Mass Components of Mesons from Lattice QCD

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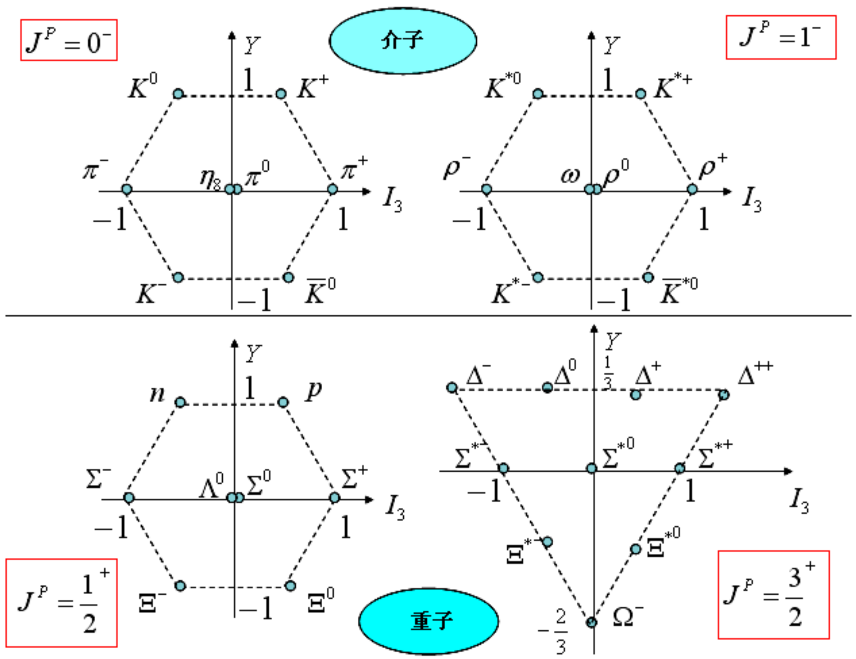
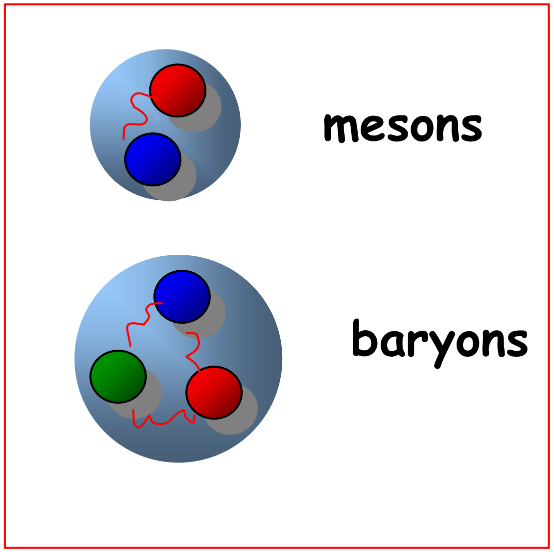
I. Motivation

Quark Model





Constituent quark mass

$$\mu_i = 2.79 \frac{e}{2M_P} e_i \quad \Rightarrow \quad m_{u,d} = M_p / 2.79 = 336 MeV$$

Deep in-elastic scattering (DIS) experments and the Parton model

$$I \equiv \int_0^1 dx x(u + \overline{u} + d + \overline{d} + s + \overline{s}) = \int_0^1 dx[9F_2^{eN} - \frac{3}{2}F_2^{\nu N}] \approx 0.5$$

However, in QCD, the current quark masses are the fundamental Parameters of the Lagrangian

$$L_{QCD} = \sum_{k=1}^{N_F} \overline{q}_k (i\gamma^\mu D_\mu - m_k) q_k - \frac{1}{4} G_{\mu\nu} G^{\mu\nu},$$

which should be determined theoretically rather than experimentally, since quarks are confined inside hadrons and are not observed as physical particles.

Current quark masses vs. hadron masses (PDG2012)

$m_u(\mu)$	$m_d(\mu)$	$m_s(\mu)$	$m_c(\mu)$	$m_b(\mu)$
$\mu = 2GeV$	$\mu = 2GeV$	$\mu = 2GeV$	$\mu = m_c$	$\mu = m_b$
$2.3^{+0.7}_{-0.5}(MeV)$	$4.8^{+0.7}_{-0.3}(MeV)$	$95(5)(MeV)$	$1.275(25)(GeV)$	$4.18(3)(GeV)$

	$I=1$			$I=1/2$		$I=1/2$		$I=0$
$J^P=0^-$	π^+	π^0	π^-	K^+	K^0	$\overline{K}^0$	K^-	η_8
M(MeV)	**139.6**	**135.0**	**139.6**	**493.7**	**497.6**	**497.6**	**493.7**	**547.5**
$J^P=1^-$	ρ^+	ρ^0	ρ^-	K^{*+}	K^{*0}	$\overline{K}^{*0}$	K^{*-}	ω
M(MeV)	**775.5**			**891.7**	**896.0**	**896.0**	**891.7**	**782.7**
$J^P=\frac{1}{2}^+$	Σ^+	Σ^0	Σ^-	p	n	Ξ^0	Ξ^-	Λ
M(MeV)	**1189.4**	**1192.6**	**1197.4**	**938.3**	**939.6**	**1314.8**	**1321.3**	**1115.7**

Open question: How do hadrons acquire their large masses ???

II. A brief introduction to Lattice QCD

Wick Rotation from Minkowski Space to Euclidean Space

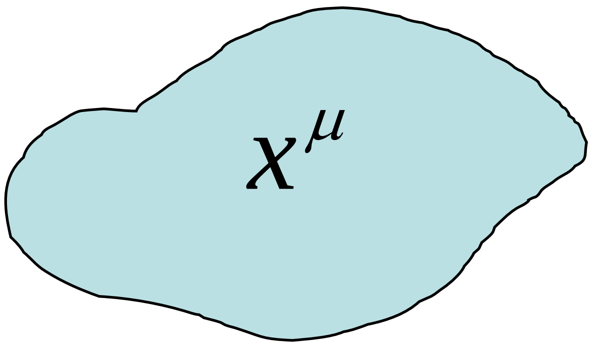
$$x_0 \equiv t \to -ix_4 \equiv -i\tau \,,$$
$$p_0 \equiv E \to ip_4 \,.$$

$$e^{iS_M} \equiv e^{i\int dx_M^4 L(x_M)} = e^{\int dx_E^4 L(x_E)} \equiv e^{-S_E}$$

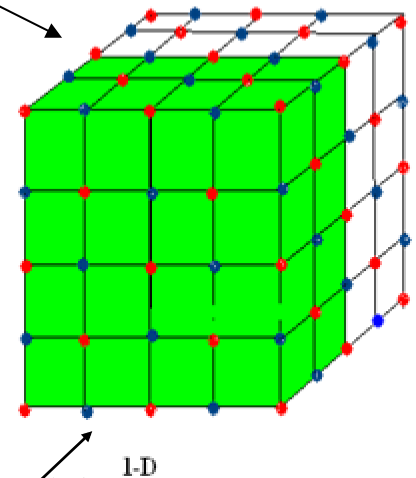
$$Z = \int \mathcal{D}A_\mu \, \mathcal{D}\psi \, \mathcal{D}\overline{\psi} \; e^{-S}$$
$$S = S_{gauge} + S_{quarks} = \int d^4x \; \left(\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\right) - \textstyle\sum_i \log(\mathrm{Det}M_i)$$
$$Z = \int \mathcal{D}A_\mu \; \det M \; e^{\int d^4x \; (-\frac{1}{4}F_{\mu\nu}F^{\mu\nu})} .$$
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}A_\mu \; \mathcal{O} \; e^{-S} \,.$$

QCD on a Euclidean Space-time Grid

- Space-time discretization



Lattice spacing a



$$A_{\mu}(x) \rightarrow U_{\mu}(x) = e^{iagA_{\mu}(x)}$$

$$\psi(x) \rightarrow \psi(x)$$

MC Simulation——Importance Sampling

- Taking $e^{-S[U,\psi,\overline{\psi}]}$ as a probability distribution, an ensemble of configurations are generated from MC simulation. This is the procedure that eat the computation resources mostly.
- After the generation of configurations, the functional integral

$$\langle O(A_\mu,\overline{\psi},\psi)\rangle = \frac{1}{Z}\int [DA_\mu D\overline{\psi} D\psi]\exp(-S(A_\mu,\overline{\psi},\psi))O(A_\mu,\overline{\psi},\psi)$$

becomes the much simpler arithmetic average:

$$\langle O \rangle_{MC} = \frac{1}{N}\sum_{i=1}^{N} O_i$$

- Generally speaking, the quantites that are most commonly calculated are Green's function, say, the vacuum expectation values of field operators defined at different space-time points.

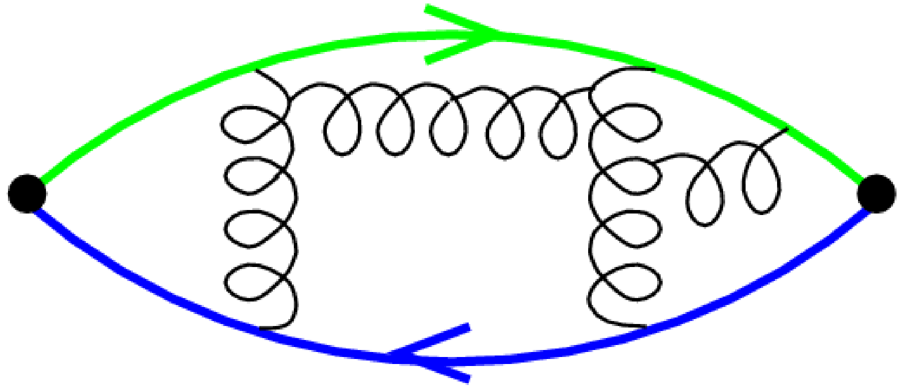
Quenched and Unquenched

$$Z = \int DUD\,\overline{\psi}D\,\psi e^{-S_g + \overline{\psi}M\psi} = \int DU \det M e^{-S_g}$$
$$= \int DUe^{-S_g + Tr\ln M}$$

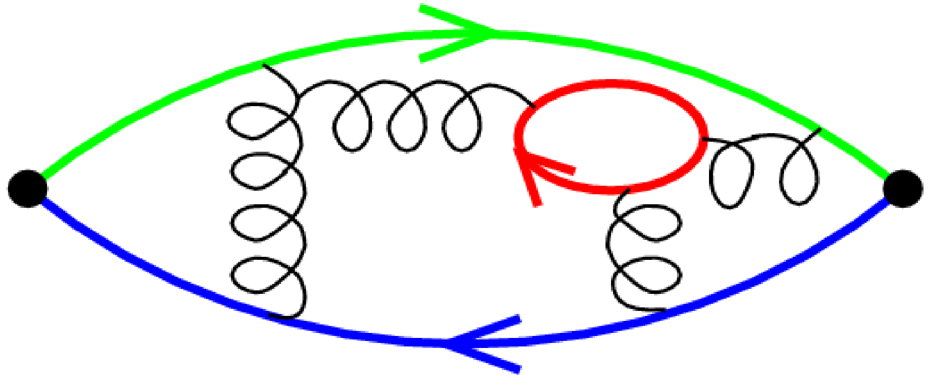
On the lattice, M is a very large matrix, such that the calculation of its trace is very expensive in the MC simulation. A way out this difficulty is to take the approximation

$$\det M[U] = const\,.$$

Theoretically, this means that we set the sea quark mass to be infinitely large such that they decouple from the gauge field. In other words, we will ignore the vacuum polarization diagram, say, the effects of sea quarks.



(A) Quenched QCD: quark loops neglected



(B) Full QCD

Chiral Fermions

- Ginsburg-Wilson relation---chiral symmetry on lattice

$$\gamma_5 D + D\gamma_5 = aD\gamma_5 D$$

- Chiral tranformation in the continuum

$$\psi \rightarrow e^{i\theta\gamma_5}\psi;$$

$$\bar{\psi} \rightarrow \bar{\psi}e^{i\theta\gamma_5}$$

- Chiral transfromation on the lattice

$$\psi \rightarrow e^{i\theta\gamma_5(1-aD/2)}\psi;$$

$$\bar{\psi} \rightarrow \bar{\psi}e^{i\theta\gamma_5(1-aD/2)}$$

The lattice action is invariant if GW relation holds for D

- Two types of fermion actions that satisfy GW relation:

overlap fermion
domain-wall fermion

- Free of fermion doubling + chiral
- But computation is much more expensive.

Monte Carlo Simulation of Lattice QCD

The equivalences between a Euclidean field theory and Classical Statistical Mechanics.

Euclidean Field Theory	Classical Statistical Mechanics
Action	Hamiltonian
unit of action h	units of energy $\beta = 1/kT$
Feynman weight for amplitudes $e^{-S/h} = e^{-\int \mathcal{L} dt/h}$	Boltzmann factor $e^{-\beta H}$
Vacuum to vacuum amplitude $\int \mathcal{D}\phi e^{-S/h}$	Partition function $\sum_{conf.} e^{-\beta H}$
Vacuum energy	Free Energy
Vacuum expectation value $\langle 0 \| \mathcal{O} \| 0 \rangle$	Canonical ensemble average $\langle \mathcal{O} \rangle$
Time ordered products	Ordinary products
Green's functions $\langle 0 \| T[\mathcal{O}_1 \ldots \mathcal{O}_n] \| 0 \rangle$	Correlation functions $\langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle$
Mass M	correlation length $\xi = 1/M$
Mass-gap	exponential decrease of correlation functions
Mass-less excitations	spin waves
Regularization: cutoff Λ	lattice spacing a
Renormalization: $\Lambda \to \infty$	continuum limit $a \to 0$
Changes in the vacuum	phase transitions

Due to the similarity, we can borrow the methods of statistical mechanics to study lattice QCD, such as Monte Carlo simulation.

Present status of Lattice QCD

$$Z = \int [DU] e^{-S_g(U) + \mathrm{Tr}\ln M[U]}$$

$\mathrm{Tr}\ln M[U] \sim \text{const.} \quad \Rightarrow$ Quenched Approximation

$m_q^{val} \neq m_q^{sea} \quad \Rightarrow$ Partially Quenched

$M_{val}[U] \neq M[U] \quad \Rightarrow$ Mixed Action

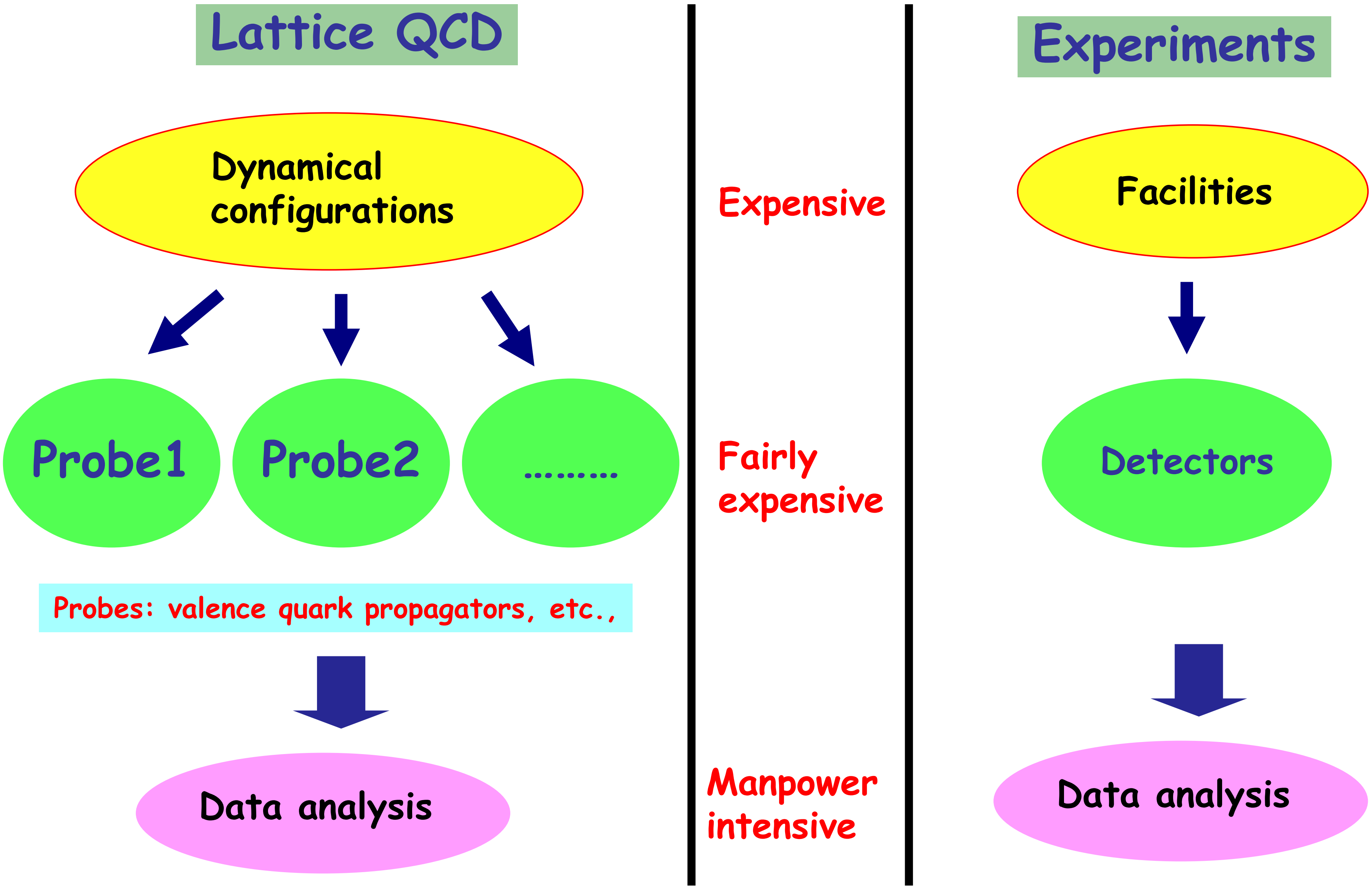
Otherwise, a unitary theory of full QCD on the lattice

} Dynamical Calculation

Observables: VEV of operators, such as Green's functons.

$$\langle O \rangle = \int [DU] O(U) e^{-S_g(U) + Tr \ln M[U]} \Rightarrow \frac{1}{N} \sum_i O_i$$

Monte Carlo simulation, importance sampling


Lattice QCD
Experiments
Dynamical configurations
Expensive
Facilities
Probe1
Probe2
………
Fairly expensive
Detectors
Probes: valence quark propagators, etc.,
Data analysis
Manpower intensive
Data analysis

Re-organization of LQCD Community

- **Large LQCD Collabortions generating dynamical configurations**

MILC: Symanzik improved gauge
(2+1) flavor staggered fermion.
CP-PACS: RG improved gauge
(2+1) flavor clover fermion
JLQCD: RG improved gauge
(2+1) flavor overlap fermion
RBC&UKQCD: DBW2 gauge
(2+1) flavor domain wall fermion
ETMC: improved gauge
(2+1) flavor twisted-mass fermion

- **Smaller groups for physical projects based on these dynamical configurations**

Large scale numerical computation on supercomputers

Large international LQCD collaborations

III. Meson mass decomposition

X. Ji, Phys. Rev. Lett. 74, 1071 (1995)

Let' start from the QCD energy-momentum density tensor,

$$
\begin{aligned}
T^{\mu\nu} &= \frac{1}{2}\bar{\psi} i \gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi + \frac{1}{2}\mathrm{Tr}(g^{\mu\nu}F^2 - F^{\mu\alpha}F^{\nu}{}_{\alpha}) \\
T^{\mu\nu} &= T^{\nu\mu}, \qquad \partial T^{\mu\nu} = 0.
\end{aligned}
$$

Due to the renormalization, it is found that the tensor has a trace part,

$$
\hat{T}^{\mu\nu} = \frac{1}{4} g^{\mu\nu} \left[\left(1 + \gamma_m \right) \bar{\psi}\psi + \frac{\beta(g)}{2g} F^2 \right) \right]
$$

The traceless part can be decomposed as,

$$
\bar{T}^{\mu\nu} = \bar{T}_q^{\mu\nu} + \bar{T}_g^{\mu\nu};
$$

$$
\begin{aligned}
\bar{T}_q^{\mu\nu} &= \frac{1}{2}\bar{\psi} i \gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \psi - \frac{1}{4} g^{\mu\nu} \bar{\psi} m \psi \\
\bar{T}_g^{\mu\nu} &= \frac{1}{4} g^{\mu\nu} F^2 - F^{\mu\alpha} F_{\alpha}^{\nu}.
\end{aligned}
$$

For a meson state |p> with the normalization,

$$\langle P|P\rangle = (2\pi)^3 2E\delta^3(0)$$

one has,

$$M = \frac{\langle P|\int d^3\vec{x}\, T^{00}(0,\vec{x})|P\rangle}{\langle P|P\rangle} \equiv \langle T^{00}\rangle;$$

$$\langle P|T^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

$$\langle P|\bar{T}^{\mu\nu}|P\rangle = 2P^\mu P^\nu - \frac{1}{2}g^{\mu\nu}M^2,$$

$$\langle P|\hat{T}^{\mu\nu}|P\rangle = \frac{1}{2}g^{\mu\nu}M^2.$$

such that,

$$\langle \bar{T}^{00}\rangle = \frac{3}{4}M,$$

$$\langle \hat{T}^{00}\rangle = \frac{1}{4}M.$$

The above expressions are general and independent of the scale. If we decompose further,

$$\langle \bar{T}_q^{00} \rangle = \frac{3}{4} M a(\mu^2),$$
$$\langle \bar{T}_g^{00} \rangle = \frac{3}{4} M (1 - a(\mu^2)).$$

Now we define,

$$\hat{T}_m^{00} = m\bar{\psi}\psi \qquad \langle \hat{T}_m^{00} \rangle \equiv Mb$$

Rearrange the QCD Hamiltonian using the equation of motion,

$$(i\gamma_\mu D^\mu - m)\psi = 0.$$

$$H_{\rm QCD} \equiv \int d^3\vec{x} T_{\rm QCD}^{00}(0,\vec{x}) = H_E + H_g + H_a = H_q + H_m + H_g + H_a$$

$$H_E = \int d^3\vec{x}\ \bar{\psi}(-i\mathbf{D_t}\gamma_4)\psi \equiv H_q + H_m,$$

$$H_q = \int d^3\vec{x}\ \bar{\psi}(-i\mathbf{D}\cdot\gamma)\psi, \qquad H_m = \int d^3\vec{x}\bar{\psi}m\psi$$

$$H_g = \int d^3\vec{x}\ \tfrac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2), \qquad H_a = \int d^3\vec{x}\ \frac{9\alpha_s}{16\pi}(\mathbf{E}^2 - \mathbf{B}^2)$$

IV. Numerical details

$$H_{\rm QCD} \equiv \int d^3\vec{x} T^{00}_{\rm QCD}(0,\vec{x}) = H_E + H_g + H_a = H_q + H_m + H_g + H_a$$

$$H_E = \int d^3\vec{x}\, \bar{\psi}(-i\mathbf{D_t}\gamma_4)\psi \equiv H_q + H_m,$$

$$H_q = \int d^3\vec{x}\, \bar{\psi}(-i\mathbf{D}\cdot\gamma)\psi, \qquad H_m = \int d^3\vec{x}\bar{\psi}m\psi$$

$$H_g = \int d^3\vec{x}\, \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2), \qquad H_a = \int d^3\vec{x}\, \frac{9\alpha_s}{16\pi}(\mathbf{E}^2 - \mathbf{B}^2)$$

- The key task is to calculated the matrix elements of these operators between hadron states
- One has to check the equation of motion of quarks in a hadron.
- The quark bilinears in the operators above should be renormalized.

1. Lattice setup

Lattice formulation:

overlap fermions as valence quarks;
2+1-flavor domain-wall fermion configurations
(generated by RBC/UKQCD Collaboration)

$L^3 \times T$	r_0/a(GeV)	$m^{res}a$	$m^l_{sea}a$	$m^s_{sea}a$
$24^3 \times 64$	4.126(11)	0.00315	0.005	0.04

r_0(fm)	a^{-1}(GeV)	Z_m	Z_A	N_{conf}
0.458(11)	1.77(5)	0.884(7)	1.110(1)	77

Overlap fermion operator for the valence quarks

$$D_c = \frac{\rho D_{ov}}{1 - D_{ov}/2} \text{ with } D_{ov} = 1 + \gamma_5 \epsilon(\gamma_5 D_w(\rho))$$

Matching the continuum form: $D_c = (1 + O(a^2)) \sum_\mu \gamma^\mu D_\mu$.

Quite a lot quark masses can be calculated simultaneously.

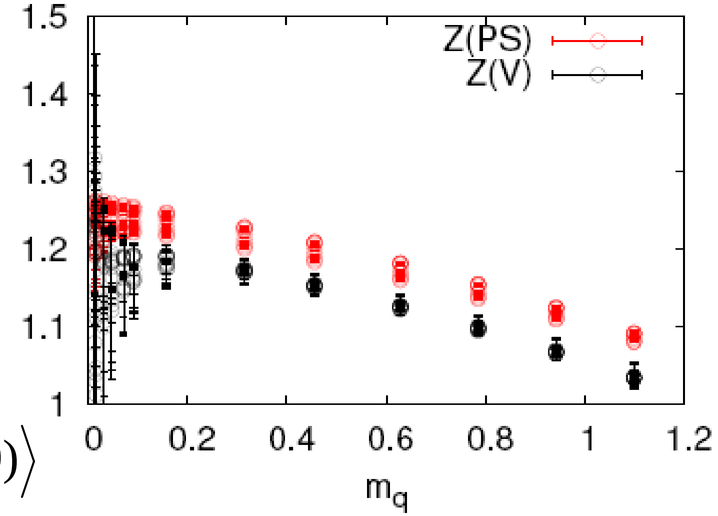
2. Test of the equation of motion of quarks in a hadron

The classical equation of motion of quarks in the presence of a background color field,

$$(i\gamma_\mu D^\mu - m)\psi = 0.$$

For a given field $\{U_\mu(x)\}$

This can be tested by



$$\langle O_H(x)[\overline{\psi}(y)(D_c+m)(x,y)\psi(z)]O_H^+(0)\rangle$$

$$= \frac{1}{Z[U_\mu]}\int D\psi D\overline{\psi} e^{-\overline{\psi}(D_c+m)\psi} O_H(x)[\overline{\psi}(y)(D_c+m)(y,z)\psi(z)]O_H^+(0)$$

$$\propto \ldots S_F(x,y)(D_c+m)(y,z)S_F(z,0)\ldots \qquad (S_F = (D_c+m)^{-1})$$

$$\propto \ldots \langle H | \overline{\psi}(y)(D_c+m)(x,y)\psi(z) | H \rangle \ldots$$

$$S_F(x,y)(D_c+m)(y,z) = \delta^4(x-y) \Longrightarrow \langle H | \overline{\psi}(y)(D_c+m)(x,y)\psi(z) | H \rangle = 0$$

$$D_c = (1+O(a^2))\gamma_\mu D_\mu \equiv Z^{-1}\gamma_\mu D_\mu$$

$$\begin{aligned} Z_E.H_E &= Z_E.H_q + H_m, \\ Z_E &= \frac{\langle H_m \rangle}{\langle H_E - H_q \rangle}, \end{aligned}$$

3. Non-perturbative renormalization of quark bilinears

The renormalization constants of quark bilinears are being calculated using a non-perturbative scheme (RI/MOM).

- The renormalization condition in the RI-MOM scheme is [G. Martinelli et al., Nucl. Phys. B **445** (1995) 81]

$$\lim_{m_q\to 0} Z_q^{-1} Z_{\mathcal{O}} \frac{1}{12} \mathrm{Tr}[\Lambda_{\mathcal{O}}(p) \Lambda_{\mathcal{O}}^{tree}(p)^{-1}]_{p^2=\mu^2} = 1,$$

  Z_q is the quark field renormalization constant: $\psi_R = Z_q^{1/2}\psi$,
  $Z_{\mathcal{O}}$ is the renormalization constant for the operator $\mathcal{O}$: $\mathcal{O}_R = Z_{\mathcal{O}}\mathcal{O}$,
  μ is the renormalization scale.

- $\Lambda_{\mathcal{O}}(p)$ is the amputated forward Green function

$$\Lambda_{\mathcal{O}}(p) = S^{-1}(p) G_{\mathcal{O}}(p) S^{-1}(p),$$

  where $S(p)$ is the quark propagator.

- The calculation has to be done in a fixed gauge, say, Landau gauge. The method is supposed to work in the window

$$\Lambda_{QCD} \ll \mu \ll \pi/a.$$

- In the RI scheme,

$$Z_q^{RI}(\mu) = \frac{-i}{48}\mathrm{Tr}\left[\gamma_\nu \frac{\partial S^{-1}(p)}{\partial p_\nu}\right]_{p^2=\mu^2}.$$

- In the RI' scheme, Z_q is given by

$$Z_q^{RI'}(\mu) = \frac{1}{12}\mathrm{Tr}\left[S^{-1}(p) S_f^{ov}(p)\right]_{p^2=\mu^2},$$

  where $S_f^{ov}(p)$ is the free overlap quark propagator.
- We obtain the renormalization constant of the local axial vector current Z_A^{WI} from Ward Identities, which equals to Z_A^{RI} in the RI scheme. Then

$$Z_q^{RI} = Z_A^{WI}\frac{1}{12}\mathrm{Tr}\left[\Lambda_A(p)\Lambda_A^{tree}(p)^{-1}\right]_{p^2=\mu^2}.$$

- Using

$$Z_A \partial_\mu A_\mu = 2 Z_m m_q Z_P P$$

and $Z_m = Z_P^{-1}$ for overlap fermions, one has

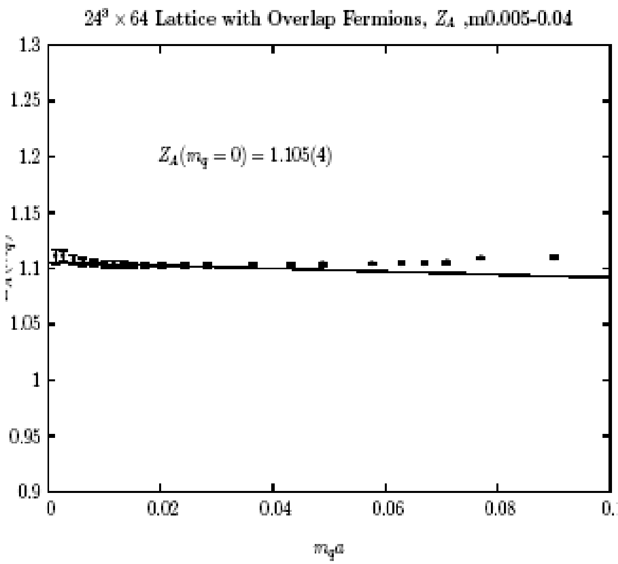
$$Z_A \partial_\mu \langle 0|A_\mu|\pi\rangle = 2 m_q \langle 0|P|\pi\rangle.$$

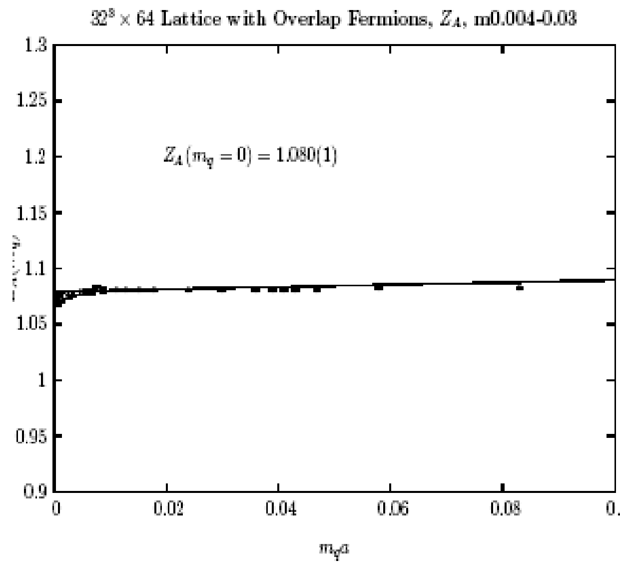
- If the pion is at rest, then from the above one gets

$$Z_A = \frac{2 m_q \langle 0|P|\pi\rangle}{m_\pi \langle 0|A_4|\pi\rangle}.$$

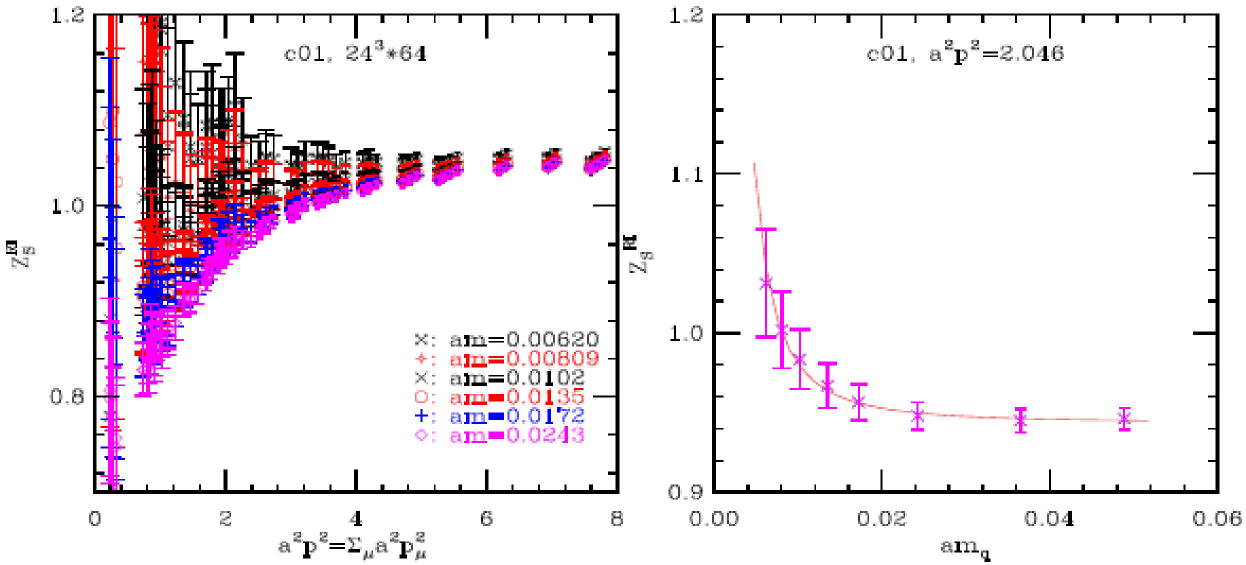
- From 2-point functions $G_{PP}(\vec{p}=0,t) = \sum_{\vec{x}} \langle 0|P(x)P(0)|0\rangle$ and $G_{A_4P}(\vec{p}=0,t) = \sum_{\vec{x}} \langle 0|A_4(x)P(0)|0\rangle$, one obtains

$$Z_A = \lim_{m_q \to 0, t \to \infty} \frac{2 m_q G_{PP}(\vec{p}=0,t)}{m_\pi G_{A_4P}(\vec{p}=0,t)}.$$





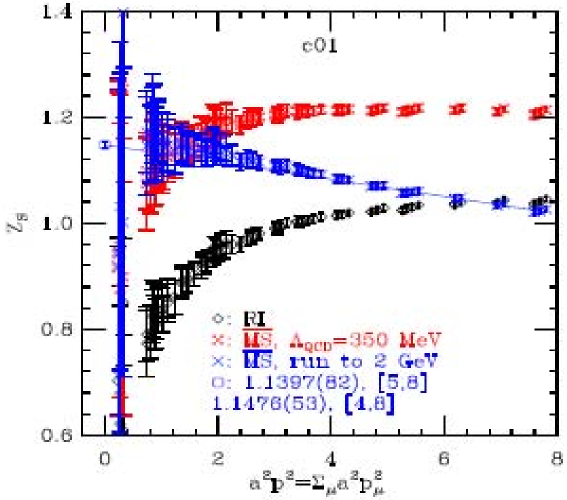
- A straight line fit in $am_q \in [0.00809, 0.02430]$ on the coarse lattice.
- $am_q \in [0.00585, 0.01520]$ on the fine lattice.



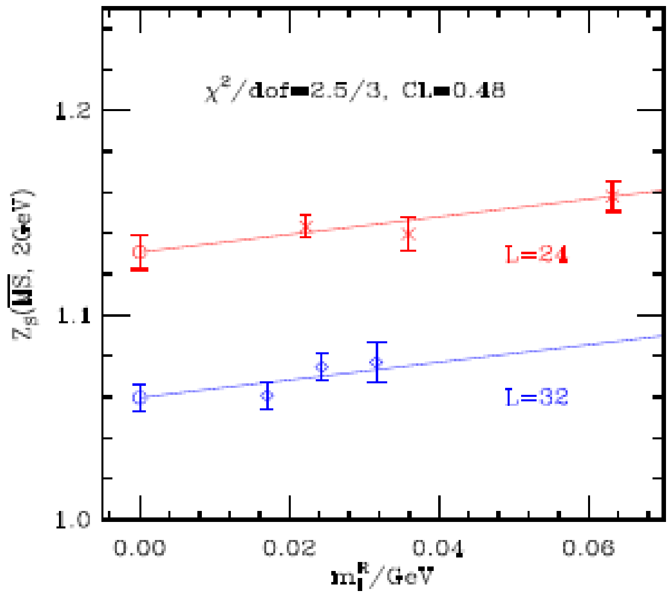
- To go to the chiral limit, we use the following to fit and take B_s:

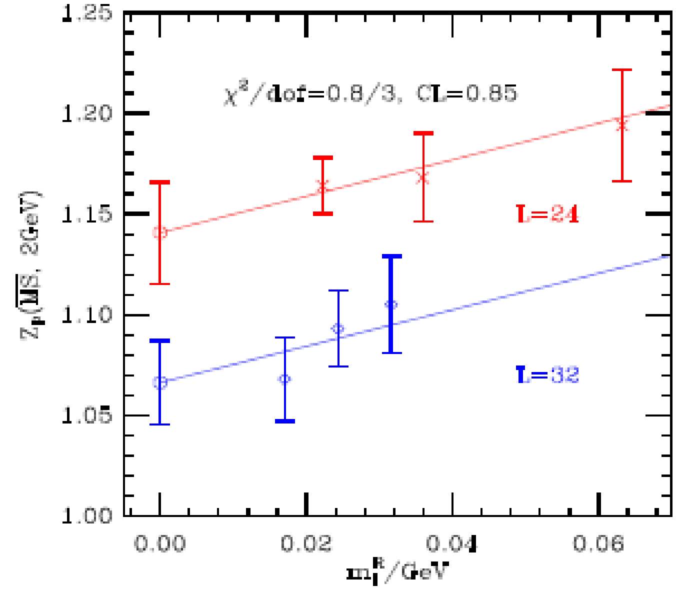
$$Z_S = \frac{A_s}{(am_q)^2} + B_s + C_s \cdot (am_q)$$

[Blum et al. 2001, Aoki et al. 2007]



- The conversion ratio from RI to $\overline{\rm MS}$ scheme is to 3-loops. [Chetyrkin and Retey 1999]
- The running of $Z_S^{\overline{\rm MS}}$ uses anomalous dimension to 4-loops.
- $Z_S = A + B \cdot a^2 p^2$ to extrapolate away $\mathcal{O}(a^2 p^2)$ discretization errors.





$$Z(m_l^R) = Z(0) + c \cdot m_l^R, \quad \text{where } m_l^R = (m_l + m_{res})Z_m^{sea}$$

ensemble	c005	c01	c02	$m_l = 0$
$Z_S^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})$	1.1433(54)	1.1397(82)	1.1581(74)	1.1308(87)
$Z_P^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})$	1.164(14)	1.168(22)	1.194(28)	1.141(25)
ensemble	f004	f006	f008	$m_l = 0$
$Z_S^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})$	1.0607(66)	1.0747(64)	1.077(10)	1.0597(64)
$Z_P^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV})$	1.068(21)	1.093(19)	1.105(24)	1.066(21)

To summarize

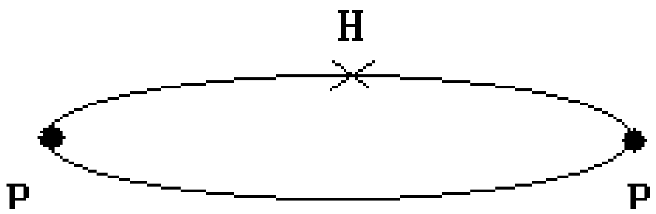
- Using the RI-MOM scheme, we calculate the renormalization constants for overlap quark bilinears on domain wall configurations.
- $Z_S = Z_P$ and $Z_V = Z_A$ are confirmed as expected.
- The conversion from the RI scheme to the $\overline{\mathrm{MS}}$ scheme is the main source of error for Z_S.
- $Z_m = 1/Z_S$ is used for the determination of strange and charm quark masses.
- We find $m_s^{\overline{\mathrm{MS}}}$(2 GeV)=0.104(9) GeV and $m_c^{\overline{\mathrm{MS}}}$(2 GeV)=1.107(38)

4. The calculation and results

$$H_E = \int d^3\vec{x}\, \bar{\psi}(-i\mathbf{D_t}\gamma_4)\psi \equiv H_q + H_m,$$

$$H_q = \int d^3\vec{x}\, \bar{\psi}(-i\mathbf{D}\cdot\gamma)\psi, \qquad H_m = \int d^3\vec{x}\bar{\psi}m\psi$$

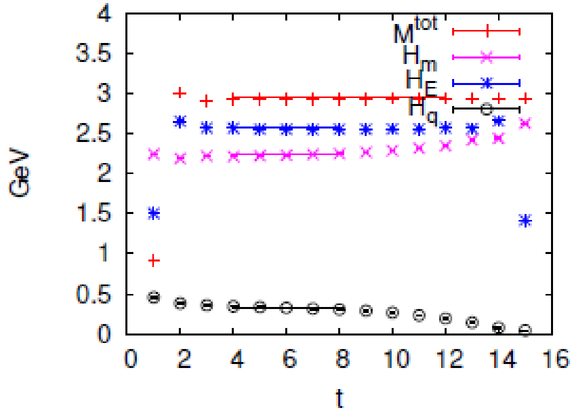
$$H_g = \int d^3\vec{x}\, \tfrac{1}{2}(\mathbf{E}^2+\mathbf{B}^2), \qquad H_a = \int d^3\vec{x}\, \frac{9\alpha_s}{16\pi}(\mathbf{E}^2-\mathbf{B}^2)$$



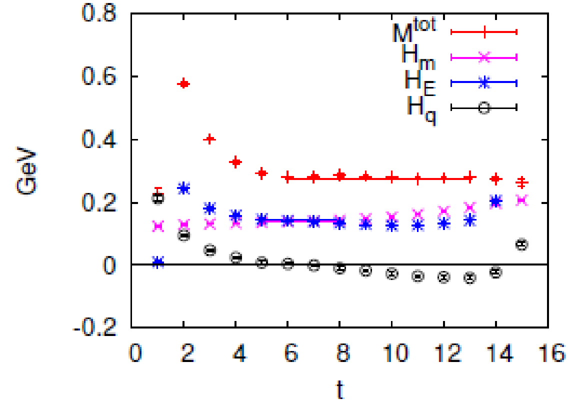
$$C^{all}(t,t',0) = \sum_{i,j} \frac{\langle 0|O_I|M^{(i)}\rangle\langle 0|O_F|M^{(j)}\rangle}{4m^{(i)}m^{(j)}}\langle M^{(j)}|T^{00}|M^{(i)}\rangle e^{-t'm^{(i)}-(t-t')m^{(j)}}$$

$$= \sum_{i} \frac{\langle 0|O_I|M^{(i)}\rangle\langle 0|O_F|M^{(i)}\rangle}{4(m^{(i)})^2}\langle M^{(i)}|T^{00}|M^{(i)}\rangle e^{-t\; m^{(i)}}$$

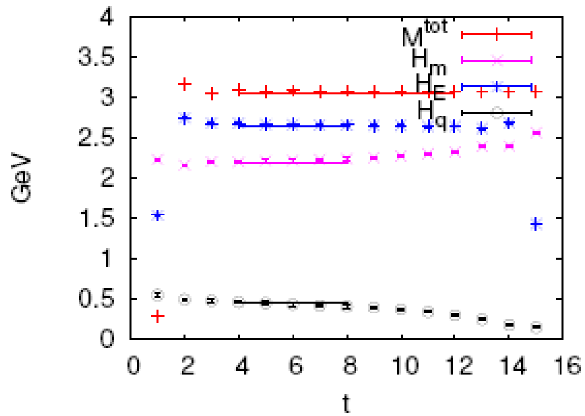
Equal mass case



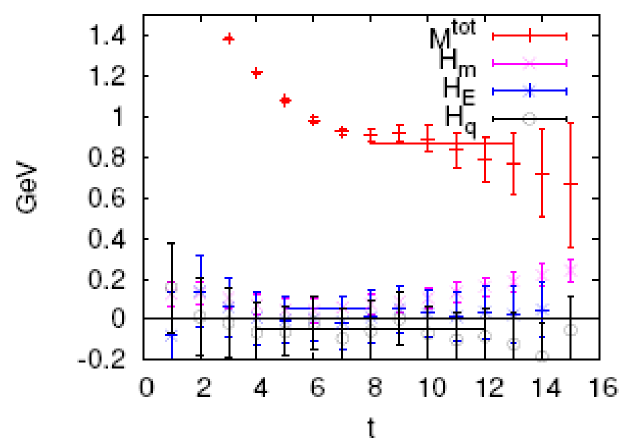
(a)η_c



(b)π

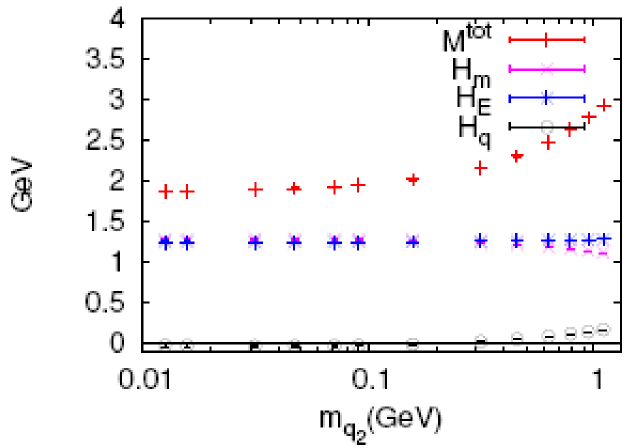


(b) J/ψ

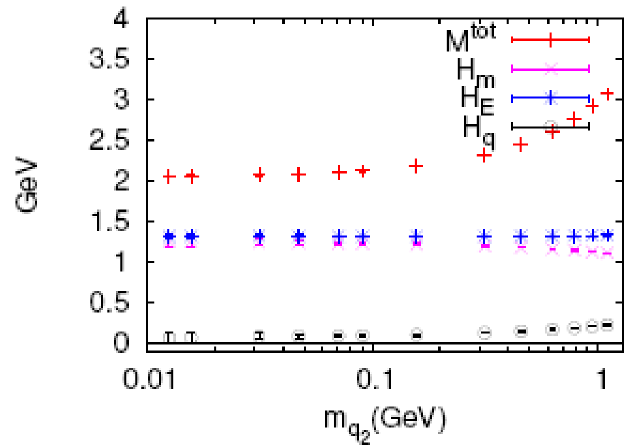


(d) ρ

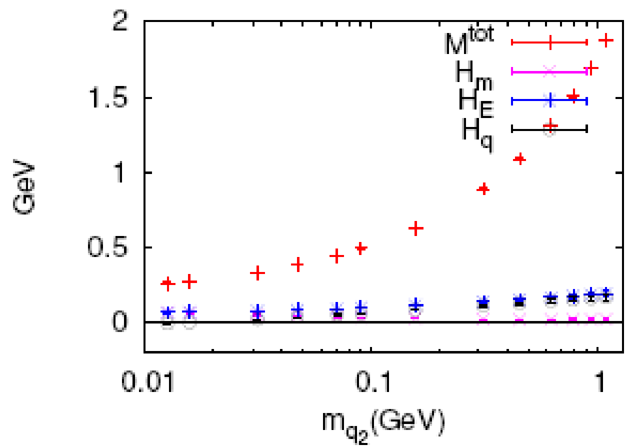
Un-equal mass case

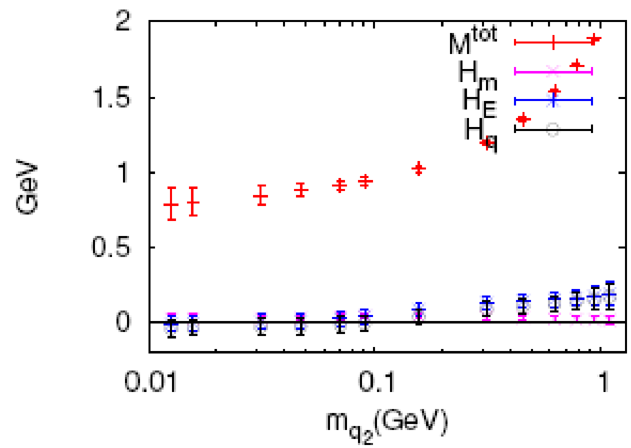


(a) heavy q_1(PS)



(b) heavy q_1(V)

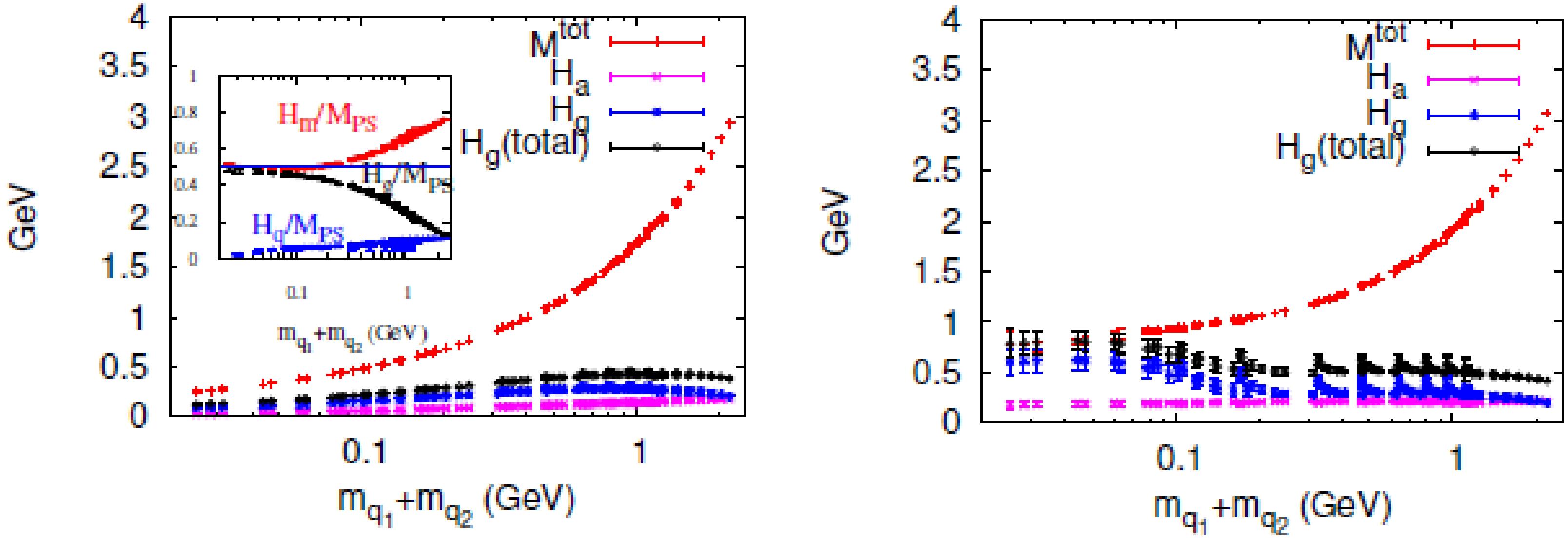




The correlation of the motion of the two quarks in a meson

The gluonic contribution to meson masses

Mass type	E_i	Mass fraction
Quark energy	E_q	$3(a-b)/4$
Quark mass	E_m	b
Quark total	$E_{q-tot.}$	$(3a+b)/4$
Gluon energy	E_g	$3(1-a)/4$
Trace anomaly	E_a	$(1-b)/4$

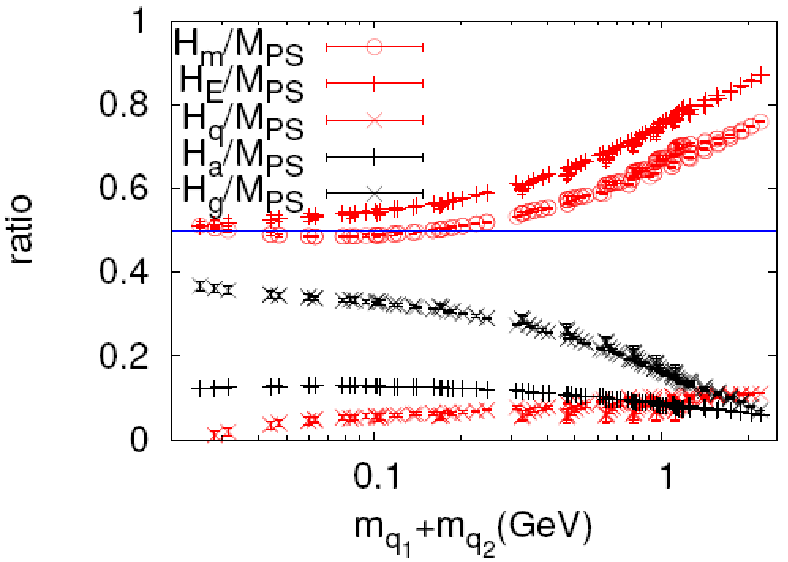


The pseudoscalar mass decomposition in the chiral limit

Theoretical analysis predicts, in the chiral limit (X.Ji, PRD52, 271(1995)),

$$H_q = 0; \qquad H_m = \frac{1}{2} m_{PS};$$

$$H_g = \frac{3}{8} m_{PS}; \qquad H_a = \frac{1}{8} m_{PS}$$



IV. Summary

1. Hadron mass can be decomposed into the components contributed from quark masses, quark kenetic energy, gluon kenetic energy, and the QCD trace anomaly.

2. The mass components of pseudoscalar and vector mesons are investigated in the framework of 2+1 full QCD lattice study.

3. In the chiral limit, the lattice result is compatible with that from theoretical analysis. Both quark components and gluon components tend to zeor when the chiral limit is approaching.

4. In constrast to the pseudoscalar, in the chiral limit, the vector meson mass is contributed predominantly from the gluon conponents.

5. For S-wave mesons made up of quarks heavier than strange quark, the total gluon component is roughly 400-500 MeV and insensitive to the quark masses.

Appendix: Direct calculation of gluon components from quenched LQCD

β	a_s(fm)	$L^3 \times T$	$Z_{H_{q-tot.}}$	Z_{H_g}	Z_{H_a}
2.4	0.222	$8^3 \times 96$	1.288(5)	1.0(2)	0.53(15)
2.8	0.138	$12^3 \times 144$	1.155(3)	–	–

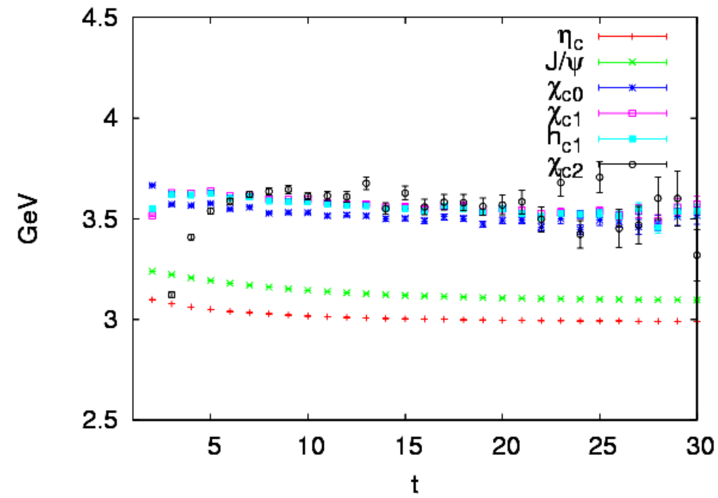
Operators defined on the lattice should be renormalized even though their continuum counterparts are scale independent.

$$O_{cont} = a^{-4} Z_O(a) O.$$

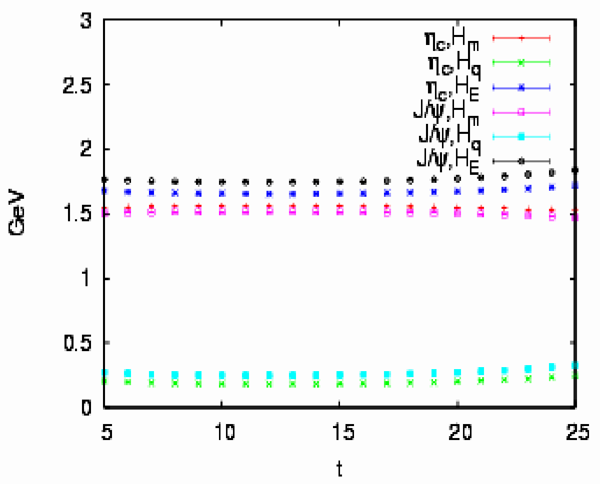
For gluonic operators, we use the glueball states to do the non-perturbative renormalization
(Y.Chen et al, Phys. Rev. D 73, 014516 (2006))

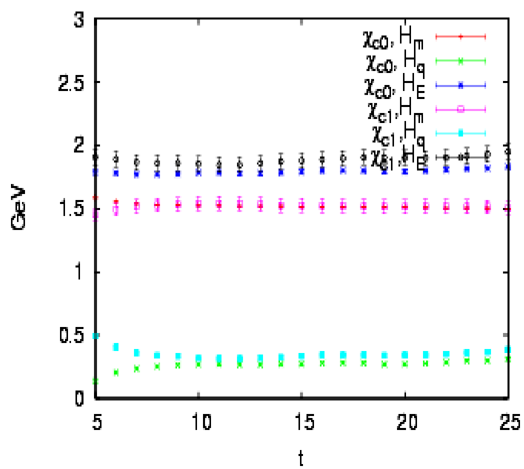
$\|G\rangle$	$\langle G\|O_+\|G\rangle_{\rm lat}$	Z_S	$\langle G\|O_-\|G\rangle_{\rm lat}$	Z_T
S	32(9)	1.1(3)	13(5)	0.7(3)
$T(E)$	102(16)	1.0(2)	51(15)	0.53(15)
$T(T_2)$	101(16)	1.0(2)	53(15)	0.51(15)

- **Effective mass plateaus of 1S, 1P charmonium states.**

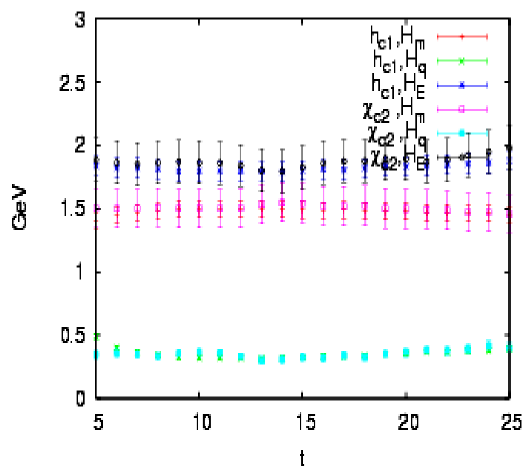


- **The quark component of the masses of 1S, 1P states.**

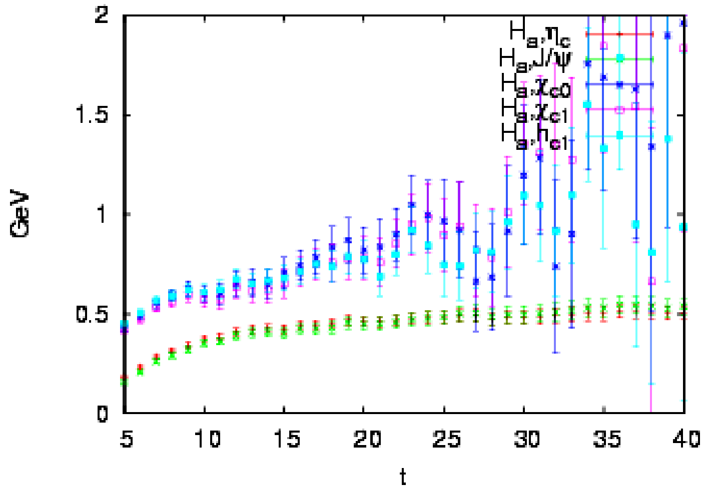




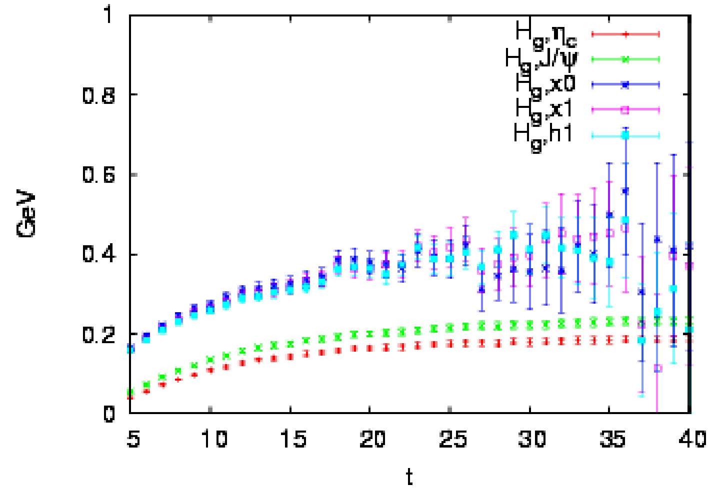
(a)



(b)



(a) H_g



(b) H_a

Some information we can get:

1. The 1S and 1P charmonium mass contributed by quarks (including the quark mass and quark kinematic energy) are around 2 GeV. This implies that the gluon component of charmonium mass is substantially large.
2. The 1P-1S mass splittings are mainly due to the different contributions of gluons.

D. The final results

- **The charmonium mass components contributed by quarks before the renormalization.**

	E_m	E_q	$E_{\mathrm{q-tot.}}$
η_c	1.554(4)	0.1836(4)	1.654(4)
J/ψ	1.509(4)	0.2507(6)	1.742(5)
χ_{c0}	1.496(24)	0.260(4)	1.751(27)
χ_{c1}	1.453(30)	0.310(6)	1.776(34)
h_{c1}	1.477(33)	0.318(7)	1.789(38)
χ_{c2}	1.62(10)	0.359(17)	1.99(11)

E_m, E_q, and E_qtot are calculated independently, and the relation E_qtot=E_m+E_q is reproduced.

- The final results of 1S,1P charmonium mass components.

	$E^r_{q-\text{tot.}}$	E^r_g	E^r_a	M	PDG
η_c	2.130(4)	0.53(12)	0.18(2)	2.84(13)	2.980
J/ψ	2.244(6)	0.56(13)	0.23(3)	3.03(14)	3.097
χ_{c0}	2.255(35)	0.95(24)	0.39(5)	3.59(24)	3.415
χ_{c1}	2.287(44)	0.90(20)	0.39(7)	3.57(20)	3.511
h_{c1}	2.304(49)	0.82(17)	0.37(4)	3.50(17)	3.525
χ_{c2}	2.56(14)	0.59(23)	0.33(6)	3.48(30)	3.556

It is impressive that the physical mass are reasonably established by their separate components.
Obviously, the gluonic contribution is very large.
The 1P-1S mass splittings come mainly from the gluonic contribution.

??? The connection to the valence charm quark mass

$$m_c^{\overline{MS}}(3GeV) = 0.987GeV(lattice)$$

$$m_c(m_c) = 1.27(11)GeV(1.273(6), lattice)$$

Thank You!