BPS M-Branes in $AdS_4 {*} Q^{1,1,1}$

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Based on work in progress with

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Outline

- Backgrounds and motivations
- BPS M2-branes dual to loop operators
- M5-branes
- Further directions

Backgrounds and Motivations

String theory

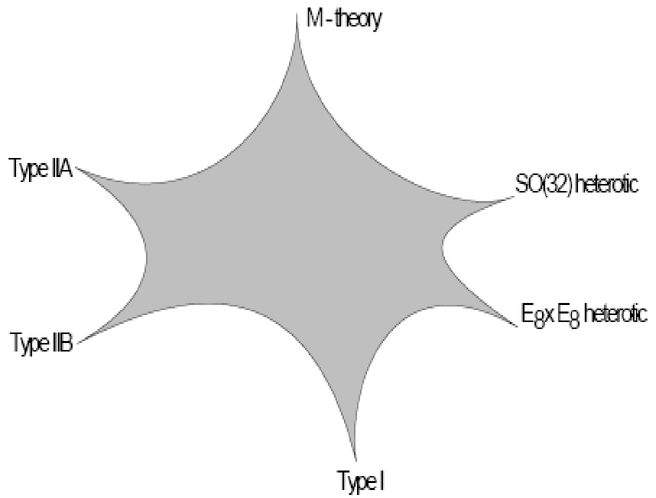
- Till 1985, five superstring theories in 10d were constructed:
- Type IIA, type IIB, type I, Heterotic SO(32), Heterotic E_8*E_8

- Their low energy effective theories are 10d SUGRA
- Type IIA, type IIB, type I with gauge group SO(32) or E_8*E_8
- 11d SUGRA was known at that time. It is also know that its compactifiaction on S^1 (keeping massless modes only) will give

  10d IIA SUGRA.
- Its relation to string theory was not know till …

M-theory

- In 1995, *Witten* and *Townsend* (independently) proposed that the strong coupling limit of IIA string theory is 11d M-theory whose LEET is 11d SUGRA.
- M-theory was proposed as a quantum gravity and related to string theory through non-perturbative dualities.
- There are two kind of branes in M-theory: M2-branes and M5-branes



M2-branes

- The LEET on multi-D2 branes in IIA string theory is (2+1)d N=8 SYM theory with gauge group SU(N). Due to RG running, this theory will run to a strongly coupling fixed point in the IR.
- The CFT on this fixed point is the LEET of multi-M2 branes.
- No other field theory descriptions were known at that time.
- This theory is dual to M-theory on $AdS_4 * S^7$
- The computations in the dual gravity description showed that (at strong coupling)the degree of freedom of this CFT scales as $N^{3/2}$.

ABJM theory

- *Aharony-Bergman-Jafferis-Maldacena* theory is a three-dimensional Chern-Simons-matter theory with $\mathcal{N}$=6 supersymmetries. The gauge group is U(N)*U(N) with Chern-Simons levels k and –k.
- They argued that this theory gives the LEET of N M2-branes at the tip of C^4/Z_k orbifold.
- The 1/6 BPS Wilson loops were constructed and studied

[Chen, JW, 08]

[Drukker, Plefka, Young]

[Rey, Suyama, Yamaguchi]

ABJM theory

Supersymmetric localization was used to compute the partition function and VEV of Wilson loops. The computation was reduced to computation in a matrix model *[Kapustin, Willett, Yaakov, 09]*

- This matrix model is solved and the $N^{3/2}$ behavior was obtained at strong coupling. *[Drukker, Marino, Putrov, 10]*

- More generally, there is correspondence between 3d Chern-Simons-matter theories and M-theory on $AdS_4 * M^7$.

Some applications

- The studies on 3d QFT may help us to understand some condensed matter systems. *[Fujita, Li, Takayanagi 09]* and other papers on FQHE.
- Applications on holographic superconductor.

- The study on M-theory on AdS_4 background may be a first step to study dS_4 in string/M theory using gauge/gravity duality. *[Polchinski, Silverstein 09]*

$Q^{1,1,1}$

- The metric on $AdS_4 * Q^{1,1,1}$ is

$$
\begin{aligned}
ds^2 &= R^2(ds_4^2 + ds_7^2), \\
ds_4^2 &= \frac{1}{4}(\cosh^2 u(-\cosh^2 \rho dt^2 + d\rho^2) + du^2 + \sinh^2 u d\phi^2), \\
ds_7^2 &= \sum_{i=1}^{3} \frac{1}{8}(d\theta_i^2 + \sin^2\theta_i d\phi_i^2) + \frac{1}{16}(d\psi + \sum_{i=1}^{3} \cos\theta_i d\phi_i)^2,
\end{aligned}
$$

- $Q^{1,1,1}$ is a seven-dimensional Sasaki-Einstein manifold. The metric cone of it is a Calabi-Yau 4-fold.

$$ds_8^2 = dr^2 + r^2 ds_7^2$$

Fluxes

- Four-form field strength:

$$H_4 = \frac{3R^3}{8} \cosh^2 u \sinh u \cosh \rho dt \wedge d\rho \wedge du \wedge d\phi.$$

- Flux quantization gives

$$\begin{aligned} R &= 2\pi l_p \left(\frac{N}{6vol(Q^{1,1,1}/Z_k)} \right)^{1/6} \\ &= l_p \left(\frac{2^8 \pi^2 k N}{3} \right)^{1/6} , \end{aligned}$$

Orbifolds

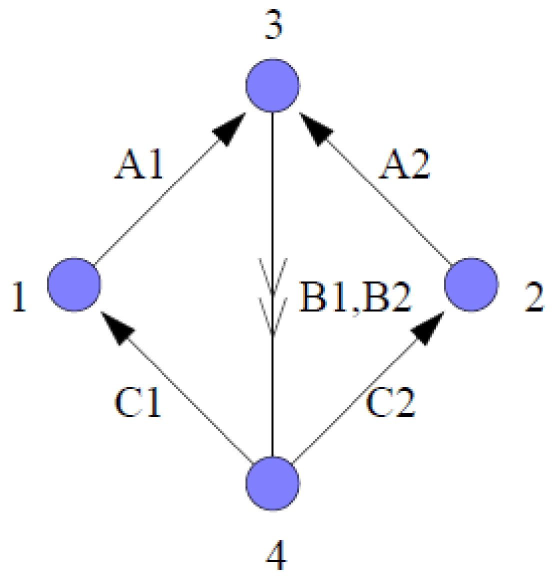
- Two kinds of orbifolds were considered

$$Q^{1,1,1}/Z_k: \quad (\phi_1, \phi_2) \sim (\phi_1 + \frac{2\pi}{k}, \phi_2 + \frac{2\pi}{k})$$
$$Q^{1,1,1}/Z'_k: \quad \phi_1 \sim \phi_1 + \frac{2\pi}{k}$$

- Various dual field theories were proposed for M-theory on AdS4*$Q^{1,1,1}/\mathbf{Z}_k$ or $Q^{1,1,1}/\mathbf{Z}_k$'. There are all 3d Chern-Simons-matter theories with 3d *N*=2 supersymmetries.

Dual field theory for $\mathbf{Z}_k$ orbifold

- *[Franco etal, 08]*



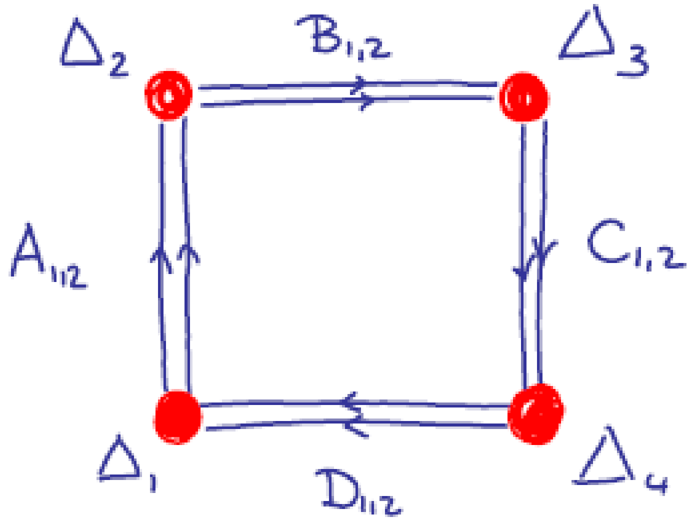
$$W = C_1A_1B_1C_2A_2B_2 - C_1A_1B_2C_2A_2B_1 \ .$$

$$S_{CS} = \int d^3x \sum_{i=1}^{4} \frac{k_i}{4\pi} \epsilon^{\mu\nu\rho} Tr(A_{i\mu}\partial_\nu A_{i\rho} + \frac{2i}{3} A_{i\mu}A_{i\nu}A_{i\rho})$$

- Chern-Simons levels are (k, k, -k, -k)

Dual field theory for for Z_k' orbifold

- *[Aganagic 09]*
- Chern-Simons levels (0, k, 0, -k).



$$W = \epsilon^{ik}\epsilon^{jl}\,\mathrm{Tr}\,A_i B_j C_k D_l.$$

BPS M2-branes

JW, Meng-Qi Zhu,

1312.3030[hep-th]

Loop operators and M2-branes

- In *N*=2 Chern-Simons-matter theories, people constructed ½-BPS Wilson loops.
- The Wilson loops in fundamental representations is to dual to certain BPS M2-branes.
- There are elegant general discussions in *[Farquet, Sparks]* used results from differential geometry.
- There are also vortex loops in these theories which dual to another class of M2-branes. (ABJM case, *[Drukker, Gomis, Young]*)
- We start with the computations of Killing spinors.

KSE

- The Killing spinor equation is

$$\nabla_{\underline{m}}\eta + \frac{1}{576}(3\Gamma_{\underline{npqr}}\Gamma_{\underline{m}} - \Gamma_{\underline{m}}\Gamma_{\underline{npqr}})H^{\underline{npqr}}\eta = 0.$$

$$\nabla_{\underline{m}}\eta = e^{\mu}_{\underline{m}}\partial_{\underline{\mu}}\eta + \frac{1}{4}\omega^{\underline{ab}}_{\underline{m}}\Gamma_{\underline{ab}}\eta$$

- The spin connection is obtained from the Cartan structure equation

$$de^{\underline{m}} + \omega^{\underline{m}}_{\underline{n}} \wedge e^{\underline{n}} = 0$$

The metric revisited

$$
\begin{aligned}
ds^2 &= R^2(ds_4^2 + ds_7^2), \\
ds_4^2 &= \frac{1}{4}(\cosh^2 u(-\cosh^2 \rho dt^2 + d\rho^2) + du^2 + \sinh^2 u d\phi^2), \\
ds_7^2 &= \sum_{i=1}^{3} \frac{1}{8}(d\theta_i^2 + \sin^2 \theta_i d\phi_i^2) + \frac{1}{16}(d\psi + \sum_{i=1}^{3} \cos \theta_i d\phi_i)^2,
\end{aligned}
$$

$$
ds^2 = g_{\mu\nu} dx^\mu dx^\nu = \eta_{ab} e_\mu^a e_\nu^b dx^\mu dx^\nu = \eta_{ab} e^a e^b.
$$

1-forms

- We introduce three set of 1-forms

$$\begin{aligned} \sigma_I^1 &= d\theta_I, \\ \sigma_I^2 &= \sin\theta_I d\phi_I, \\ \sigma_I^3 &= \cos\theta_I d\phi_I, \end{aligned} \qquad \begin{aligned} w^1 &= -\cos\psi\sin\theta_3 d\phi_3 + \sin\psi d\theta_3, \\ w^2 &= \sin\psi\sin\theta_3 d\phi_3 + \cos\psi d\theta_3, \\ w^3 &= d\psi + \cos\theta_3 d\phi_3, \end{aligned}$$

with I=1, 2

$$\begin{aligned} d\sigma_I^i + \frac{1}{2}\epsilon^{ijk}\sigma_I^j \wedge \sigma_I^k &= 0, \\ dw^i + \frac{1}{2}\epsilon^{ijk} w^j \wedge w^k &= 0. \end{aligned}$$

Vielbeins

$$ds_7^2 = \frac{1}{8}[\sum_{I=1}^{2}((\sigma_I^1)^2 + (\sigma_I^2)^2) + (w^1)^2 + (w^2)^2] + \frac{1}{16}(\sigma_1^3 + \sigma_2^3 + w^3)^2.$$

$$\begin{aligned}
e^{\underline{0}} &= \frac{R}{2}\cosh u \cosh \rho dt,\ e^{\underline{1}} = \frac{R}{2}\cosh u d\rho, \\
e^{\underline{2}} &= \frac{R}{2} du,\ e^{\underline{3}} = \frac{R}{2}\sinh u d\phi, \\
e^{\underline{4}} &= \frac{R}{2\sqrt{2}}\sigma_1^1,\ e^{\underline{5}} = \frac{R}{2\sqrt{2}}\sigma_1^2, \\
e^{\underline{6}} &= \frac{R}{2\sqrt{2}}\sigma_2^1,\ e^{\underline{7}} = \frac{R}{2\sqrt{2}}\sigma_2^2, \\
e^{\underline{8}} &= \frac{R}{2\sqrt{2}} w^1,\ e^{\underline{9}} = \frac{R}{2\sqrt{2}} w^2, \\
e^{\underline{\sharp}} &= \frac{R}{4}(\sigma_1^3 + \sigma_2^3 + w^3).
\end{aligned}$$

Killing spinor equation

- Now the solution to the Killing spinor equation

$$\nabla_{\underline{m}}\eta + \frac{1}{576}(3\Gamma_{\underline{npqr}}\Gamma_{\underline{m}} - \Gamma_{\underline{m}}\Gamma_{\underline{npqr}})H^{\underline{npqr}}\eta = 0.$$

is

$$\eta = e^{\frac{u}{2}\Gamma_{\underline{2}}\hat{\Gamma}} e^{\frac{\rho}{2}\Gamma_{\underline{1}}\hat{\Gamma}} e^{\frac{t}{2}\Gamma_{\underline{0}}\hat{\Gamma}} e^{\frac{\phi}{2}\Gamma_{\underline{23}}}\eta_0, \qquad \hat{\Gamma} = \Gamma_{\underline{0123}}.$$

$$\Gamma^{\underline{45}}\eta_0 = \Gamma^{\underline{67}}\eta_0 = \Gamma^{\underline{89}}\eta_0,$$

η_0 is independent of the coordinates. (For KSE in $T^{1,1}$, *[Arean etal, 04]*.)

Killing spinors of the orbifolds

- To check the supersymmetries preserved by the orbifolds, we confirm that

$$\mathcal{L}_{K_i}\eta \equiv (K_i)^{\underline{m}}\nabla_{\underline{m}}\eta + \frac{1}{4}(\nabla_{\underline{m}}(K_i)_{\underline{n}})\Gamma^{\underline{mn}}\eta.$$

vanishes, with $K_i \equiv \frac{\partial}{\partial \phi_i}$

$$\nabla_{\underline{m}}\eta = e^{\mu}_{\underline{m}}\partial_{\underline{\mu}}\eta + \frac{1}{4}\omega^{\underline{ab}}_{\underline{m}}\Gamma_{\underline{ab}}\eta$$

- It is not enough to show that η is independent of \phi_i.

Probe M2-branes

- Bosonic part of M2 action

$$S_{M2} = T_2 \left(\int d^3\xi \sqrt{-\det g_{\mu\nu}} - \int P[C_3] \right),$$

- Equations of motion

$$\begin{aligned} & \frac{1}{\sqrt{-g}} \partial_m \left(\sqrt{-g} g^{mn} \partial_n X^{\underline{N}} \right) G_{\underline{MN}} + g^{mn} \partial_m X^{\underline{N}} \partial_n X^{\underline{P}} \Gamma^{\underline{Q}}_{\underline{NP}} G_{\underline{QM}} \\ = \quad & \frac{1}{3!} \epsilon^{mnp} (P[H_4])_{\underline{M}mnp}. \end{aligned}$$

BPS condition

$$\Gamma_{M2}\eta = \eta,$$

with

$$\Gamma_{M2} = \frac{1}{\sqrt{-\det g_{\mu\nu}}}\partial_\tau X^{\mu_1}\partial_\xi X^{\mu_2}\partial_\sigma X^{\mu_3} e^{\underline{m}_1}_{\mu_1} e^{\underline{m}_2}_{\mu_2} e^{\underline{m}_3}_{\mu_3} \Gamma_{\underline{m}_1\underline{m}_2\underline{m}_3},$$

M2 branes – the first class

- Ansatz (AdS_2 in AdS_4, S^1 in M_7):

$$t = \tau, \rho = \xi, \psi = \psi(\sigma), \phi_i = \phi_i(\sigma), i = 1, 2, 3$$

- Equations of motion give

$$u = 0,$$

$$\sin\theta_i \phi_i' (\psi' + \sum_{i=1}^{3} \cos\theta_j \phi_j' - 2\cos\theta_i \phi_i') = 0.$$

$$\psi = m_\psi \sigma, \ \phi_i = m_i \sigma.$$

BPS M2-branes

- BPS conditions give that for each *i,*

$$\sin\theta_i = 0,$$

- *or*

$$\phi_i' = 0.$$

$$\Gamma_{\underline{01}\underline{\natural}}\eta_0 = \pm\eta_0,$$

M2-branes dual to Wilson loops

- For M2-branes in AdS4*$Q^{1,1,1}/Z_k$ dual to Wilson loops in fundamental representation

$$m_\psi = 0, m_1 = m_2 = \frac{1}{k}, m_3 = 0, (\theta_1, \theta_2) = (0,0), (0,\pi), (\pi,0), (\pi,\pi).$$

$$S_{M2} = -2\pi\sqrt{\frac{N}{3k}},$$

when $(\theta_1, \theta_2) = (0, 0)$ or (π, π);

$$S_{M2} = 0.$$

when $(\theta_1, \theta_2) = (\pi, 0)$ or $(0, \pi)$. Finally we get

$$< W > \sim \exp(2\pi\sqrt{\frac{N}{3k}})$$

M2-branes dual to Wilson loops

- For M2-branes in AdS4*$Q^{1,1,1}/Z_k$' dual to Wilson loops in fundamental representation

$$m_\psi = 0, m_1 = \frac{1}{k}, m_2 = m_3 = 0, \theta_1 = 0, \pi$$

$$S_{M2} = -\pi\sqrt{\frac{N}{3k}},$$

$$< W >\sim \exp(\pi\sqrt{\frac{N}{3k}}).$$

The second class of M2-branes

- Ansatz (AdS_2 in AdS_4, S^1 in AdS_4 *M_7)

$$t = \tau, \rho = \xi, \phi = \sigma,$$

$$\psi = \psi(\sigma), \phi_i = \phi_i(\sigma),$$

with u and θ_i being constants.

EOM for u gives

$$2\cosh u \sinh u \sqrt{\sinh^2 u + c} + \frac{\cosh^3 u \sinh u}{\sqrt{\sinh^2 u + c}} - 3 \sinh u \cosh^2 u = 0.$$

$$c = \frac{1}{2}\sum_{i=1}^{3} \sin^2 \theta_i^2 \phi_i'^2 + \frac{1}{4}(\psi' + \sum_{i=1}^{3} \cos\theta_i \phi_i')^2.$$

- This lead to c=1 or c=-3/4*cosh^{2}u+1.
- Only c=1 gives BPS solutions.
- Other EOM's give

$$\sin\theta_i \phi_i'(\psi' + \sum_{j=1}^{3} \cos\theta_j \phi_j' - 2\cos\theta_i \phi_i') = 0.$$

$$\psi = m_\psi \sigma,\ \phi_i = m_i \sigma.$$

BPS M2-branes

- BPS conditions give that for each *i,*

$$\sin\theta_i = 0,$$

- *or*

$$\phi_i' = 0.$$

$$\Gamma_{\underline{01}\underline{\natural}}\eta_0 = \pm\eta_0,$$
$$S_{M2} = -2\pi\sqrt{\frac{kN}{3}},$$

Probe M5-branes

D.-S. Li, Z.-W. Liu and JW

1ymm.nnnn[hep-th]

Ansatz 1

- Let us use the Poincare coordinates on AdS_4, then the metric is:

$$ds_4^2 = \frac{1}{4}\frac{dy^2 - dt^2 + dx_1^2 + dx_2^2}{y^2}$$

- Consider the embedding:

$$\xi^0 = t, \xi^1 = y, \xi^2 = x_1, x_2 = x_2(y),$$
$$\xi^3 = \theta_1, \xi^4 = \phi_1, \xi^5 = \psi.$$

$$h_3 = b(\xi)(1+*)\frac{R^3\sqrt{1+(\frac{dx_2}{dy})^2}dy \wedge dt \wedge dx_1}{8y^3}$$

- Similar solution in AdS_4*S^7 was considered in *[Chen, 07]*.
- We are checking if our solutions are special case of solutions in *[Yamaguchi 07]*.
- Such brane with x_2=const was considered in *[Ahn, 99]*. He did not show that the equations of motion are satisfied and he did not check whether it is BPS.
- *Ahn* discussed that such branes are dual to some domain wall, while at that time people only discussed proposed field theory dual at UV (some Yang-Mills-matter theory) not in terms of Chern-Simons-matter theory.

Ansatz 2

- We switch back to global coordinates

$$ds_4^2 = \frac{1}{4}(-\cosh^2 u dt^2 + du^2 + \sinh^2 u d\Omega_2^2)$$

- We consider the ansatz

$$\xi^0 = t,$$
$$\xi^1 = \theta_1, \xi^2 = \phi_1, \xi^3 = \theta_2, \xi^4 = \phi_2, \xi^5 = \psi.$$
$$h_3 = 0$$

with u=0.

- This is dual to a baryonic operator *[Ahn, 99] [Benishti etal 10]*.

Further directions

Further directions

- The Wilson loop in other representations

Representation	M-theory description	IIA string description
Fundamental	M2-branes	Fundamental strings
Symmetric	M2-branes	D2-branes
Anti-symmetric	Kaluza-Klein monopoles	D6-branes

- Constructions of BPS vortex loops in the field theory side.

- Similar studies in other 7d Sasaki-Einstein manifolds.

Thank you very much for your time!