# Form factors and observables for $B \longrightarrow K^* l^+ l^-$ and $B_s \longrightarrow \phi l^+ l^-$ from lattice QCD[1]

Zhaofeng Liu[2]

IHEP

Peking Univ., March 27, 2014

[1] arXiv:1310.3722, 1310.3887

[2] Collaborators: Ron R. Horgan, Stefan Meinel and Matthew Wingate

# Outline

- Motivation
- Lattice setup and method
- Form factors
- Observables for $B^0 \longrightarrow K^{*0}\mu^+\mu^-$ and $B_s^0 \longrightarrow \phi\mu^+\mu^-$
- Summary

# Motivation

- The Standard Model (SM) is quite successful. The Higgs boson was discovered at LHC in 2012.
- However physics beyond the SM is needed: neutrino oscillations, matter-antimatter asymmetry, dark matter and dark energy, etc.

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- The Standard Model (SM) is quite successful. The Higgs boson was discovered at LHC in 2012.
- However physics beyond the SM is needed: neutrino oscillations, matter-antimatter asymmetry, dark matter and dark energy, etc.
- The direct way to look for new physics is searching for new particles at even higher energies.
- The indirect way is searching for deviations from the SM by precise measurements.
- Processes suppressed in the SM are sensitive to new physics.

# Motivation

- $B \to K^* l^+ l^-$ and $B_s \to \phi l^+ l^-$ are flavor changing neutral current (FCNC) processes.
- FCNC transition $b \to s$ is suppressed in the Standard Model.

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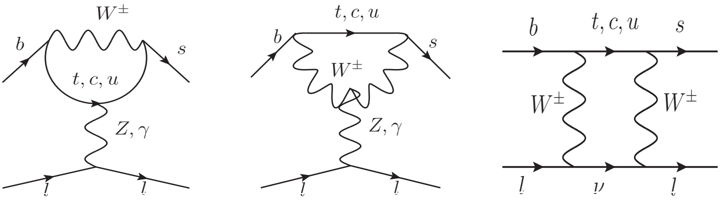
- $B \to K^* l^+ l^-$ and $B_s \to \phi l^+ l^-$ are flavor changing neutral current (FCNC) processes.
- FCNC transition $b \to s$ is suppressed in the Standard Model.
- Dominant contributions are from penguin and box diagrams ($b \to s\ ll$):




- Sensitive to supersymmetry, extra dimensions, ... (see e.g., NPB830(2010)17 and arXiv:0907.5386[hep-ph])

# Rare $B$ decays

- $B$ meson decays are studied by effective field theory.

$$m_t, m_Z, m_W \gg m_b \gg \Lambda_{\mathrm{QCD}} \gg m_u, m_d$$

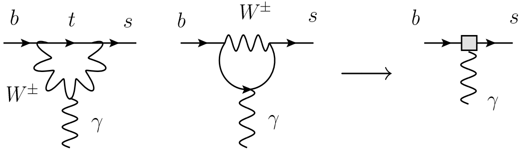
- $b \to s$ effective weak Hamiltonian in the Standard Model ($V_{ub}V_{us}^* \ll V_{tb}V_{ts}^*$ and CKM unitarity)

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}} \sum_{i=1}^{10} V_{tb}V_{ts}^* C_i(\mu) Q_i(\mu)$$

- Quarks are confined in hadrons. Non-perturbative effects of QCD.
- The hadronic matrix elements $\langle F|Q_i|B_{(s)}\rangle$ have to be computed non-perturbatively. We work on $F = K^*, \phi$ by using lattice QCD.

# Local operators in our calculation

- $Q_7 = m_b \bar{s}\sigma^{\mu\nu}(1+\gamma_5)bF_{\mu\nu}$

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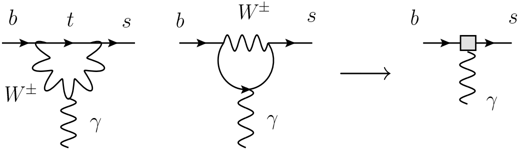
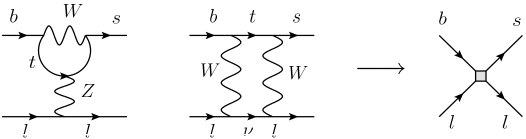



Relevant for $B \to K^*\gamma$, $B \to K^{(*)}l^+l^-$, $B_s \to \phi\gamma$, $B_s \to \phi l^+l^-$.

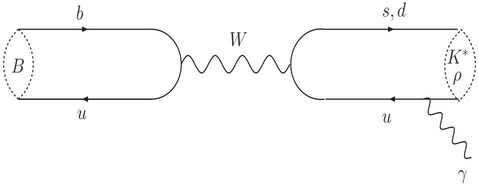
- $Q_9 = \bar{s}\gamma^\mu(1-\gamma_5)b\bar{l}\gamma_\mu l$, $\quad Q_{10} = \bar{s}\gamma^\mu(1-\gamma_5)b\bar{l}\gamma_\mu\gamma_5 l$.



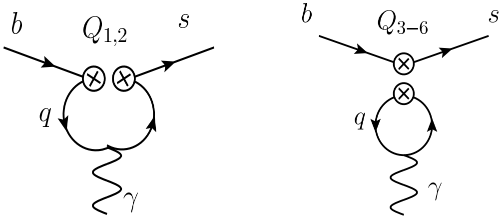

Relevant for $B \to K^{(*)}l^+l^-$, $B_s \to \phi l^+l^-$.

# Other contributions

- Weak annihilation is doubly suppressed ($V_{ub}V_{us}^* \ll V_{tb}V_{ts}^*$) for $B \to K^*$.




- Contributions from $Q_1$ and $Q_{3-6}$ are loop-suppressed.



# Other contributions

- Charmonium resonances from $Q_2 = (\bar{s}c)_{V-A}(\bar{c}b)_{V-A}$. $q^2 \sim m^2_{c\bar{c}}$ regions are avoided.

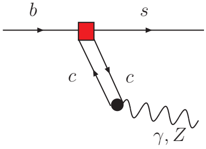



Figure: Contributions from charmonium resonances

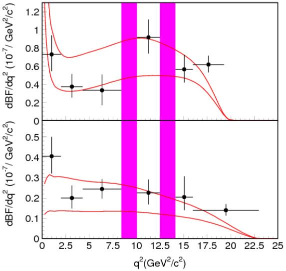



Figure: Differential branching fractions for $B \to K^* ll$ (upper plot) and $B \to Kll$ (lower plot) versus $q^2$. $J/\psi$ and $\psi(2S)$ events are rejected. PRL103,171801(2009),BELLE

- Theoretical uncertainties are mainly from hadronic matrix elements.

# Parametrization of matrix elements

$B \to K^*\gamma, \quad B_s \to \phi\gamma, \quad B \to K^* l^+ l^-$

$$q^\nu \langle K^*(p', \lambda)|\bar{s}\sigma_{\mu\nu}b|B(p)\rangle = 2T_1(q^2)\epsilon_{\mu\nu\rho\sigma}e_\lambda^{*\nu}p^\rho p'^\sigma, \quad (e_\lambda^\nu : \text{polarization}),$$

$$q^\nu \langle K^*(p', \lambda)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|B(p)\rangle = iT_2(q^2)\left[e_{\lambda\mu}^*(M_B^2 - M_{K^*}^2) - \right.$$

$$\left.(e_\lambda^* \cdot q)(p+p')_\mu\right] + iT_3(q^2)(e_\lambda^* \cdot q)\left[q_\mu - \frac{q^2}{M_B^2 - M_{K^*}^2}(p+p')_\mu\right].$$

- The physical range of $t \equiv q^2 = (p - p')^2$ is $\left[0, (M_{B_{(s)}} - M_F)^2\right]$.
- The statistical error of $T_3$ in our calculation is large. We compute

$$T_{23} = \frac{M_B + M_F}{8M_B M_F^2}[(M_B^2 + 3M_F^2 - q^2)T_2 - \frac{\lambda T_3}{M_B^2 - M_F^2}],$$

where $\lambda = (t_+ - t)(t_- - t), t_\pm = (M_B \pm M_F)^2$.

# Parametrization of matrix elements

$B \to K^* l^+ l^-$

$$\langle K^*(p',\lambda)|\bar{s}\gamma^\mu b|B(p)\rangle = \frac{2iV(q^2)}{M_B + M_{K^*}} \epsilon^{\mu\nu\rho\sigma} e^*_{\lambda\nu} p'_\rho p_\sigma,$$

$$\langle K^*(p',\lambda)|\bar{s}\gamma^\mu\gamma_5 b|B(p)\rangle = 2M_{K^*} A_0(q^2) \frac{e^*_\lambda \cdot q}{q^2} q^\mu$$
$$+(M_B + M_{K^*}) A_1(q^2) \left[e^{*\mu}_\lambda - \frac{e^*_\lambda \cdot q}{q^2} q^\mu\right]$$
$$-A_2(q^2) \frac{e^*_\lambda \cdot q}{M_B + M_{K^*}} \left[p^\mu + p'^\mu - \frac{M_B^2 - M_{K^*}^2}{q^2} q^\mu\right].$$

- $A_2$ has large statistical uncertainty. We compute

$$A_{12} = \frac{(M_B + M_F)^2 (M_B^2 - M_F^2 - q^2) A_1 - \lambda A_2}{16 M_B M_F^2 (M_B + M_F)}$$

# Form factors

- We determine the 7 linearly independent form factors

$$V, A_0, A_1, A_{12}, T_1, T_2, T_{23}$$

for $B \to K^*$, $B_s \to \phi$ and $B_s \to K^*$ ($b \to u/d$).

- We also give

$$\begin{aligned} V_\pm(q^2) &= \frac{1}{2}\left[\left(1+\frac{m_V}{m_B}\right)A_1(q^2) \mp \frac{\sqrt{\lambda}}{m_B(m_B+m_V)}V(q^2)\right] \\ T_\pm(q^2) &= \frac{1}{2m_B^2}\left[(m_B^2-m_V^2)T_2(q^2) \mp \sqrt{\lambda}T_1(q^2)\right]. \end{aligned}$$

- $A_0, A_{12}, T_{23}, V_\pm, T_\pm$ form the helicity basis.
- We give the first unquenched lattice QCD results of these form factors.

# Lattice calculations of form factors in rare $B$ decays

Dynamical simulations (2+1)

- $B \to K l^+ l^-$ form factors, HPQCD, PRD2013
- $B \to K$ vector form factors, Fermilab and MILC, Lattice2011

Quenched simulations

- D. Becirevic, V. Lubicz and F. Mescia, Nucl. Phys. B **769**, 31 (2007)
- L. Del Debbio, J. M. Flynn, L. Lellouch and J. Nieves [UKQCD Collaboration], Phys. Lett. B **416**, 392 (1998)
- A. Abada *et al.* [APE Collaboration], Phys. Lett. B **365**, 275 (1996)
- T. Bhattacharya and R. Gupta, Nucl. Phys. Proc. Suppl. **42**, 935 (1995)
- K. C. Bowler *et al.* [UKQCD Collaboration], Phys. Rev. Lett. **72**, 1398 (1994)
- C. W. Bernard, P. Hsieh and A. Soni, Phys. Rev. Lett. **72**, 1402 (1994)

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- Using non-relativistic QCD(NRQCD) for the $b$ quark and Staggered fermions for light quarks.
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- The pion masses are 313.4(1), 519.2(1), 344.3(1) MeV respectively.

| label | $1/a$(GeV) | $am_{sea}$ | Volume | $N_{conf} \times N_{src}$ | $am_{val}$ |
|---|---|---|---|---|---|
| c007 | 1.660(12) | 0.007/0.05 | $20^3 \times 64$ | $2109 \times 8$ | 0.007/0.04 |
| c02 | 1.665(12) | 0.02/0.05 | $20^3 \times 64$ | $2052 \times 8$ | 0.02/0.04 |
| f0062 | 2.330(17) | 0.0062/0.031 | $28^3 \times 96$ | $1910 \times 8$ | 0.0062/0.031 |

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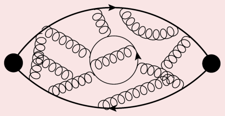
- Matchings to QCD for the vector and tensor (at the scale $\mu = m_b$) currents were calculated in Müller, Hart and Horgan, PRD83(2011) 034501 [arXiv:1011.1215 [hep-lat]].

# 2-point functions

- Meson 2-point correlators:



$$C_{FF}(x_t, \vec{p}') = \sum_{\vec{x}} \langle \Omega | \Phi_F(x) \Phi_F^\dagger(0) | \Omega \rangle e^{-i\vec{p}' \cdot \vec{x}}, \quad F = K, K^*, \text{ or } \phi.$$

Using $\sum_{n,\vec{k}} \frac{1}{2E_{\vec{k}} V_3} |n, \vec{k}\rangle \langle n, \vec{k}| = 1$, ($V_3 = L^3$), we find

$$C_{FF}(x_t, \vec{p}') = \frac{1}{2E_{\vec{p}'} V_3} |\langle \Omega | \Phi_F | \vec{p}' \rangle|^2 e^{-E_{\vec{p}'} x_t} + \text{ (excited state contributions).}$$

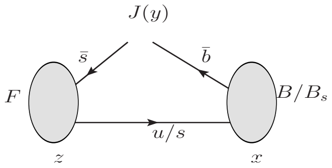
- Similarly,

$$C_{BB}(x_t, \vec{p}) = \sum_{\vec{x}} \langle \Omega | \Phi_B(x) \Phi_B^\dagger(0) | \Omega \rangle e^{-i\vec{p} \cdot \vec{x}}$$

can give us $\langle \Omega | \Phi_B | B(B_s) \rangle$ and $E_{B_{(s)}}$ when $x_t$ is big ($x_t \gg 0$).

# 3-point correlators

- 3-point correlators



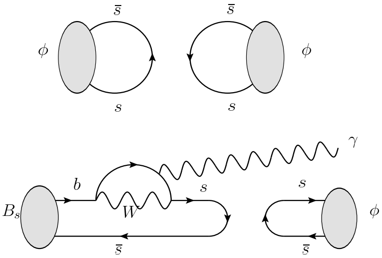

$$C_{FJB}(\vec{p},\vec{p}',T,t)=\sum_{\vec{x}}\sum_{\vec{y}}\langle\Omega|\Phi_B(\vec{x},T)J(\vec{y},t)\Phi_F^\dagger(0)|\Omega\rangle e^{-i\vec{p}\cdot\vec{x}}e^{i\vec{q}\cdot\vec{y}},$$

$q=p-p'$, $\quad q^2_{max}=(M_B-M_F)^2$ when both $B$ and $F$ are at rest.

- By using the completeness relation twice, one sees that $C_{FJB}$ can give us $\langle B(p)|J(q)|F(p')\rangle$ at $0\ll t\ll T$ once we know $\langle\Omega|\Phi_{B(F)}|B(F)\rangle$ and $E_{B(F)}$ from the 2-point correlators.

# Some details

- For $B_s \to \phi$, disconnected diagrams are ignored (time consuming, OZI suppressed).




- As the 3-momentum of mesons increases, the 2/3-point functions become noisier in LQCD calculations.
- Thus we work at high-$q^2$ region: $q^2 \sim q^2_{max} = (M_B - M_F)^2$.

# $q^2$ dependence of form factors

- To get form factors at low $q^2$, we need extrapolations.
- Dispersion relations relate form factors to resonances $R$ and multiparticle states above the threshold at $t_+ = (M_B + M_F)^2$:

$$F(q^2) = \sum_R \frac{\mathrm{Res}_{q^2=M_R^2} F(q^2)}{M_R^2 - q^2} + \frac{1}{\pi}\int_{t_+}^{\infty} dt \frac{\mathrm{Im}F(t)}{t - q^2 - i\epsilon}.$$

- The poles between $t_- = (M_B - M_F)^2$ and $t_+$ can be fixed by experiment measurements, e.g., $M^2_{B_s^*}$ for $T_1$ of $B \to K^*$.
- The poles above $t_+$ from higher resonances and multiparticle states can be modeled by an effective pole.

# Extrapolation to low $q^2$

- Or they can be described by a Series Expansion of a variable $z$ ($z$-expansion or SE).
- Remember: our calculation is at unphysical pion masses.
- Also, there are discretization errors.
- We use the simplified series expansion, modified to account for lattice spacing and quark mass dependence. [Bourrely et al.(2008), Na et al.(2010)]
- Define $z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_\pm = (M_B \pm M_F)^2$.
- $z(q^2 = t_0, t_0) = 0$. $t_0$ is chosen such that the physical region ($0 \leq q^2 \leq t_-$) is around $z = 0$.

# Extrapolations

- The form factors $V, A_0, A_1, A_{12}, T_1, T_2, T_{23}$ are fitted to:

$$F(t) = \frac{1}{P(t;\Delta m)}[1 + b_1(aE_F)^2 + \ldots]\sum_n a_n d_n z^n.$$

- The pole factor is given as

$$P(t;\Delta m) = 1 - \frac{t}{(m_{B_{(s)}} + \Delta m)^2}.$$

- Quark mass dependence is taken into account by the $d_n$ terms

$$d_n = [1 + c_{n1}\Delta x + c_{n2}(\Delta x)^2 + \ldots]$$

with $\Delta x = (m_\pi^2 - m_{\pi,\mathrm{phys}}^2)/(4\pi f_\pi)^2$ acting as a proxy for the difference away from physical $u/d$ quark mass.

# Extrapolations

- Varying the $\Delta m$ values by 20% has no effect on the final results for the form factor curves.
- We find the lattice spacing dependence to be negligible and the quark mass dependence to be very mild, often negligible.
- Therefore we use a 4 parameter fit

$$F(t) \;=\; \frac{1}{P(t)}[a_0(1 + c_{01}\Delta x + c_{01}^s \Delta x_s) + a_1 z]\,.$$

- $T_1(q^2 = 0) = T_2(q^2 = 0)$ is used as a constraint.

# Systematic uncertainties

- Unphysical $b$ quark mass: our $B$ and $B_s$ masses are 5% too heavy.
- In the $m_B \to \infty$ limit the form factors scale like [Isgur & Wise 1990]

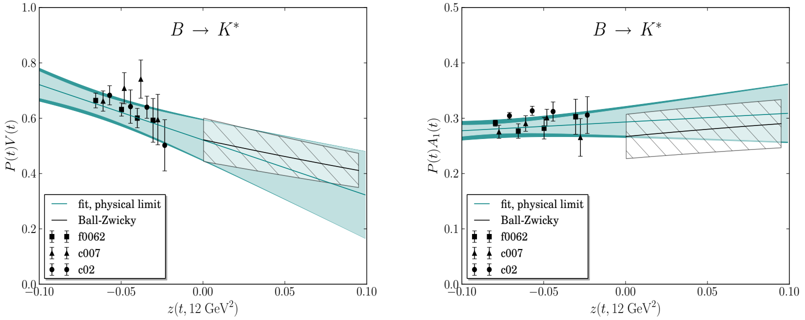
$$V, A_0, T_1, T_{23} \propto m_B^{1/2}$$
$$A_1, A_{12}, T_2 \propto m_B^{-1/2}.$$

  Scaling the central values by 0.976 ($V$ *et al.*) and 1.025 ($A_1$ *et al.*).
- The remaining error is suppressed by a factor of $\Lambda_{\mathrm{QCD}}/m_b$: well below 1% and is treated as negligible.

# Systematic uncertainties

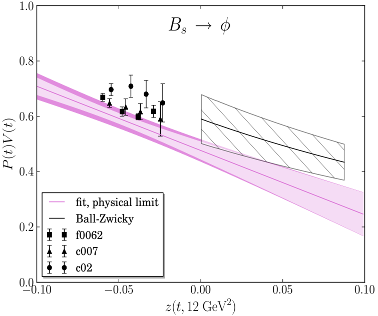
- Matching factors of currents are calculated by 1-loop lattice perturbation theory.
- The truncation of $O(\alpha_s^2)$ terms in the perturbative matching of operators from lattice NRQCD to the continuum gives the largest uncertainty: 4%.
- $O(\alpha_s \Lambda_{\mathrm{QCD}}/m_b)$ terms in the heavy quark expansion: 2%.
- $O(\Lambda_{\mathrm{QCD}}^2/m_b^2)$ terms in the heavy quark expansion: 1%.
- Adding all systematic uncertainties in quadrature: 5%.

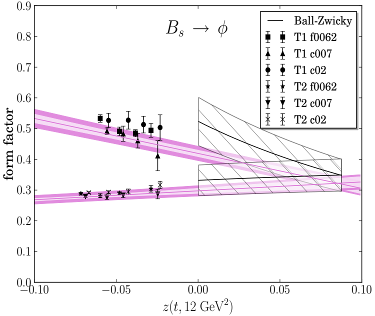
# $B \longrightarrow K^*$ form factors $P(t)V(t)$ and $P(t)A_1(t)$ against $z$




- For comparison, the LCSR results are shown with a 15% uncertainty (hatched band) [Ball & Zwicky 2004].

# $B_s \to \phi$ form factors $P(t)V(t)$ and $P(t)T_{1,2}(t)$ against $z$



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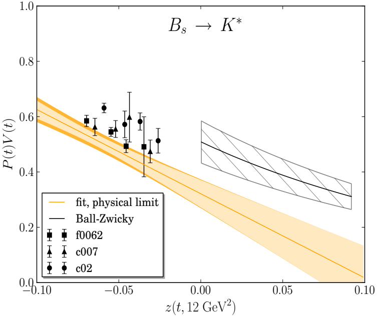
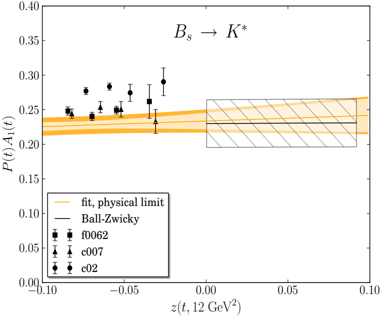



- The correlation matrices of the fit parameters are given in arXiv:1310.3722.

# $B^0 \to K^{*0}\mu^+\mu^-$ and $B_s^0 \to \phi\,\mu^+\mu^-$ observables

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \left[\, C_i O_i + C_i' O_i' \,\right],$$

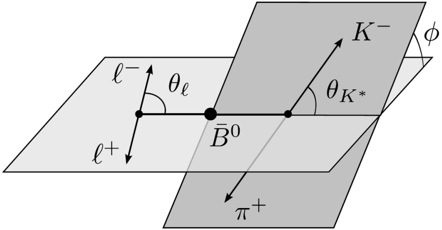
where $O_i^{(\prime)}$ are local operators and $C_i^{(\prime)}$ are the corresponding Wilson coefficients, encoding the physics at the electroweak energy scale and beyond. The operators ($P_{R,L} = (1 \pm \gamma_5)/2$)

$$\begin{aligned} O_7^{(\prime)} &= e\, m_b/(16\pi^2)\ \bar{s}\sigma_{\mu\nu} P_{R(L)} b\ \ F^{\mu\nu}, \\ O_9^{(\prime)} &= e^2/(16\pi^2)\ \bar{s}\gamma_\mu P_{L(R)} b\ \ \bar{\ell}\gamma^\mu \ell, \\ O_{10}^{(\prime)} &= e^2/(16\pi^2)\ \bar{s}\gamma_\mu P_{L(R)} b\ \ \bar{\ell}\gamma^\mu \gamma_5 \ell, \end{aligned}$$

give the leading contributions to the decays $B^0 \to K^{*0}\mu^+\mu^-$ and $B_s^0 \to \phi\,\mu^+\mu^-$.

# Observables

In the narrow-width approximation [Krüger et al. 1999, Kim et al. 2000], $\bar{B}^0 \to \bar{K}^{*0}(\to K^-\pi^+)\ell^+\ell^-$ is described by four variables: the invariant mass of the lepton pair, $q^2$, three angles $\theta_\ell$, $\theta_{K^*}$, $\phi$, defined as in [Altmannshofer et al. 2008].



# Observables

- The decay distribution is

$$\frac{\mathrm{d}^4\Gamma}{\mathrm{d}q^2\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_{K^*}\,\mathrm{d}\phi} = \frac{9}{32\pi}\Big[\ I_1^s \sin^2\theta_{K^*} + I_1^c \cos^2\theta_{K^*}$$
$$+ (I_2^s \sin^2\theta_{K^*} + I_2^c \cos^2\theta_{K^*})\cos 2\theta_\ell + \ I_3 \sin^2\theta_{K^*} \sin^2\theta_\ell \cos 2\phi$$
$$+ I_4 \sin 2\theta_{K^*} \sin 2\theta_\ell \cos\phi + \ I_5 \sin 2\theta_{K^*} \sin\theta_\ell \cos\phi$$
$$+ (I_6^s \sin^2\theta_{K^*} + I_6^c \cos^2\theta_{K^*})\cos\theta_\ell + \ I_7 \sin 2\theta_{K^*} \sin\theta_\ell \sin\phi$$
$$+ I_8 \sin 2\theta_{K^*} \sin 2\theta_\ell \sin\phi + \ I_9 \sin^2\theta_{K^*} \sin^2\theta_\ell \sin 2\phi\Big], \qquad (1)$$

  where the coefficients $I_i^{(a)}$ depend only on $q^2$.

- Integrating over the angles, one obtains

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{3}{4}(2I_1^s + I_1^c) - \frac{1}{4}(2I_2^s + I_2^c).$$

# Observables

- The angular distribution of the CP-conjugated mode $B^0 \to K^{*0}(\to K^+\pi^-)\ell^+\ell^-$ is obtained from Eq. (1) by

$$I^{(a)}_{1,2,3,4,7} \to \bar{I}^{(a)}_{1,2,3,4,7}, \qquad I^{(a)}_{5,6,8,9} \to -\bar{I}^{(a)}_{5,6,8,9}.$$

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- Normalized CP averages and CP asymmetries of the angular coefficients are defined as

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$$F_L = -S_2^c, \quad A_{FB} = (-3/8)(2S_6^s + S_6^c).$$

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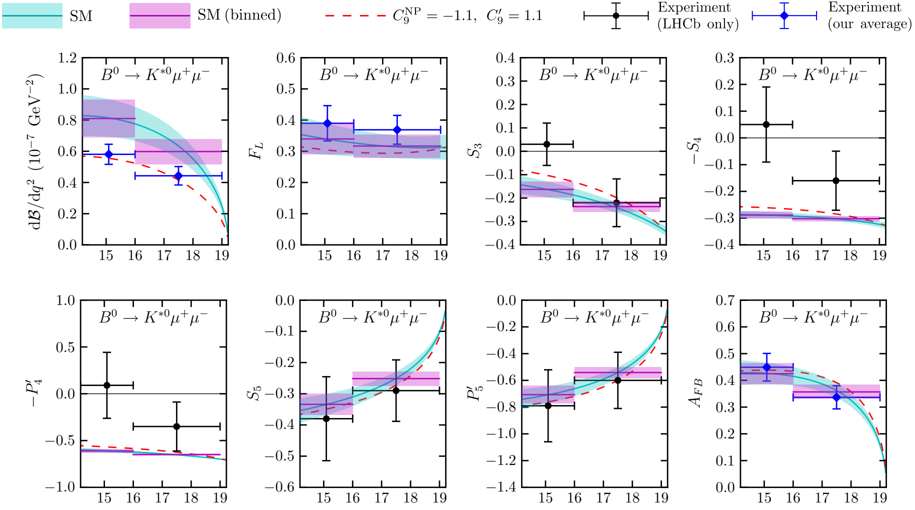
- Experiments give results for binned observables $\langle S^{(a)}_i\rangle$ and $\langle A^{(a)}_i\rangle$, $q^2$-integrals of numerator and denominator.
- $\langle S_{4,5,7,8}\rangle$ and $\langle P'_{4,5,6,8}\rangle = \frac{\langle S_{4,5,7,8}\rangle}{2\sqrt{-\langle S^c_2\rangle\langle S^s_2\rangle}}$ have been measured for the first time by the LHCb Collaboration ($B \to K^*$, 1308.1707).

# Theory versus experiment ($q^2 > 14.18$ GeV$^2$)



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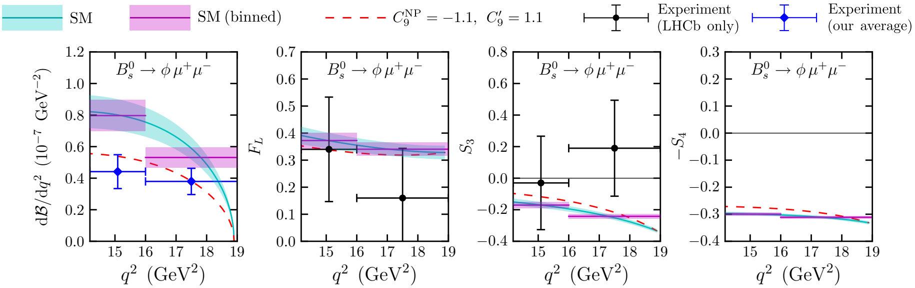


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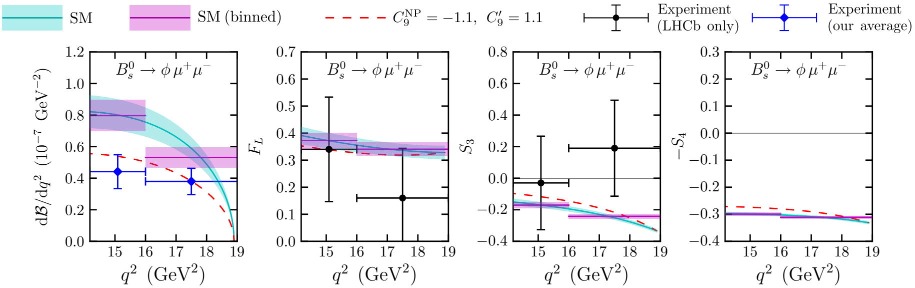


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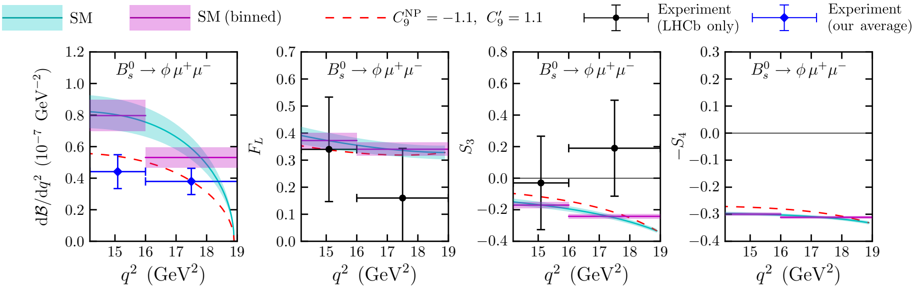


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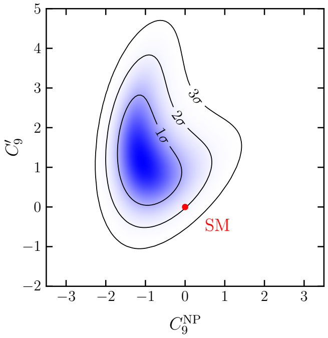
- Our results for $F_L$, $S_5$, $P_5'$, and $A_{FB}$ are in agreement with experiment.
- For the $B^0 \to K^{*0}\mu^+\mu^-$ observables $S_3$, $S_4$, and $P_4'$, we see deviations between the LHCb data and our results in the lower bin.

# Theory versus experiment ($q^2 > 14.18$ GeV$^2$)

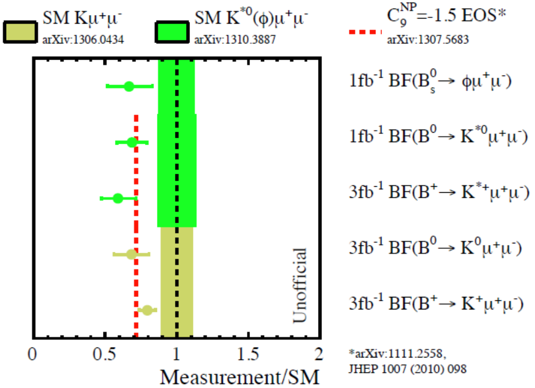
- To study the possibility of new physics in $C_9$ and $C_9'$, we fit these two parameters to the experimental data ($d\mathcal{B}/dq^2, F_L, S_3, S_4, S_5, A_{FB}$ for $B^0 \to K^{*0}$. $d\mathcal{B}/dq^2, F_L, S_3$ for $B_s^0 \to \phi$).
- The best-fit values are $C_9^{\rm NP} = -1.1 \pm 0.5$, $C_9' = 1.1 \pm 0.9$.




- $C_9^{\rm NP} = -1.1 \pm 0.7, \quad C_9' = 0.4 \pm 0.7 \quad$ (higher bin only).

# From Mitesh Patel@Moriond EW 2014, LHCb results

## High-$q^2$ diff. branching fractions




- **High $q^2$** branching fraction measurements are below the latest SM (lattice) predictions
- Better consistency with $C_9^{NP}=-1.5$ suggested by (**low $q^2$**) anomalous angular data

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# Summary

- We calculate all 7 form factors relevant to rare $B/B_s$ decays using 2+1 flavor lattice configurations.
- With NRQCD, we work directly at the (almost) physical $b$ quark mass.
- Our calculations are most precise in the low recoil region $q^2 \approx q^2_{max}$.
- The statistical error is the largest source of uncertainties.
- Form factors for $B_s \to K^*$ are also obtained.
- Observables for $B^0 \to K^{*0}\mu^+\mu^-$ and $B^0_s \to \phi\mu^+\mu^-$ are calculated with the above form factors and are compared with experiments.

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Thanks for your attention!

BACKUP

# Lattice setup

- Tadpole improved $\mathcal{O}(1/m_b^2, v_{rel}^4)$ moving NRQCD action. Discretisation error starts at $\mathcal{O}(\alpha_s a^2)$ (tree-level errors begin at $\mathcal{O}(a^5)$).
- The bare $b$ quark mass is determined from the physical $\Upsilon$ masses using NRQCD.

  [A. Gray *et al.*, Phys. Rev. D **72**, 094507 (2005)]
- Lüscher-Weisz gluon action. AsqTad fermion action (sea and light valence quarks).
- The local operators (currents) are expanded to $\mathcal{O}(1/m_b)$ (included).
- Operator matching factors are calculated by tadpole-improved 1-loop lattice perturbation theory.

$$J^{cont} = (1 + \alpha_s\, c_+)\, J_+^{(0)} + \alpha_s\, c_-\ J_-^{(0)}\ + \frac{1}{m_b}\, J_+^{(1)}.$$

$\mathcal{O}(\alpha_s/m_b, \alpha_s^2, 1/m_b^2)$ ignored.

# Correlators

Interpolating fields:

- Light mesons: $\Phi_F = \bar{q}\Gamma s, \quad q = u, s, \quad \Gamma = \gamma_5, \gamma_i.$
- $B/B_s$ mesons: $\Phi_B = \bar{q}\gamma_5\Psi_b, \quad q = u, s.$

3-point correlators

- $T = x_t - z_t$ is varied between 11 and 26 on the coarse lattice, 15 and 36 on the fine lattice. (About 1.3 to 3.2 fm.)
- $t = y_t - z_t = 0, 1, \cdots, T$. Fit both $t$ and $T$.

# Systematic uncertainties

- $c^s_{01}$ is estimated from a simultaneous fit which treats the $B \to K^*$ and $B_s \to K^*$ form factors as the $B_s \to \phi$ data, but with a mistuned spectator or offspring quark mass.

$$F(t) \ = \ \frac{1}{P(t)}[a_0(1 + f_{01}\Delta y + g_{01}\Delta w) + a_1 z]$$

where

$$\Delta y = \frac{1}{(4\pi f_\pi)^2}(m^2_{offspr} - m^2_{\eta_s,phys}),$$

$$\Delta w = \frac{1}{(4\pi f_\pi)^2}(m^2_{spect} - m^2_{\eta_s,phys}).$$

- $\eta_s$ is a fictional, $\bar{s}s$ pseudoscalar meson. Its “physical” mass is obtained from chiral perturbation theory and lattice data. HPQCD 2010, Sharpe and Shoresh 2000