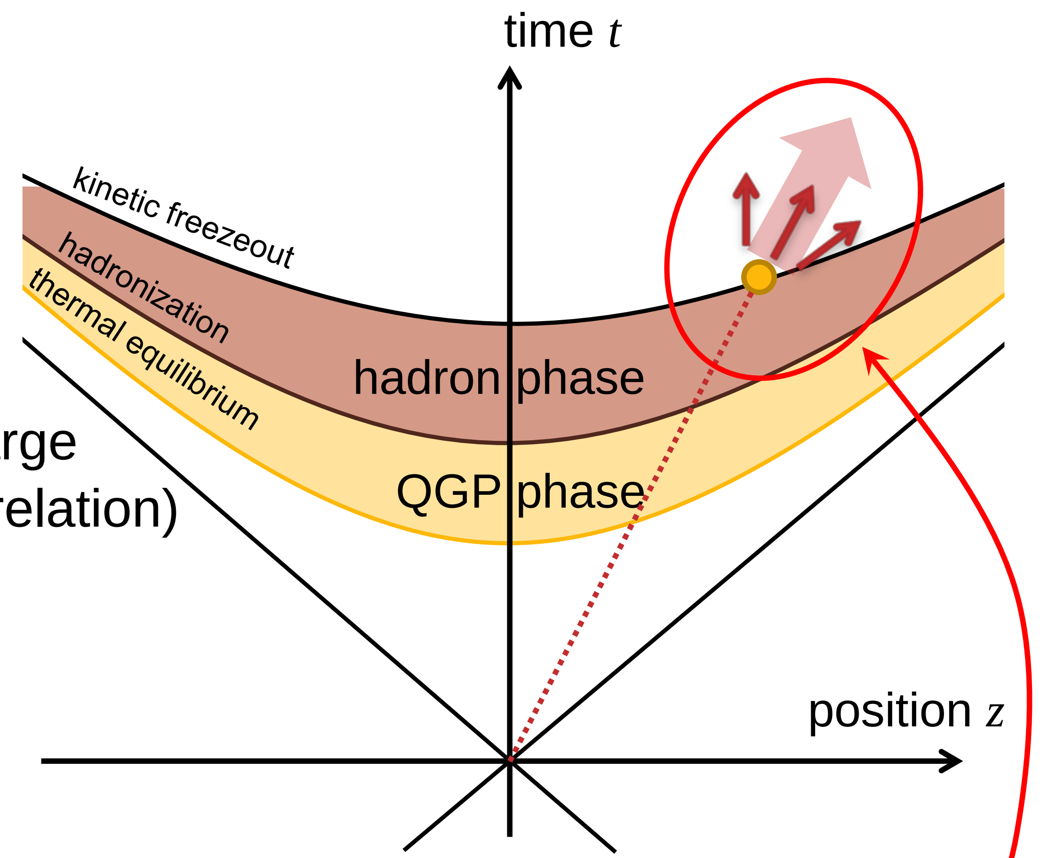
Blurring: loss of y-η correspondence

y: (momentum space) rapidity, η: space-time rapidity

$y = \eta$ in Bjorken picture

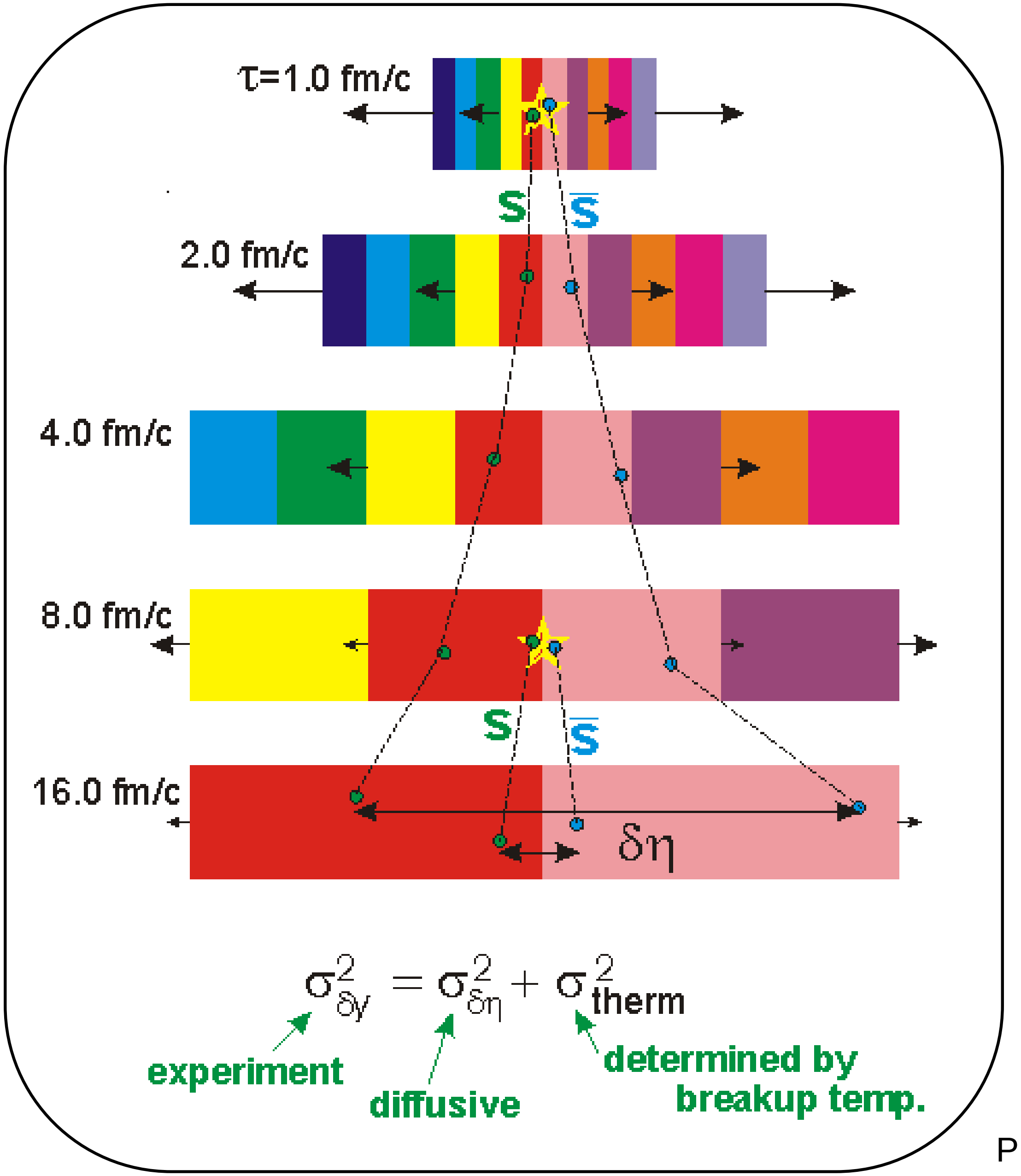
is blurred in one particle distribution owing to thermal motion

Accordingly, conserved charge fluctuation (two particle correlation) is modified



Blurring in rapidity space takes place !

Similarity with Balance Function



Information at Larger Δy
= Earlier Stage

Local Charge Conservation

Diffusion

Information at Smaller Δy
= Later Stage

Pratt, QM2002

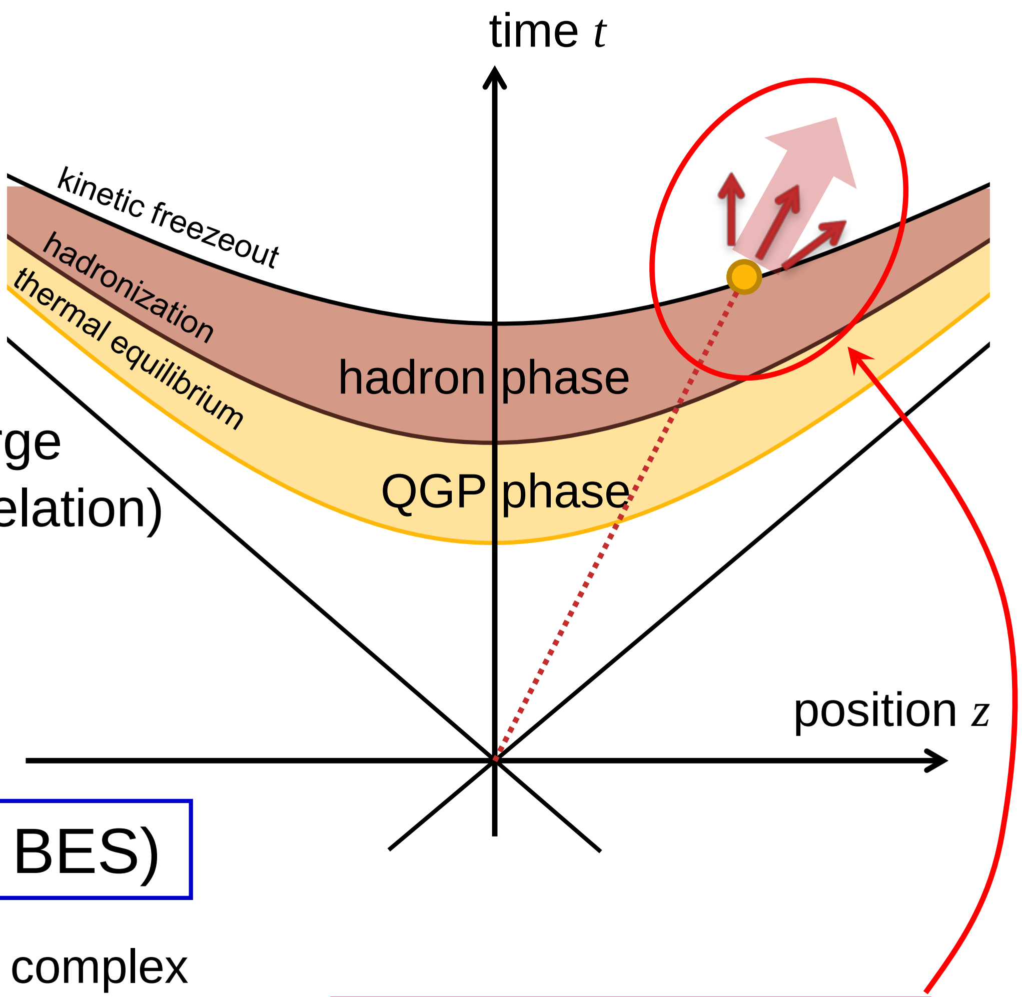
Blurring: loss of y-η correspondence

y: (momentum space) rapidity, η: space-time rapidity

$y = \eta$ in Bjorken picture

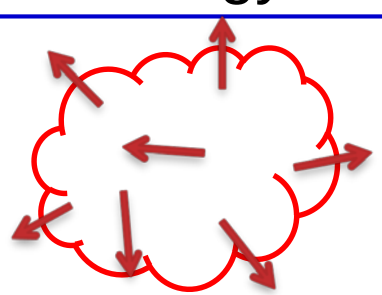
is blurred in one particle distribution owing to thermal motion

Accordingly, conserved charge fluctuation (two particle correlation) is modified



Blurring in rapidity space takes place !

Low energy collision (such as BES)



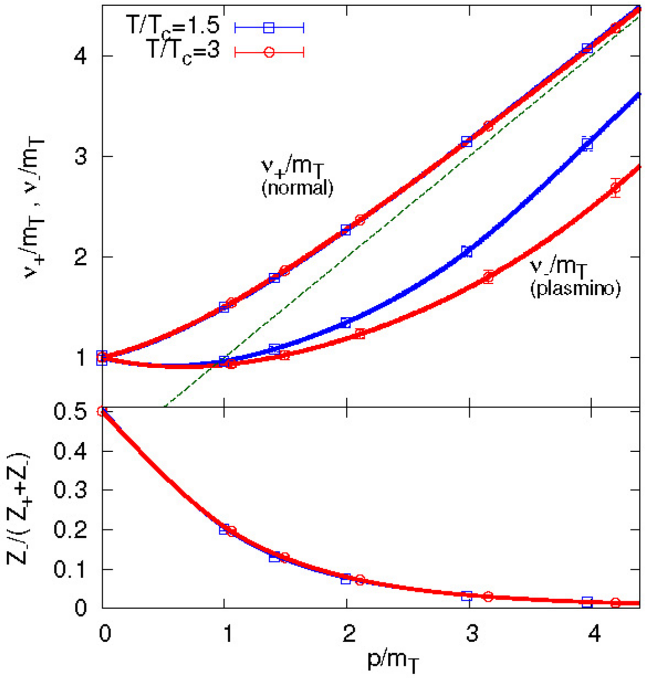
y-η relation: more complex

This should be taken into account in interpretation

Future 1: from Lattice to Dileptons

Nonperturbative calculation of dilepton production

1. Quark dispersion relation on the Lattice



➡ Nonperturbative

$$m_T/T = 0.768(0.725) \quad \text{at } T = 1.5T_c(3T_c)$$

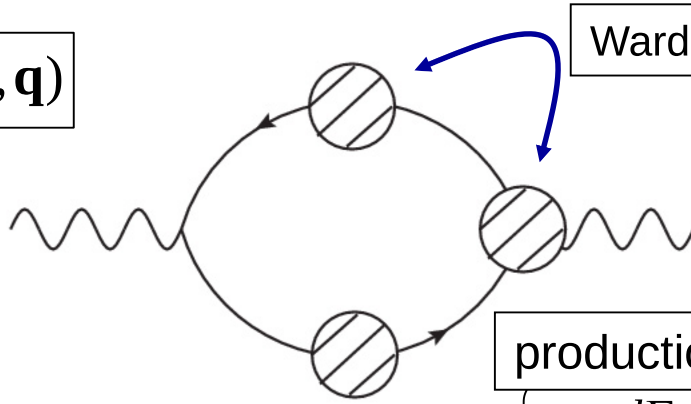
Kaczmarek et al. (2012)

➢ 2 pole fit with widths (Not HTL calculation)

Future 1: from Lattice to Dileptons

2. Ward Identity and Dilepton production rate

$\Pi_{\mu}^{R,\mu}(\omega,\mathbf{q})$



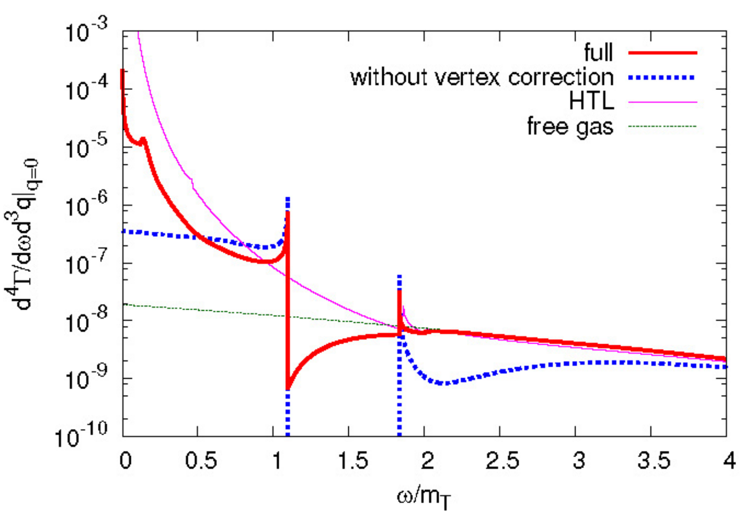
Ward identity

Gauge Symmetry

production rate

$$\frac{d\Gamma}{d\omega d^3q} = \frac{\alpha}{12\pi^4}\frac{1}{Q^2}\frac{1}{e^{\beta\omega}-1}\mathrm{Im}\,\Pi_{\mu}^{R,\mu}(\omega,\mathbf{q})$$

3. Dilepton Yield

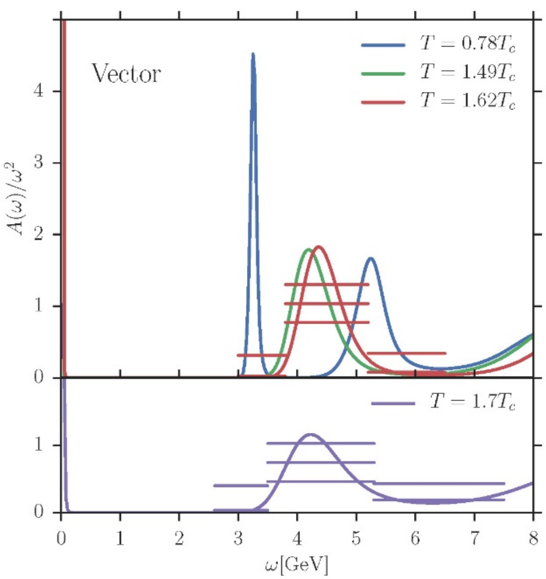


- The result happened to be close to that by HTL
- Vertex correction: important

Kim, Kitazawa, M.A. (2015)

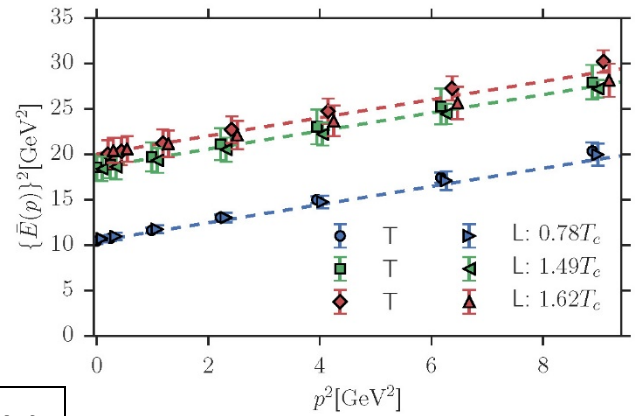
Future 2: Coupling of Heavy Quarks and Matter

Lattice Result (J/ψ)



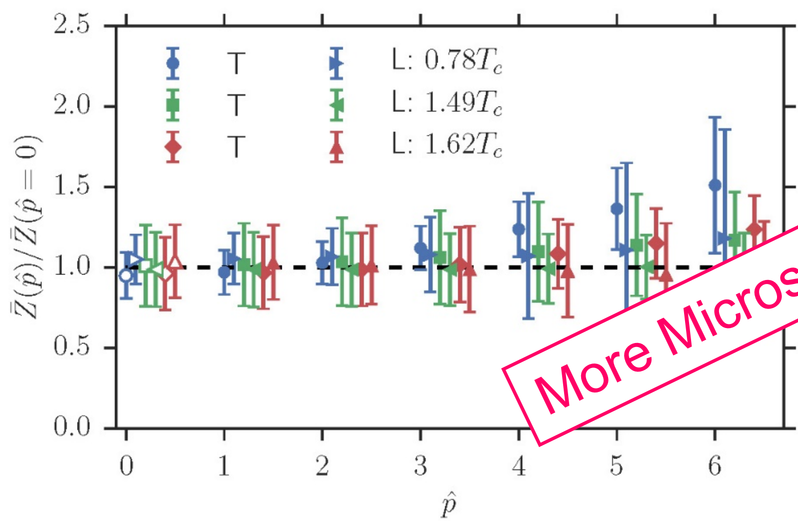
Spectral Function at p=0

Shift of peak position?



Dispersion Relation for T and L modes

$E(\mathbf{p})^2 = E(0)^2 + \mathbf{p}^2$ also at finite T?



More Microscopic Understanding Wanted!

Residue unchanged up to T=1.62T_c

$$p_i = \frac{2}{a_\sigma}\sin\left(\frac{\pi \hat{p}_i}{N_\sigma}\right)$$

Ikeda, Kitazawa, M.A. (2017)

Future 2: Coupling of Heavy Quarks and Matter

➢ Coupling of Heavy (anti)Quarks with Matter?

Note: Debye Screening assumes *small $e\varphi(r)$ and static matter*

- Stress Tensor Distribution around Quark-Antiquark Pair?

On Lattice, Translational Invariance is lost

Energy Momentum Tensors: Noether Currents of Translational Symmetry

⇒ A lot of difficulty met on Lattice:

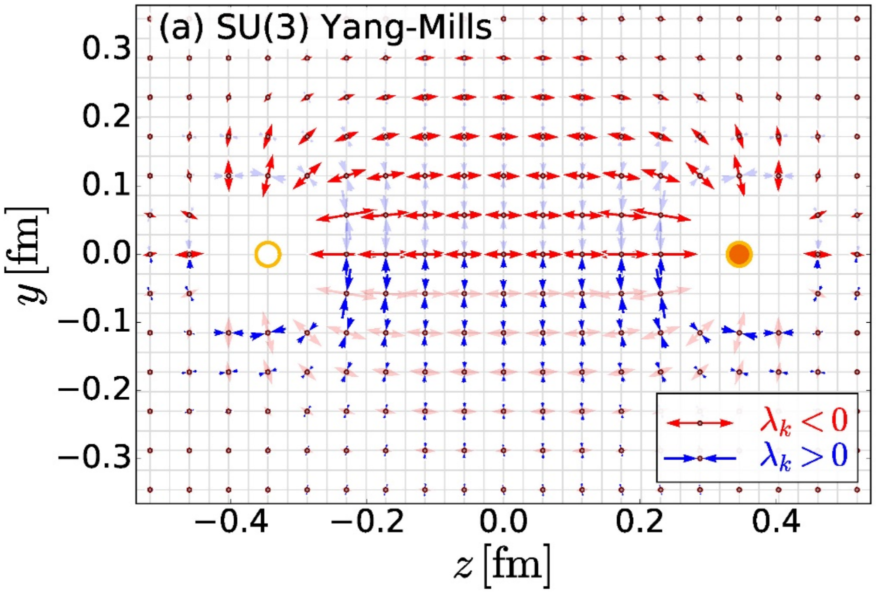
Definition of Energy Momentum Tensor on Lattice?
Supersymmetry on Lattice?

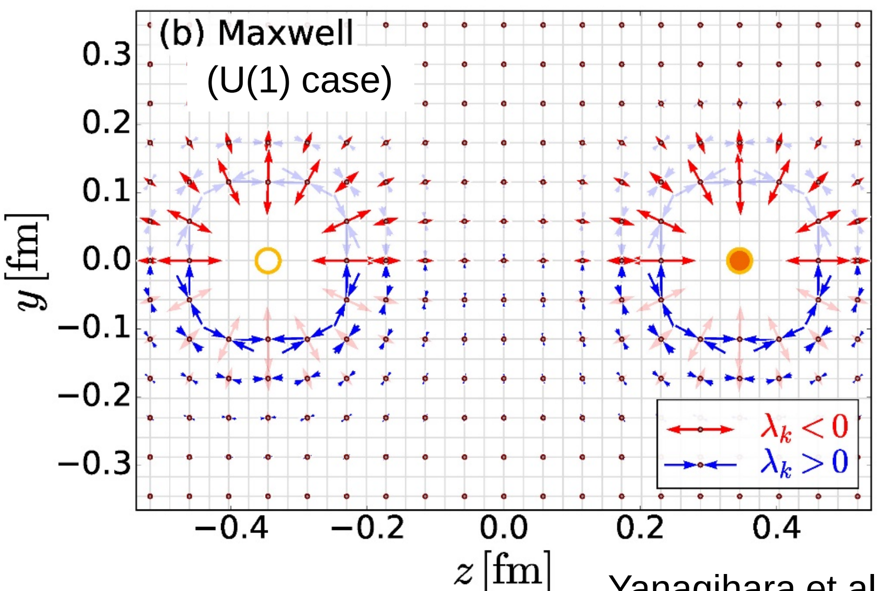
A way to restore Translational Invariance: *Gradient Flow*

Lüscher (2010)

Future 2: Coupling of Heavy Quarks and Matter

T=0 Heavy quark and antiquark





Yanagihara et al. (2018)

Eigenvectors of T_{ij} (i,j=1,2,3) and Signs of Eigenvalues

One eigenvector: perpendicular to yz-plane

✓ Flux Tube is visible

Finite T analysis: in progress

In QGP Physics

In Condensed Matter Physics

➢ 1st — Macroscopic Properties

- bulk quantities: heat capacity, heat conductivity, transport coefficients, …
- phase structure

← *We are HERE*

➢ 2nd — Microscopic Properties

correct understanding of data: needed

- effective mass, band structure
- gap structure
- various correlations
- spectral function

← *We are HERE*

➢ 3rd — Microscopic Understanding

← *Next Decade(s)*

- (Normal) Superconductor: BCS theory
- Fractal Quantum Hall Effect: Laughlin wave function