Patterns of Strong Coupling for LHC Searches

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Precision measurement at the LHC

A general $2 \to 2$ scattering at low energy:

$$\mathcal{A}(\phi\phi \to \phi\phi) \sim g_{SM}^2 \left(1 + \frac{g_*^n}{g_{SM}^n}\frac{E^2}{m_*^2} + \cdots\right)$$

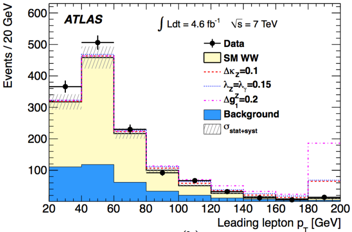
where $n \leq 2$, for weakly coupled theory $g_* \sim g_{SM}$:

$$\frac{\delta\sigma}{\sigma} < 1$$

for the expansion to make sense.

But the present several searches (VH, VV) at the LHC sensitive to $\mathcal{O}(1)$ effects

Precision measurement at the LHC



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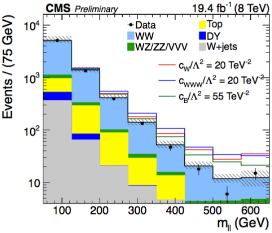


One can thinking of the LHC open a new door to strong coupling!

Power counting of $\hbar$

Natural units:

$$\hbar = c = 1$$

Let's restore $\hbar$ in our action for the path-integration:

$$e^{iS/\hbar} = e^{i\int d^4x \mathcal{L}/\hbar}$$

For the non-canonically normalized fields:

$$\mathcal{L}/\hbar = \frac{1}{g_\phi^2 \hbar}\left(\frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}\phi^2 + \cdots\right)$$

So that:

$$[g_\phi] = \hbar^{-1/2}, \qquad \mathcal{A}_n \propto g_\phi^{n-2}$$

SILH scenario

SILH can be thinking of as a set of power-counting rules associated with following considerations:

- Two sectors: the elementary sector (including SM gauge bosons and fermions), the composite (strong) sector (including the Higgs).
- Higgs are further assumed as pseudo-Goldstone bosons for naturalness consideration.
- The physics of the new sector is broadly characterized by one scale m_* and one coupling g_*.
- The elementary fields are assumed linearly coupled to the strong sector according to the hypothesis of partial compositeness.

Partial Compositeness

The mixing Lagrangian in the UV:

$$\mathcal{L}_{mix} = \epsilon_A A_\mu J^\mu + \epsilon_\psi \psi\, \mathcal{O}_\psi + h.c.\,,$$

leading to the effective Lagrangian below the scale m_*:

$$\mathcal{L}_{eff} = \frac{1}{g_*^2}\left\{ m_*^4 L\left(\frac{\Phi}{m_*}, \frac{D_\mu}{m_*}, \frac{\epsilon_A \hat{F}^i_{\mu\nu}}{m_*^2}, \frac{\epsilon_\psi \hat{\psi}}{m_*^{3/2}}\right) - \frac{1}{4}(\hat{F}^i_{\mu\nu})^2 + i\bar{\hat{\psi}}\gamma^\mu D_\mu \hat{\psi}\right\}\,,$$

$$\begin{aligned} D_\mu &\equiv \partial_\mu + i\epsilon_A T_i \hat{A}^i_\mu\,, \\ \hat{F}^i_{\mu\nu} &\equiv \partial_\mu \hat{A}^i_\nu - \partial_\nu \hat{A}^i_\mu - \epsilon_A f^{ijk}\hat{A}^j_\mu \hat{A}^k_\nu\,, \end{aligned}$$

Partial Compositeness

- To go to canonically normalized fields: $\hat{A}_\mu = g_* A_\mu, \hat{\psi} = g_* \psi$
- The gauge coupling: $g \equiv g_* \epsilon_A$
- ϵ measures the degree of the compositeness of the SM fields.
- $\epsilon \sim 1$ means fully composite, which can be achieved for the right-handed top quark, if $3/2 < \text{dim}\mathcal{O}_{t_R} < 5/2$.

Partial Compositeness

The composite fields can be continuously deformed to the elementary fields.

SILH operators

$$\mathcal{L}_6 = \frac{1}{m_*^2} \sum_i c_i \, \mathcal{O}_i \, .$$

$\mathcal{O}_H = \frac{1}{2}(\partial^\mu \|H\|^2)^2$ $\mathcal{O}_T = \frac{1}{2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2$ $\mathcal{O}_6 = \|H\|^6$	$\mathcal{O}_{y_\psi} = \|H\|^2 \bar{\psi}_L H \psi_R$
$\mathcal{O}_W = \frac{i}{2}\left(H^\dagger \sigma^a \overleftrightarrow{D^\mu} H\right) D^\nu W^a_{\mu\nu}$ $\mathcal{O}_B = \frac{i}{2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right) \partial^\nu B_{\mu\nu}$	$\mathcal{O}_{2B} = -\frac{1}{2}(\partial_\rho B_{\mu\nu})^2$ $\mathcal{O}_{2W} = -\frac{1}{2}(D_\rho W^a_{\mu\nu})^2$ $\mathcal{O}_{2G} = -\frac{1}{2}(D_\rho G^A_{\mu\nu})^2$
$\mathcal{O}_{HW} = i(D^\mu H)^\dagger \sigma^a (D^\nu H) W^a_{\mu\nu}$ $\mathcal{O}_{HB} = i(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$	$\mathcal{O}_{3W} = \frac{1}{3!}\epsilon_{abc} W^{a\,\nu}_\mu W^b_{\nu\rho} W^{c\,\rho\mu}$ $\mathcal{O}_{3G} = \frac{1}{3!} f_{ABC} G^{A\,\nu}_\mu G^B_{\nu\rho} G^{C\,\rho\mu}$
$\mathcal{O}_{BB} = \|H\|^2 B_{\mu\nu} B^{\mu\nu}$ $\mathcal{O}_{GG} = \|H\|^2 G^A_{\mu\nu} G^{A\mu\nu}$	$\mathcal{O}^\psi_{L,R} = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{\psi}_{L,R}\gamma^\mu \psi_{L,R})$ $\mathcal{O}^{(3)\,\psi}_L = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H)(\bar{\psi}_L \sigma^a \gamma^\mu \psi_L)$
	$\mathcal{O}_{4\psi} = \bar{\psi}\gamma_\mu \psi \bar{\psi}\gamma^\mu \psi$

Table 1: *Dimension-6 operators used in our analysis.*

SILH operators

- For purely bosonic operators, only O_W, O_B, O_{2V} can be generated at tree level by exchange massive vectors in minimally coupled theory (Holographic composite Higgs model and little Higgs model).

- $\mathcal{O}_{GG}, \mathcal{O}_{BB}$ subject to the same selection rule for the Higgs potential, will have extra suppression y_t^2/g_*^2.

- In the general case *GSILH*, the minimal coupling condition is relaxed.

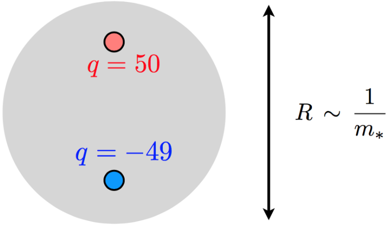
SILH operators

	$\|H\|^2$	$\|H\|^4$	$\mathcal{O}_H$	$\mathcal{O}_6$	$\mathcal{O}_V$	$\mathcal{O}_{2V}$	$\mathcal{O}_{3V}$
ALH	m_*^2	g_*^2	g_*^2	g_*^4	g_V	$\frac{g_V^2}{g_*^2}$	$\frac{g_V^2}{g_*^2} g_V$
GSILH	$\frac{y_t^2}{16\pi^2} m_*^2$	$\frac{y_t^2}{16\pi^2} g_*^2$	g_*^2	$\frac{y_t^2}{16\pi^2} g_*^4$	g_V	$\frac{g_V^2}{g_*^2}$	$\frac{g_V^2}{g_*^2} g_V$
SILH	$\frac{y_t^2}{16\pi^2} m_*^2$	$\frac{y_t^2}{16\pi^2} g_*^2$	g_*^2	$\frac{y_t^2}{16\pi^2} g_*^4$	g_V	$\frac{g_V^2}{g_*^2}$	$\frac{g_V^2}{16\pi^2} g_V$

	$\mathcal{O}_{HV}$	$\mathcal{O}_{VV}$	$\mathcal{O}_{y_\psi}$				
ALH	g_V	g_V^2	$y_\psi g_*^2$				
GSILH	g_V	$\frac{y_t^2}{16\pi^2} g_V^2$	$y_\psi g_*^2$				
SILH	$\frac{g_*^2}{16\pi^2} g_V$	$\frac{y_t^2}{16\pi^2} g_V^2$	$y_\psi g_*^2$				

Strong multi-polar interactions

An object can have large multi-pole and small monopole:



$$q = 1, \qquad |\vec{d}| \sim 50R$$

Strong multi-polar interactions

- Two couplings involved: gauge coupling (monopole) g and the strong coupling g_* controlling the multipole interactions of the resonances.
- The small parameter $\epsilon = g/g_*$ is technically natural, since $\epsilon = 0$ is a stable point by deformed symmetry (not enhanced).

Abelian case

$$\mathcal{L}_{eff} = \frac{m_*^4}{g_*^2} L \left(\frac{\hat{F}_{\mu\nu}}{m_*^2}, \frac{\partial_\mu}{m_*}, \frac{\hat{\Phi}}{m_*} \right) ,$$

The effective Lagrangian can be deformed by including small charges:

$$\partial_\mu \Phi \to (\partial_\mu + i\epsilon q_\Phi A_\mu)\Phi ,$$

Strong multi-polar interactions

The situation can be generalized to non-Abelian cases by requiring:

- There are N_A composite $U(1)^{N_A}$ gauge bosons.
- The $U(1)^{N_A}$ photons transform in the adjoint under the global symmetry $\mathcal{G}$ of the strong sector.

Inonu-Wigner (IW) contraction

Small charges are included by deforming the symmetry:

$$[\mathcal{G}]_{global} \rtimes [U(1)^{N_A}]_{local} \rightarrow [\mathcal{G}]_{local} \, .$$

leading to the following effective Lagrangian:

$$\mathcal{L}_{eff} = \frac{m_*^4}{g_*^2} L \left(\frac{\hat{F}^i_{\mu\nu}}{m_*^2}, \frac{D_\mu}{m_*} \right) \, ,$$

Strong multi-pole and Partial Compositeness

Can we obtain Strong multi-pole interactions from Partial Compositeness?

$$\Delta\mathcal{L}_{mix} = \epsilon_F F_{\mu\nu}\mathcal{O}^{\mu\nu} \, ,$$

$$L \to L\left(\frac{\hat{\Phi}}{m_*}, \frac{D_\mu}{m_*}, \frac{\epsilon_F \hat{F}_{\mu\nu}}{m_*^2}, \frac{\epsilon_\psi \hat{\psi}}{m_*^{3/2}}\right) \, .$$

We can define a effective coupling:

$$g_{eff} \sim \epsilon_F g_* \frac{E}{m_*} \, .$$

However,

Unitarity of CFT require dim $\mathcal{O}^{\mu\nu} \geq 2$, the mixing is irrelevant except $\mathcal{O}^{\mu\nu}$ is a free field.

Remedios

If only the gauge bosons are involved in the strong dynamics, the following operators are enhanced:

$$c_{3W}, c_{3G} \sim g_* \ , \qquad c_{2W}, c_{2B}, c_{2G} \sim 1 \, .$$

The phenomenological consequences:

$$c_{3W} \sim g_* \qquad \Rightarrow \qquad \delta\mathcal{A}(\bar\psi\psi \to V_T V_T) \sim g g_* \frac{E^2}{m_*^2} \ ,$$

$$\delta\mathcal{A}(V_T V_T \to V_T V_T) \sim g g_* \frac{E^2}{m_*^2}, g_*^2 \frac{E^4}{m_*^4} \ ,$$

$$c_{2W}, c_{2B} \sim 1 \qquad \Rightarrow \qquad \delta\mathcal{A}(\psi\bar\psi \to V_T^* \to \psi\bar\psi) \sim g^2 \frac{E^2}{m_*^2} \, .$$

Remedios

Note that,

- As long as $g_* \frac{E^2}{m_*^2} > g$, dimension-8 operators are needed for consistent analysis of WW scattering.
- The anomalous TGC:

$$\lambda_\gamma \equiv \frac{c_{3W}}{g} \frac{m_W^2}{m_*^2} \sim \frac{g_*}{g} \frac{m_W^2}{m_*^2} \, ,$$

- The high precision of LEP makes $c_{2W,2B}$ more relevant.

Remedios

The modification of the gauge propagator can be traded as the W, Y parameters:

$$W, Y \equiv c_{2W,2B}\frac{m_W^2}{m_*^2} \sim \frac{m_W^2}{m_*^2}\,.$$

$$W, Y \lesssim 10^{-3} \Rightarrow m_* \gtrsim 3\text{TeV}$$

Compared with:

$$\lambda_\gamma \lesssim 10^{-2} \Rightarrow m_* \gtrsim 1.5\sqrt{\frac{g_*}{4\pi}}\,\text{TeV}$$

Remedios + MCHM

It is more motivated to include the Higgs as Pseudo-Goldstone bosons of the strong sector:

$$\mathcal{G} = [SO(5) \times \widetilde{SU}(2) \times U(1)_X]_{global} \times [U(1)^4]_{local}$$

An extra global $\widetilde{SU}(2)$ is needed to make the Higgs mass stable.

The effective Lagrangian

$$\mathcal{L}_{eff} = \frac{m_*^4}{g_*^2} L \left(U, \frac{\hat{F}^i_{\mu\nu}}{m_*^2}, \frac{D_\mu}{m_*} \right)$$

Remedios + MCHM

- In the limit $g = g' = 0$, the extra $\widetilde{SU}(2)$ forbids the operators involving both gauge fields and the Higgs bosons $\mathcal{O}_{W,HW}$
- $B_{\mu\nu}$ is a singlet of the global symmetry
- $SO(4)$ symmetry further kills $\mathcal{O}_{B,HB}$

One extra operator

$$\mathcal{O}_H \sim g_*^2$$

Dimension-8 operators enhanced by g_*^2

$$_8\mathcal{O}_{HWW} = D_\mu H^\dagger D_\nu H \, W_\rho^{a\,\mu} W^{a\nu\rho} \,, \quad _8\mathcal{O}_{HBB} = D_\mu H^\dagger D_\nu H \, B^\mu_{\ \rho} B^{\nu\rho}$$

Remedios + ISO(4)

If we give up UV completion within QFT, the non-compact group can be considered:

$$\mathcal{G} = [ISO(4)]_{global} \rtimes [U(1)^4]_{local} ,$$

The Higgs are living in the flat coset $ISO(4)/SO(4)$:

$$H \to H + c, \quad H \to RH$$

which kills $\mathcal{O}_H$.

$(3, 1)$ is an irreducible representation of $SO(4)$

$$\mathcal{O}_{HW} \sim g_*^2$$

Remedios + ISO(4)

The phenomenology:

$$
\begin{aligned}
\delta g_1^Z &= \frac{\delta\kappa_\gamma}{\cos^2\theta_W} = \frac{\delta g_{hZ\gamma}}{\sin\theta_W\cos\theta_W} = -\frac{m_Z^2}{m_*^2}\frac{c_{HW}}{g} \sim \frac{m_Z^2}{m_*^2}\frac{g_*}{g} \\
\lambda_\gamma &= \frac{m_W^2}{m_*^2}\frac{c_{3W}}{g} \sim \frac{m_W^2}{m_*^2}\frac{g_*}{g}
\end{aligned}
$$

where our convention

$$
\delta\mathcal{L}_{hZ\gamma} = \delta g_{hZ\gamma}\frac{h}{v}Z_{\mu\nu}A^{\mu\nu}
$$

$\mathcal{G}$ breaking effects

The source of breaking:

$$\mathcal{L}_{break} = -\epsilon_t g_* \left[\bar{Q}_L \tilde{H} t_R + \ldots\right] + \epsilon_2 m_*^2 \left[|H|^2 + \ldots\right] - \epsilon_4 \frac{g_*^2}{2} \left[|H|^4 + \ldots\right]$$

with the following identification:

$$y_t \equiv \epsilon_t g_*, \quad \epsilon_2 m_*^2 \equiv m_H^2, \quad \epsilon_4 g_*^2 \equiv \lambda_h$$

The normalization of the couplings:

$$\Delta\mathcal{L}^h_{\psi\psi} = (h/v)(\delta g_{h\psi\psi} m_\psi \bar{\psi}\psi + \mathrm{h.c.})$$
$$\Delta\mathcal{L}^h_{\gamma\gamma} = (h/v)\delta g_{h\gamma\gamma} F_{\mu\nu} F^{\mu\nu}$$
$$\Delta\mathcal{L}^h_{VV} = (h/v)\delta g_{hVV} m_W^2 \left(W^{+\mu} W^-_\mu + \frac{Z^\mu Z_\mu}{2\cos^2\theta_W}\right)$$

$\mathcal{G}$ breaking effects

The first class: $H^\dagger \partial^4 H / m_*^2$

- No field strength $|\Box H|^2/m_*^2$: by field redefinition,

$$c_6 \sim \lambda_h^2, c_{4\psi} \sim y_\psi^2$$

$$c_{y_\psi} \sim y_\psi \lambda_h \quad \Rightarrow \quad \delta g_{h\psi\psi} \sim \frac{m_h^2}{m_*^2}$$

- One field strength:

$$c_B \sim g' \,, \quad c_W \sim g \quad \Rightarrow \quad \delta \widehat{S} \sim \frac{m_W^2}{m_*^2}$$

- Two field strengths:

$$c_{BB} \sim g'^{\,2} \quad \Rightarrow \quad \delta g_{h\gamma\gamma} \sim \frac{e^2 v^2}{m_*^2}$$

$\mathcal{G}$ breaking effects

The second class: SM operators + derivatives

$$c_H \sim \lambda_h \quad \Rightarrow \quad \delta g_{hVV} \sim \frac{m_h^2}{m_*^2}$$

The third class: Loops of SM fields ($\Delta I_c = 2$)

$$c_T \sim \left(\frac{g_*}{4\pi}\right)^2 \times g'^2 \quad \Rightarrow \quad \delta \widehat{T} \sim \left(\frac{g_*}{4\pi}\right)^2 \times \tan^2\theta_W \frac{m_W^2}{m_*^2}$$

$$c_T \sim \frac{y_t^4}{16\pi^2} \quad \Rightarrow \quad \delta \widehat{T} \sim \left(\frac{y_t}{4\pi}\right)^2 \times \frac{m_t^2}{m_*^2}$$

Remedios Scenario

In summary:

Model	$\mathcal{O}_{2V}$	$\mathcal{O}_{3V}$	$\mathcal{O}_{HW}$	$\mathcal{O}_{HB}$	$\mathcal{O}_{V}$	$\mathcal{O}_{VV}$	$\mathcal{O}_{H}$	$\mathcal{O}_{y_\psi}$
Remedios	1	g_*						
Remedios+MCHM	1	g_*	g	g'	g_V	g_V^2	g_*^2	$y_\psi g_*^2$
Remedios+$ISO(4)$	1	g_*	g_*	g'	g_V	g_V^2	λ_h	$y_\psi \lambda_h$

Partially Composite Fermions

Assuming the family symmetry, the best way to look at the fermion compositeness is $\psi\psi \to \psi\psi$:

$$\delta\mathcal{A}(\psi\psi \to \psi\psi) \simeq \epsilon_\psi^4 g_*^2 \frac{E^2}{m_*^2}\,,$$

The bound from LHC Run1 (arXiv:1201.6510):

$$m_* \gtrsim (g_* \epsilon_\psi^2 / 4\pi) \times 60\,\mathrm{TeV}$$

It seems difficult to have fully composite fermions:

$$\epsilon_\psi \sim 1$$

Partially Composite Fermions + Higgs compositeness

If Higgs is also composite, processes like:

$$\bar{\psi}\psi \to V_L V_T / V_L h$$

are also relevant to probe the scenario.

But, the operators

$$\mathcal{O}^{\psi}_{L,R} = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{\psi}_{L,R}\gamma^\mu \psi_{L,R}), \quad \mathcal{O}^{(3)\,\psi}_{L} = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H)(\bar{\psi}_L \sigma^a \gamma^\mu \psi_L)$$

are constrained by LEP-I Z-pole physics:

$$m_* \gtrsim (g_* \epsilon_\psi / 4\pi) \times 40\,\mathrm{TeV}$$

Fermions as composite Pseudo-Goldstini

Can we have soft-IR fermions?

A first attempt:

$$\psi \to \psi + \xi$$

The operators starting from dimension-10, the amplitude growing as:

$$\delta\mathcal{A} \propto s^3$$

disfavored by basic principles (unitarity and analyticity).

A non-linearly realized SUSY can do the job!

$$\delta\psi = \xi + \frac{i}{2F^2}\partial_\mu\psi(\bar{\psi}\gamma^\mu\xi - \bar{\xi}\gamma^\mu\psi)$$

Fermions as composite Pseudo-Goldstini

The operators starting from dimension-8

$$\frac{i}{F^2}\bar{\psi}(\gamma^\mu \overleftrightarrow{\partial^\nu} + \gamma^\nu \overleftrightarrow{\partial^\mu})\psi F_{\mu\rho}F_\nu{}^\rho\,, \qquad \frac{i}{F^2}\partial_\mu\phi^\dagger\partial_\nu\phi\,\bar{\psi}(\gamma^\mu \overleftrightarrow{\partial^\nu} + \gamma^\nu \overleftrightarrow{\partial^\mu})\psi\,,$$
$$\frac{1}{F^2}\bar{\psi}^2\partial^2\psi^2\,, \qquad \frac{1}{F^2}\partial_\nu\bar{\psi}\gamma^\mu\psi\;\bar{\psi}_q\gamma_\mu\partial^\nu\psi_q\,, \qquad \frac{1}{F^2}\partial_\nu\bar{\psi}_q\gamma^\mu\psi\;\bar{\psi}\gamma_\mu\partial^\nu\psi_q\,.$$

We can identify:

$$F \sim m_*^2/g_*$$

Generations to $\mathcal{N} > 1$ is also possible.

Fermions as composite Pseudo-Goldstini

The phenomenological consequences:

$$\delta\mathcal{A}(\psi\psi \to \psi\psi) \simeq g_*^2 \frac{E^4}{m_*^4}\,,$$

$$\delta\mathcal{A}(\bar{\psi}\psi \to V_L V_L) \simeq g_*^2 \frac{E^4}{m_*^4}\left(g^2 \frac{E^2}{m_*^2}\right)\,.$$

$$\delta\mathcal{A}(\bar{\psi}\psi \to V_T V_T) \simeq g_*^2 \frac{E^4}{m_*^4}\left(g g_* \frac{E^2}{m_*^2}\right)\,.$$

The dimension-8 dominates over dimension-6 whenever

$$E \gtrsim \sqrt{g/g_*}\,m_*$$

More importantly, they give sizable contribution to neutral diboson pair production !

Conclusion

- It is still possible to make the SM degrees of freedom emerging from a strong dynamics above the TeV scale.
- We have constructed the effective Lagrangians for the transverse gauge bosons involving in the strong dynamics through multi-pole interactions.
- We also combined the scenario (Remedios) with the composite Higgs models, motivated by naturalness consideration.
- The Fermions can also get involved as pseudo-Goldstini.
- Our scenario motivated several precision measurements (VH,VV) at the LHC, where dimension-8 operators dominates over dimension-6.

Dimension-8 operators

$(X_{\mu\nu})^4$

$SU(2)_L:$

$${}_8\mathcal{O}_{4W} = W^a_{\mu\nu} W^{a\,\mu\nu} W^b_{\rho\sigma} W^{b\,\rho\sigma} \qquad {}_8\mathcal{O}'_{4W} = W^a_{\mu\nu} W^{b\,\mu\nu} W^a_{\rho\sigma} W^{b\,\rho\sigma}$$

$${}_8\mathcal{O}_{4\widetilde{W}} = W^a_{\mu\nu} W^{a\,\nu\rho} W^b_{\rho\sigma} W^{b\,\sigma\mu} \qquad {}_8\mathcal{O}'_{4\widetilde{W}} = W^a_{\mu\nu} W^{b\,\nu\rho} W^a_{\rho\sigma} W^{b\,\sigma\mu}$$

$U(1)_Y:$

$${}_8\mathcal{O}_{4B} = B_{\mu\nu} B^{\mu\nu} B_{\rho\sigma} B^{\rho\sigma} \qquad {}_8\mathcal{O}_{4\widetilde{B}} = B_{\mu\nu} B^{\nu\rho} B_{\rho\sigma} B^{\sigma\mu}$$

$SU(2)_L \times U(1)_Y:$

$${}_8\mathcal{O}_{2WB} = W^a_{\mu\nu} W^{a\,\mu\nu} B_{\rho\sigma} B^{\rho\sigma} \qquad {}_8\mathcal{O}'_{2WB} = W^a_{\mu\nu} B^{\mu\nu} W^a_{\rho\sigma} B^{\rho\sigma}$$

$${}_8\mathcal{O}_{2\widetilde{W}\widetilde{B}} = W^a_{\mu\nu} W^{a\,\nu\rho} B_{\rho\sigma} B^{\sigma\mu} \qquad {}_8\mathcal{O}'_{2\widetilde{W}\widetilde{B}} = W^a_{\mu\nu} B^{\nu\rho} W^a_{\rho\sigma} B^{\sigma\mu}\,.$$

Dimension-8 operators

$D\psi^2(X_{\mu\nu})^2$ Strongly interacting fermions and vectors generate

$$ {}_8\mathcal{O}_{TWW} = \mathcal{T}^{\mu\nu} W^a_{\mu\rho} W^{a\,\rho}_{\nu} \qquad {}_8\mathcal{O}_{TBB} = \mathcal{T}^{\mu\nu} B_{\mu\rho} B_{\nu}^{\ \rho} $$
$$ {}_8\mathcal{O}_{TWB} = \mathcal{T}^{a\,\mu\nu} W^a_{\mu\rho} B_{\nu}^{\ \rho} $$

where $\mathcal{T}^{\mu\nu} = \frac{i}{4}\bar{\psi}(\gamma^\mu \overleftrightarrow{D}^\nu + \gamma^\nu \overleftrightarrow{D}^\mu)\psi$ and
$\mathcal{T}^{a,\,\mu\nu} = \frac{i}{4}\bar{\psi}(\gamma^\mu \overleftrightarrow{D}^\nu + \gamma^\nu \overleftrightarrow{D}^\mu)\sigma^a\psi$ for $SU(2)_L$ doublets

D^4H^4 In models where the Higgs is composite,

$$ {}_8\mathcal{O}_{\{D\}H} = (D_{\{\mu} H^\dagger D_{\nu\}} H)^2 \qquad {}_8\mathcal{O}_{DH} = (D_\mu H^\dagger D^\mu H)^2 $$

Dimension-8 operators

$\boxed{D^2 H^2 (X_{\mu\nu})^2}$ On the other hand,

$$ {}_8\mathcal{O}_{HWW} = D_\mu H^\dagger D_\nu H \, W^{a\,\mu}_{\rho} W^{a\nu\rho} \,, \quad {}_8\mathcal{O}_{HBB} = D_\mu H^\dagger D_\nu H \, B^{\mu}_{\ \rho} B^{\nu\rho} $$

$$ {}_8\mathcal{O}'_{HWW} = D_\mu H^\dagger \sigma^a D_\nu H \, W^{b\,\mu}_{\rho} W^{c\nu\rho} \epsilon^{abc} $$

$$ {}_8\mathcal{O}_{HWB} = D_\mu H^\dagger \sigma^a D_\nu H \, W^{a\,\mu}_{\rho} B^{\nu\rho} $$

$\boxed{D^3 H^2 \psi^2}$ If the fermions are pseudo-Goldstini,

$$ {}_8\mathcal{O}_{TH} \;=\; \mathcal{T}^{\mu\nu} D_\mu H^\dagger D_\nu H $$